Partial Response Signaling

So far, we have considered pulse shaping with pulses of duration $T_{\text{symbol period}}$.

$\implies$ full-response signaling

Good: Power/energy associated with transmission of $n$-th is confined to the $n$-th symbol period

$\implies$ no overlap $\implies$ no interference between symbols

Example: Half-sine pulse

Not so good: Limited pulse duration implies infinite bandwidth

Example: Half-sine pulse has 99% containment $Bw \approx 24 \cdot T$

Idea: Consider pulses that are longer than a symbol duration $T$

Objective: Truly bandlimited PSD $\implies$ partial response signaling
Example: Gaussian Pulse

\[ p(t) = \exp\left(-\frac{1}{2}\left(\frac{t}{\tau}\right)^2\right) \]

* pulse is wider than \( T \)

**GOOD**: * PSD is very narrow
  99% containment BW \( \approx 0.58T \)

**BAD**: * power/energy of \( n \)-th symbol "spills" into adjacent symbol periods

\[ \Rightarrow \text{Signal does not go through the symbols any more} \]

This is called intersymbol interference (ISI)

ISI prevents reliable recovery of symbols by receiver.

\[ \Rightarrow \text{VERY BAD} \]

We must avoid ISI!!
\[ p(t) = e^{-\frac{t^2}{2T^2}} \]
$99\% \text{ BW} \leq 0.58 \frac{1}{f}$
Recall:

Linear Modulation:

\[ s(t) = \sum_{n} b_n \cdot p(t - nT) \]

Want: \( s(mT) = b_m \) for no ISI

\[ s(mT) = \sum_{n} b_n \cdot p(mT - nT) \]

\[ = \sum_{n} b_n \cdot p((m-n) \cdot T) \]

\[ = \sum_{n} b_n \cdot p((m-n) \cdot T) = b_m \]

\[ \Rightarrow \text{we need: } p(mT) = \begin{cases} 1 & m = 0 \\ 0 & \text{else} \end{cases} \]

(for no ISI at \( t = mT \))

\[ \text{[Nyquist condition]} \]

- Pulses that satisfy this condition are called Nyquist pulses

- Equivalent frequency domain condition:

\[ \frac{1}{T} \cdot \sum_{k} P(f + \frac{k}{T}) = 1 \]

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Example: \( P(t) = \text{sinc} \left( \frac{t}{T} \right) \)

This is the minimum BW Nyquist pulse.

Only issue with sinc pulses:

- When sampling times are not perfect (time synchronization), then ISI returns.

- ISI with timing errors is in theory unbounded.

- Large errors are result of slow decay of sinc function \( \left( \frac{t}{T} \right) \).
With sample timing errors, ISI reappears.

Sinc

Transient

No ISI
There are Nyquist that trade off:

- Small increase in BW (excess BW)
- Increased robustness to timing errors

Example: Trapezoidal Pulse:

\[
BW = \frac{1+q}{T}
\]

Excess BW (over sinc pulse): \(\frac{a}{T}\)

Excess BW factor \(a\) (e.g., 50%)

Time domain: \(p(t) = \text{sinc}(\xi/T) \cdot \text{sinc}(\alpha t/T)\)

\(\xi\) guarantees no ISI

\(p(t)\) decays like \(\xi^2\) \(\Rightarrow\) better robustness

Note: when \(a=0\) \(\Rightarrow p(t) = \text{sinc}(\xi/T)\)
Example: Raised Cosine Pulse

\[ P(t) = \begin{cases} 
  \frac{1}{2} \cdot \left(1 + \cos\left(\frac{\pi t}{\frac{1}{2}}\right) \cdot \frac{\pi t}{2} \right) & \frac{1}{T} \leq \frac{1}{2} \\
 0 & \frac{1}{2} < \frac{1}{T} < \frac{1}{4} + \frac{9}{8} \\
 1 & \frac{1}{4} + \frac{9}{8} \leq \frac{1}{T} \end{cases} \]

Time domain:

\[ p(t) = \text{sinc}\left(\frac{t}{T}\right) \cdot \frac{\cos(\pi t + \pi/4)}{1 - (2at^2)^2} \]

decays like \( \frac{1}{t^3} \)

See demo_pulseshape.m experiment with