Baseband signal \( u(t) = u_c(t) + j u_s(t) \)  
complex envelope 

Passband signal \( s(t) = \text{Re} \{ u(t) e^{j \omega_c t} \} = u_c(t) \cdot \cos(2 \pi f_c t) - u_s(t) \cdot \sin(2 \pi f_c t) \)

Frequency Domain:

Passband Signal
Complex passband
\( c(t) = u(t) e^{j \omega_c t} \)

2 Applications for complex envelope:

1. Dealing with phase + frequency offsets
2. Passband signals through passband filters
Phase and Frequency Offset

\[ m_c(t) \xrightarrow{\cos(2\pi f_c t)} s(t) \xrightarrow{\cos(2\pi f_c t + \Theta(t))} \hat{m}_c(t) \]

\[ m_s(t) \xrightarrow{\sin(2\pi f_c t)} \xrightarrow{\text{LPF}} m_s(t) \]

**Transmitter**
- Transmitter and receiver oscillators differ by \( \Theta(t) \)
- Complex Envelope techniques greatly simplify analysis.

**Trick:** change to receiver's oscillator as the reference with transmitter as reference:

\[ s(t) = \text{Re}\left(u(t) \cdot e^{j2\pi f_c t}\right) \]

where \( u(t) = u_c(t) + j \cdot u_s(t) \)
In the receiver, the reference carrier is \( e^{j(2\pi ft + \Theta(t))} \) (not \( e^{j2\pi ft} \)).

\[
S(t) = \text{Re}\{u(t) \cdot e^{j2\pi ft}\} \\
= \text{Re}\{u(t) \cdot e^{-j\Theta(t)} \cdot e^{j\Theta(t)} \cdot e^{j2\pi ft}\} \\
= \text{Re}\{u(t) \cdot e^{-j\Theta(t)} \cdot e^{j(2\pi ft + \Theta(t))}\}
\]

Baseband equivalent signal in receiver's reference.

\( \Rightarrow I \) and \( Q \) signals at receiver:

\[
\hat{m}_c(t) = \text{Re}\{u_c(t) \cdot e^{-j\Theta(t)}\} \\
= u_c(t) \cdot \cos(\Theta(t)) + u_s(t) \cdot \sin(\Theta(t))
\]

\[
\hat{m}_s(t) = \text{Im}\{u(t) \cdot e^{-j\Theta(t)}\} \\
= -u_c(t) \cdot \sin(\Theta(t)) + u_s(t) \cdot \cos(\Theta(t))
\]

With phase or frequency offset, \( I \) and \( Q \) channels mix.
Examples:

- $\Theta(t) = 0$: $\hat{m}_c(t) = m_c(t)$  
  \[
  m_s(t) = m_s(t) \text{    \textit{Ideal: perfect sync}}
  \]

- $\Theta(t) = \frac{\pi}{2}$: $\hat{m}_c(t) = m_s(t)$  
  \[
  \hat{m}_s(t) = -m_c(t) \text{    \textit{90° phase error: channels flip}}
  \]

- $\Theta(t) = 2\pi ft$: 
  \[
  \hat{m}_c(t) = m_c(t) \cdot \cos(2\pi ft) + m_s(t) \cdot \sin(2\pi ft)
  \]
  \[
  \hat{m}_s(t) = m_s(t) \cdot \cos(2\pi ft) - m_c(t) \cdot \sin(2\pi ft)
  \]
  \[
  \text{Frequency Error: channels roll in and out}
  \]
Passband Filtering

Filtering of passband signals occurs at:

- **Transmitter**: Frequency shaping to meet emission mask

- **Receiver**: Channel selection; isolate signal of interest

- **Channel**: e.g. multipath

Fundamentally, it is possible to do filtering at passband:

- All LTI relationships hold:

\[ Y_p(f) = S_p(f) \cdot H_p(f) \]

\[ y_p(t) = S_p(t) \ast h_p(t) \]

**Example**: 

\[ s_p(t) = \int_{-1}^{1} f(t) \cos(2\pi 50t) \]

\[ h_p(t) = \int_{0}^{3} f(t) \sin(2\pi 50t) \]

\[ \Rightarrow y_p(t) = \int_{-\infty}^{\infty} s_p(\tau) \cdot h_p(t-\tau) d\tau = \int_{-\infty}^{\infty} \left[ \int_{-1}^{1} f(\tau) \cdot \cos(2\pi 50(\tau-t)) \right] \cdot \sin(2\pi 50t) d\tau \]

\[ \text{(derived solution)} \]
\[ y_p(f) = S_p(f) \cdot H_p(f) \]
\[ y_c(f) = \frac{1}{2} S_c(f) \cdot H_c(f) \]
\[ y_b(f) = \frac{1}{2} S_b(f) \cdot H_b(f) \]

**Passband**

**Complex Passband**

**Baseband**