Estimating Parameter of a Sinusoid from Samples
Problem Statement

- **Given:**
  - A set of samples from a sinusoidal signal, e.g., $\text{sig} = [1.6180, 1.6164, 1.6147, \ldots]$;
  - A sampling frequency $f_s$, e.g., $f_s = 44100$;

- **Objective:**
  - Determine signal’s amplitude, frequency, and phase.

\[ x(t) = A \cos(2\pi ft + \phi) \]
Amplitude Estimation

- For amplitude estimation, the overall maximum value of a samples is used.
- In MATLAB, this is accomplished via the `max` function:
  \[ \text{amp} = \max(\text{sig}); \]

The maxima of a sinusoid equal the amplitude of a sinusoid.
Phase Estimation

- The phase can be determined from the first sample.
  - First sample, \( \text{sig}(1) \), corresponds to \( x(0) \).
  - We have \( x(0) = A \cos(\phi) \).
  - MATLAB:
    \[
    \text{phi} = \acos(\text{sig}(1)/\text{amp});
    \]
    This is always positive!
- Phase \( \phi \) may be negative.
  - Occurs when signal is initially increasing.
  - MATLAB:
    \[
    \text{if sig}(2) > \text{sig}(1)
    \]
    \[
    \text{phi} = -\text{phi};
    \]
    end
Frequency Estimation

- Frequency estimation is based on the slope of a sinusoid at a zero crossing.
- First step: find first zero crossing.
  - Using MATLAB’s `find` function.
- Second step: determine the slope near the zero-crossing.
  - Compute “rise-over-run”
- Solve for frequency.

\[ \text{The slope at a zero-crossing equals: } 2A \pi f \]
Summary and Conclusions

- Presented methods for estimating amplitude, frequency, and phase from samples of a signal.
- Estimates are reasonable as they are derived directly from mathematical properties of sinusoidal signals.
- Room for Improvement:
  - Estimates should use all samples – phase and frequency estimates are based on very, few samples.
    - This will likely lead to poor results if sinusoids are not perfect (e.g., corrupted by noise).
  - Phase estimate relies on amplitude estimate – if amplitude is wrong, then phase will be wrong.