Modeling of Wireless Communication Systems using MATLAB

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Part I

The Wireless Channel

The Wireless Channel

Characterization of the wireless channel and its impact on digitally modulated signals.

- From the physics of propagation to multi-path fading channels.
- Statistical characterization of wireless channels:
  - Doppler spectrum,
  - Delay spread
  - Coherence time
  - Coherence bandwidth
- Simulating multi-path, fading channels in MATLAB.
- Lumped-parameter models:
  - discrete-time equivalent channel.
- Path loss models, link budgets, shadowing.

Outline

Learning Objectives

- Understand models describing the nature of typical wireless communication channels.
  - The origin of multi-path and fading.
  - Concise characterization of multi-path and fading in both the time and frequency domain.
    - Doppler spectrum and time-coherence
    - Multi-path delay spread and frequency coherence
- Appreciate the impact of wireless channels on transmitted signals.
  - Distortion from multi-path: frequency-selective fading and inter-symbol interference.
  - The consequences of time-varying channels.

Outline

Path Loss

- Path loss L_P relates the received signal power P_r to the transmitted signal power P_t:

$$P_r = P_t \cdot \frac{G_r \cdot G_t}{L_P},$$

where G_t and G_r are antenna gains.

- Path loss is very important for cell and frequency planning or range predictions.
  - Not needed when designing signal sets, receiver, etc.

Path Loss

- Path loss modeling is “more an art than a science.”
  - Standard approach: fit model to empirical data.
  - Parameters of model:
    - d - distance between transmitter and receiver,
    - f_c - carrier frequency,
    - h_b, h_m - antenna heights,
    - Terrain type, building density, ...

Example: Free Space Propagation

- In free space, path loss L_P is given by Friis's formula:

$$L_P = \left(\frac{4\pi d}{\lambda_c}\right)^2 = \left(\frac{4\pi f_c d}{c}\right)^2.$$

  - Path loss increases proportional to the square of distance d and frequency f_c.
- In dB:

$$L_{P(dB)} = -20\log_{10}(\frac{c}{4\pi}) + 20\log_{10}(f_c) + 20\log_{10}(d).$$

  - Example: $f_c = 1$GHz and $d = 1$km

$$L_{P(dB)} = -146 \text{ dB} + 180 \text{ dB} + 60 \text{ dB} = 94 \text{ dB}.$$

Example: Two-Ray Channel

- Antenna heights: h_b and h_m.
- Two propagation paths:
  1. direct path, free space propagation,
  2. reflected path, free space with perfect reflection.
- Depending on distance d, the signals received along the two paths will add constructively or destructively.
- Path loss:

$$L_P = \frac{1}{4} \cdot \left(\frac{4\pi f_c d}{c}\right)^2 \cdot \left(\frac{1}{\sin(\frac{2\pi c h_b h_m}{f_c d})}\right)^2.$$

- For $d \gg h_b h_m$, path loss is approximately equal to:

$$L_P \approx \left(\frac{d^2}{h_b h_m}\right)^2$$

  - Path loss proportional to d^4 is typical for urban environment.

Okumura-Hata Model for Urban Area

- Okumura and Hata derived empirical path loss models from extensive path loss measurements.
  - Models differ between urban, suburban, and open areas, large, medium, and small cities, etc.
- Illustrative example: Model for Urban area (small or medium city)

$$L_{P(dB)} = A + B \log_{10}(d),$$

where

$$\begin{aligned} A &= 69.55 + 26.16 \log_{10}(f_c) - 13.82 \log_{10}(h_b) - a(h_m) \\ B &= 44.9 - 6.55 \log_{10}(h_b) \\ a(h_m) &= (1.1 \log_{10}(f_c) - 0.7) \cdot h_m - (1.56 \log_{10}(f_c) - 0.8) \end{aligned}$$

Signal and Noise Power

- Received Signal Power:

$$P_r = P_t \cdot \frac{G_r \cdot G_t}{L_P \cdot L_R},$$

where L_R is implementation loss, typically 2-3 dB.
- (Thermal) Noise Power:

$$P_N = kT_0 \cdot B_W \cdot F, \text{ where}$$

  - k - Boltzmann's constant ($1.38 \cdot 10^{-23}$ Ws/K),
  - T_0 - temperature in K (typical room temperature, $T_0 = 290$ K),
  - $\Rightarrow kT_0 = 4 \cdot 10^{-21}$ W/Hz $= 4 \cdot 10^{-18}$ mW/Hz $=$ -174 dBm/Hz,
- B_W - signal bandwidth,
- F - noise figure, figure of merit for receiver (typical value: 5dB).

Signal-to-Noise Ratio

- The ratio of received signal power and noise power is denoted by SNR.
- From the above, SNR equals:

$$\text{SNR} = \frac{P_t G_r \cdot G_t}{kT_0 \cdot B_W \cdot F \cdot L_P \cdot L_R}.$$

  - SNR increases with transmitted power P_t and antenna gains.
  - SNR decreases with bandwidth B_W, noise figure F, and path loss L_P.

E_s / N_0

- For the symbol error rate performance of communications system the ratio of signal energy E_s and noise power spectral density N_0 is more relevant than SNR.
- Since $E_s = P_r \cdot T_s = \frac{P_r}{R_s}$ and $N_0 = kT_0 \cdot F = P_N / B_W$, it follows that

$$\frac{E_s}{N_0} = \text{SNR} \cdot \frac{B_W}{R_s},$$

  where T_s and R_s denote the symbol period and symbol rate, respectively.

E_s / N_0

- Thus, E_s / N_0 is given by:

$$\frac{E_s}{N_0} = \frac{P_t G_r \cdot G_t}{kT_0 \cdot R_s \cdot F \cdot L_P \cdot L_R}.$$

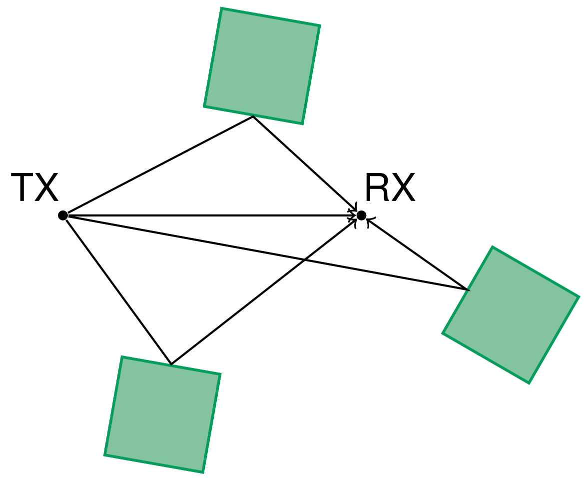
- in dB:

$$\begin{aligned}(\tfrac{E_s}{N_0})_{(dB)} = \; & P_{t(dBm)} + G_{t(dB)} + Gr(dB) \\ & -(kT_0)_{(dBm/Hz)} - R_{s(dBHz)} - F_{(dB)} - L_{R(dB)}.\end{aligned}$$

Outline

Multi-path Propagation

- The transmitted signal propagates from the transmitter to the receiver along many different paths.
- These paths have different
  - path attenuation a_k,
  - path delay τ_k,
  - phase shift ϕ_k,
  - angle of arrival θ_k.
    - For simplicity, we assume a 2-D model, so that the angle of arrival is the azimuth.
    - In 3-D models, the elevation angle of arrival is an additional parameter.



Channel Impulse Response

- From the above parameters, one can easily determine the channel's (baseband equivalent) impulse response.
- **Impulse Response:**

$$h(t) = \sum_{k=1}^{K} a_k \cdot e^{j\phi_k} \cdot e^{-j2\pi f_c \tau_k} \cdot \delta(t - \tau_k)$$

  - Note that the delays τ_k contribute to the phase shifts ϕ_k.

Received Signal

- Ignoring noise for a moment, the received signal is the convolution of the transmitted signal $s(t)$ and the impulse response

$$R(t) = s(t) * h(t) = \sum_{k=1}^{K} a_k \cdot e^{j\phi_k} \cdot e^{-j2\pi f_c \tau_k} \cdot s(t - \tau_k).$$

  - The received signal consists of multiple
    - scaled (by $a_k \cdot e^{j\phi_k} \cdot e^{-j2\pi f_c \tau_k}$),
    - delayed (by τ_k)

  copies of the transmitted signal.

Channel Frequency Response

- Similarly, one can compute the frequency response of the channel.
- Direct Fourier transformation of the expression for the impulse response yields

$$H(f) = \sum_{k=1}^{K} a_k \cdot e^{j\phi_k} \cdot e^{-j2\pi f_c \tau_k} \cdot e^{-j2\pi f \tau_k}.$$

  - For any given frequency f, the frequency response is a sum of complex numbers.
  - When these terms add destructively, the frequency response is very small or even zero at that frequency.
  - These nulls in the channel's frequency response are typical for wireless communications and are refered to as frequency-selective fading.

Frequency Response in One Line of MATLAB

- The Frequency response

$$H(f) = \sum_{k=1}^{K} a_k \cdot e^{j\phi_k} \cdot e^{-j2\pi f_c \tau_k} \cdot e^{-j2\pi f \tau_k}.$$

can be computed in MATLAB via the one-liner

```
HH = PropData.Field.*exp(-j*2*pi*fc*tau) * exp(-j*2*pi*tau'*ff);
```

- Note that `tau'*ff` is an inner product; it produces a matrix (with K rows and as many columns as `ff`).
- Similarly, the product preceding the second complex exponential is an inner product; it generates the sum in the expression above.

Example: Ray Tracing

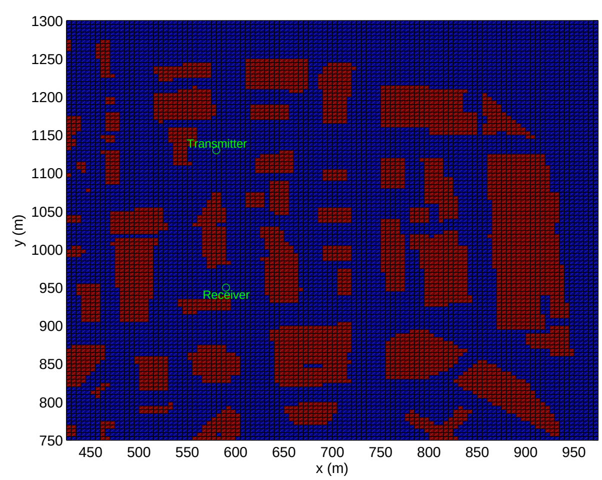


Figure: All propagation paths between the transmitter and receiver in the indicated located were determined through ray tracing.

Impulse Response

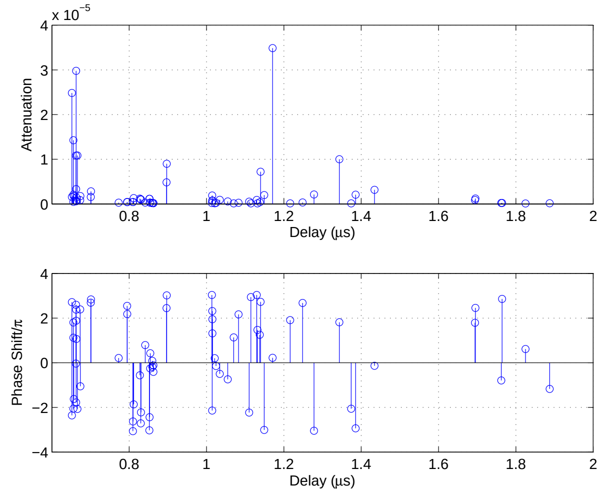


Figure: (Baseband equivalent) Impulse response shows attenuation, delay, and phase for each of the paths between receiver and transmitter.

Frequency Response

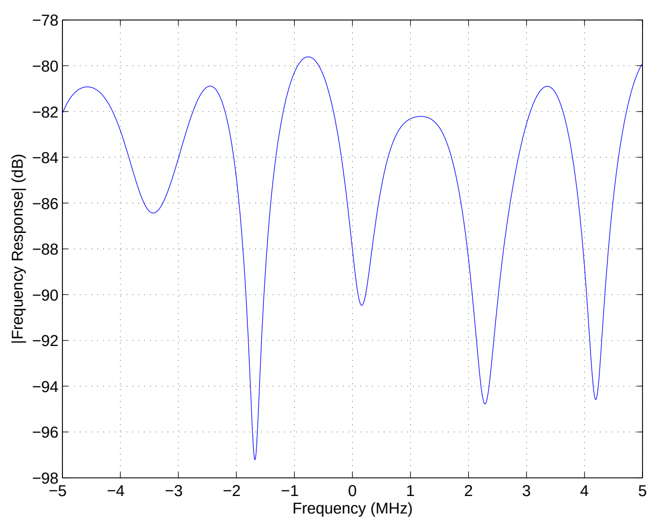


Figure: (Baseband equivalent) Frequency response for a multi-path channel is characterized by deep "notches".

Implications of Multi-path

- Multi-path leads to signal distortion.
  - The received signal “looks different” from the transmitted signal.
  - This is true, in particular, for wide-band signals.
- Multi-path propagation is equivalent to *undesired* filtering with a linear filter.
  - The impulse response of this undesired filter is the impulse response $h(t)$ of the channel.
- The effects of multi-path can be described in terms of both time-domain and frequency-domain concepts.
  - In either case, it is useful to distinguish between narrow-band and wide-band signals.

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Example: Transmission of a Linearly Modulated Signal

- Transmission of a linearly modulated signal through the above channel is simulated.
  - BPSK,
  - (full response) raised-cosine pulse.
- Symbol period is varied; the following values are considered
  - $T_s = 30\mu s$ (bandwidth approximately 60 KHz)
  - $T_s = 3\mu s$ (bandwidth approximately 600 KHz)
  - $T_s = 0.3\mu s$ (bandwidth approximately 6 MHz)
- For each case, the transmitted and (suitably scaled) received signal is plotted.
  - Look for distortion.
  - Note that the received signal is complex valued; real and imaginary part are plotted.

Example: Transmission of a Linearly Modulated Signal

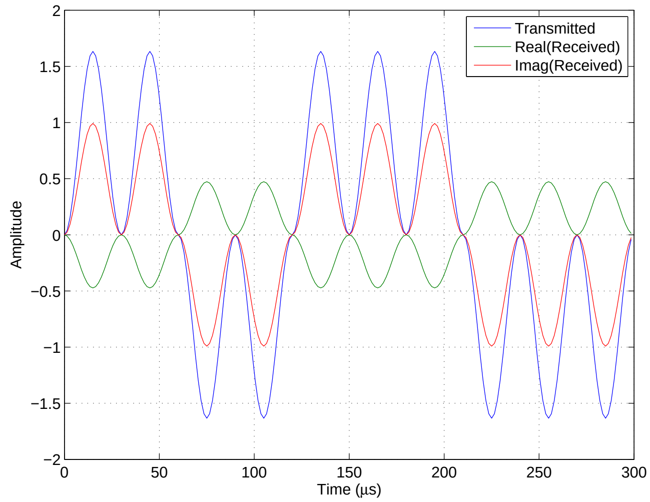

Figure: Transmitted and received signal; $T_s = 30\mu s$. No distortion is evident.

Example: Transmission of a Linearly Modulated Signal

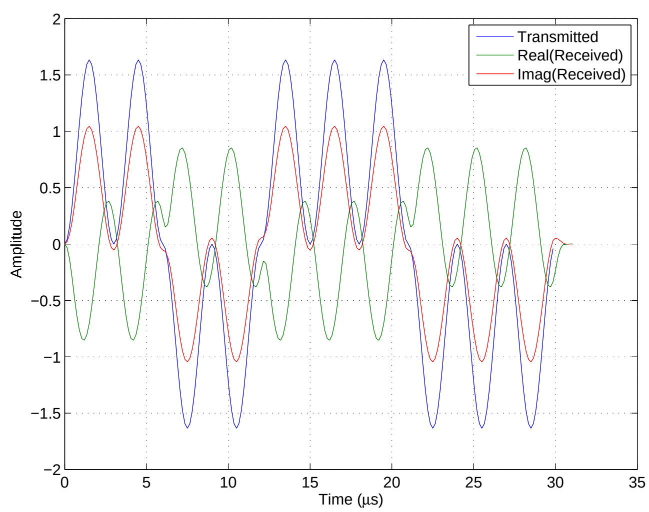

Figure: Transmitted and received signal; $T_s = 3\mu s$. Some distortion is visible near the symbol boundaries.

Example: Transmission of a Linearly Modulated Signal

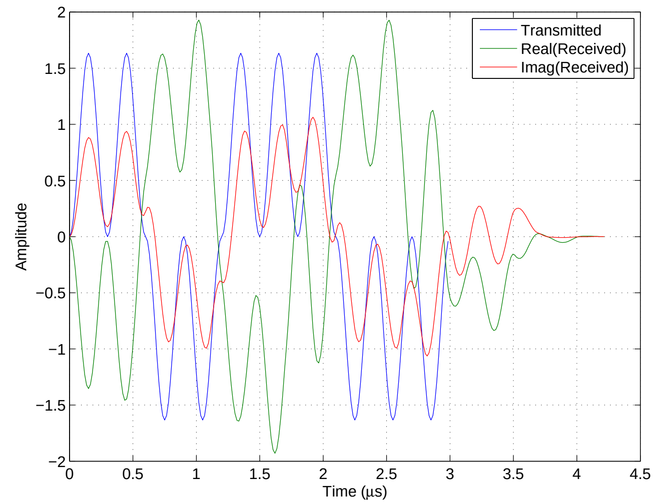

Figure: Transmitted and received signal; $T_s = 0.3\mu s$. Distortion is clearly visible and spans multiple symbol periods.

Eye Diagrams for Visualizing Distortion

- An eye diagram is a simple but useful tool for quickly gaining an appreciation for the amount of distortion present in a received signal.
- An eye diagram is obtained by plotting many segments of the received signal on top of each other.
  - The segments span two symbol periods.
- This can be accomplished in MATLAB via the command

```
plot( tt(1:2*fsT), real(reshape(Received(1:Ns*fsT), 2*fsT, [ ])))
```

  - `Ns` - number of symbols; should be large (e.g., 1000),
  - `Received` - vector of received samples.
  - The **`reshape`** command turns the vector into a matrix with `2*fsT` rows, and
  - the **`plot`** command plots each column of the resulting matrix individually.

Eye Diagram without Distortion

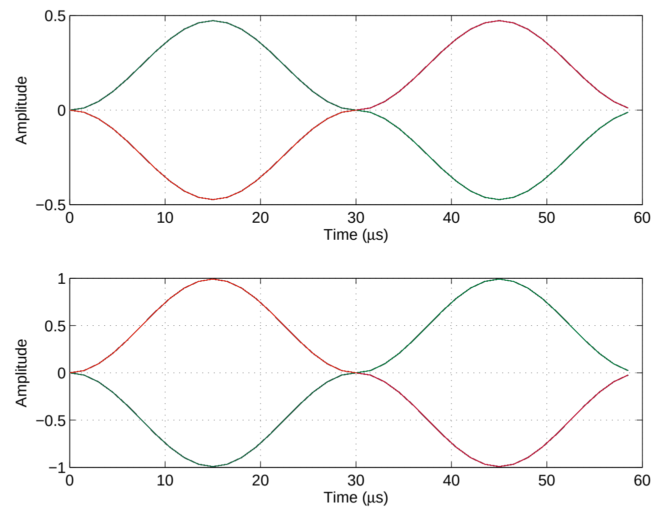

Figure: Eye diagram for received signal; $T_s = 30\mu s$. No distortion: "the eye is fully open".

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Eye Diagram with Distortion

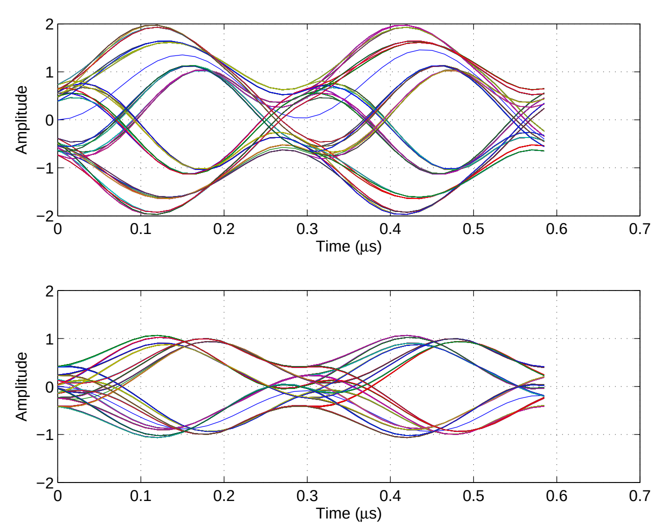

Figure: Eye diagram for received signal; $T_s = 0.3\mu s$. Significant distortion: "the eye is partially open".

Inter-Symbol Interference

- The distortion described above is referred to as inter-symbol interference (ISI).
  - As the name implies, the undesired filtering by the channel causes energy to be spread from one transmitted symbol across several adjacent symbols.
- This interference makes detection mored difficult and must be compensated for at the receiver.
  - Devices that perform this compensation are called equalizers.

Inter-Symbol Interference

- **Question:** Under what conditions does ISI occur?
- **Answer:** depends on the channel and the symbol rate.
  - The difference between the longest and the shortest delay of the channel is called the delay spread T_d of the channel.
  - The delay spread indicates the length of the impulse response of the channel.
  - Consequently, a transmitted symbol of length T_s will be spread out by the channel.
  - When received, its length will be the symbol period plus the delay spread, $T_s + T_d$.
- **Rule of thumb:**
  - if the delay spread is much smaller than the symbol period ($T_d \ll T_s$), then ISI is negligible.
  - If delay is similar to or greater than the symbol period, then ISI must be compensated at the receiver.

Frequency-Domain Perspective

- It is interesting to compare the bandwidth of the transmitted signals to the frequency response of the channel.
  - In particular, the bandwidth of the transmitted signal relative to variations in the frequency response is important.
  - The bandwidth over which the channel's frequency response remains approximately constant is called the coherence bandwidth.
- When the frequency response of the channel remains approximately constant over the bandwidth of the transmitted signal, the channel is said to be flat fading.
- Conversely, if the channel's frequency response varies significantly over the bandwidth of the signal, the channel is called a frequency-selective fading channel.

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Statistical Characterization of Channels

Example: Narrow-Band Signal

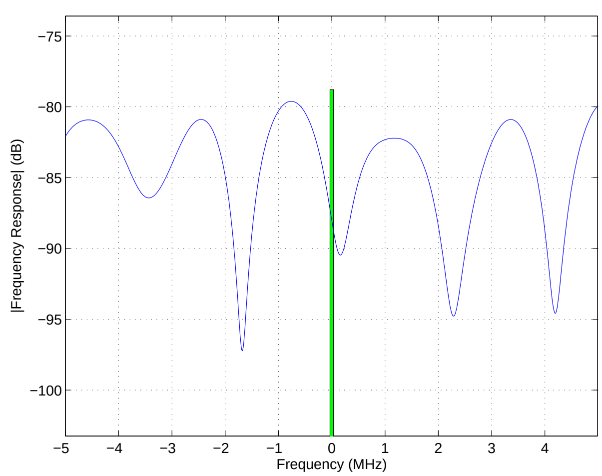


Figure: Frequency Response of Channel and bandwidth of signal; $T_s = 30\mu s$, Bandwidth $\approx$ 60 KHz; the channel's frequency response is approximately constant over the bandwidth of the signal.

Example: Wide-Band Signal

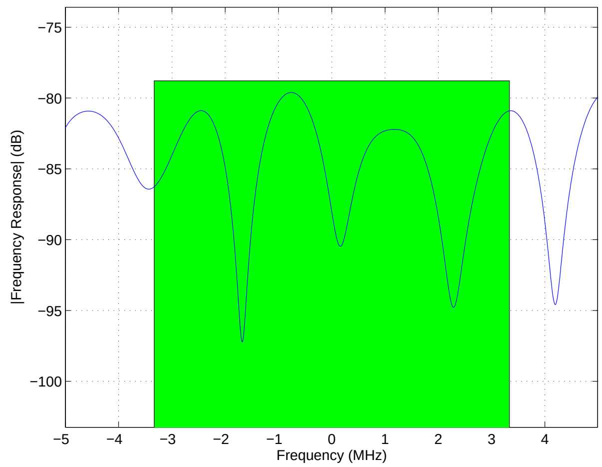


Figure: Frequency Response of Channel and bandwidth of signal; $T_s = 0.3\mu s$, Bandwidth $\approx$ 6 MHz; the channel's frequency response varies significantly over the bandwidth of the channel.

Frequency-Selective Fading and ISI

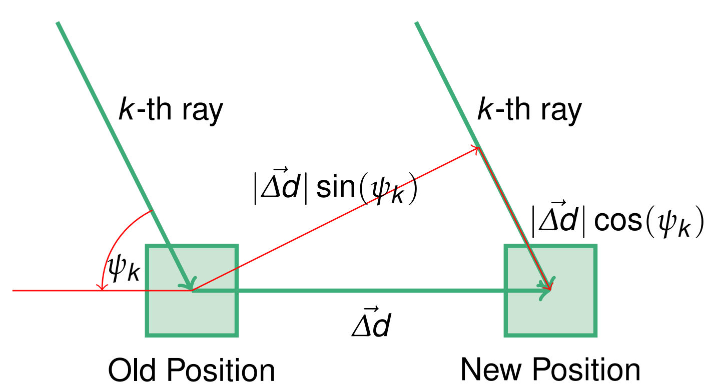
- Frequency-selective fading and ISI are dual concepts.
  - ISI is a time-domain characterization for significant distortion.
  - Frequency-selective fading captures the same idea in the frequency domain.
- **Wide-band signals** experience ISI and frequency-selective fading.
  - Such signals require an equalizer in the receiver.
  - Wide-band signals provide built-in diversity.
    - Not the entire signal will be subject to fading.
- **Narrow-band signals** experience flat fading (no ISI).
  - Simple receiver; no equalizer required.
  - Entire signal may be in a deep fade; no diversity.

Time-Varying Channel

- Beyond multi-path propagation, a second characteristic of many wireless communication channels is their time variability.
  - The channel is time-varying primarily because users are mobile.
- As mobile users change their position, the characteristics of each propagation path changes correspondingly.
  - Consider the impact a change in position has on
    - path gain,
    - path delay.
  - Will see that angle of arrival θ_k for k-th path is a factor.

Path-Changes Induced by Mobility

- Mobile moves by $\vec{\Delta d}$ from old position to new position.
  - distance: $|\vec{\Delta d}|$
  - angle: $\angle \vec{\Delta d} = \delta$
- Angle between k-th ray and $\vec{\Delta d}$ is denoted $\psi_k = \theta_k - \delta$.
- Length of k-th path increases by $|\vec{\Delta d}| \cos(\psi_k)$.



Impact of Change in Path Length

- We conclude that the length of each path changes by $|\vec{\Delta d}|\cos(\psi_k)$, where
  - ψ_k denotes the angle between the direction of the mobile and the k-th incoming ray.
- **Question:** how large is a typical distance $|\vec{\Delta d}|$ between the old and new position is?
  - The distance depends on
    - the velocity v of the mobile, and
    - the time-scale ΔT of interest.
- In many modern communication system, the transmission of a frame of symbols takes on the order of 1 to 10 ms.
- Typical velocities in mobile systems range from pedestrian speeds ($\approx$ 1m/s) to vehicle speeds of 150km/h($\approx$ 40m/s).
- Distances of interest $|\vec{\Delta d}|$ range from 1mm to 400mm.

Impact of Change in Path Length

- **Question:** What is the impact of this change in path length on the parameters of each path?
  - We denote the length of the path to the old position by d_k.
  - Clearly, $d_k = c \cdot \tau_k$, where c denotes the speed of light.
  - Typically, d_k is much larger than $|\vec{\Delta d}|$.
- **Path gain a_k:** Assume that path gain a_k decays inversely proportional with the square of the distance, $a_k \sim d_k^{-2}$.
- Then, the relative change in path gain is proportional to $(|\vec{\Delta d}|/d_k)^2$ (e.g., $|\vec{\Delta d}| = 0.1$m and $d_k = 100$m, then path gain changes by approximately 0.0001%).
  - **Conclusion:** The change in path gain is generally small enough to be negligible.

Impact of Change in Path Length

- **Delay τ_k:** By similar arguments, the delay for the k-th path changes by at most $|\vec{\Delta d}| / c$.
- The relative change in delay is $|\vec{\Delta d}| / d_k$ (e.g., 0.1% with the values above.)
  - **Question:** Is this change in delay also negligible?

Relating Delay Changes to Phase Changes

- **Recall:** the impulse response of the multi-path channel is

$$h(t) = \sum_{k=1}^{K} a_k \cdot e^{j\phi_k} \cdot e^{-j2\pi f_c \tau_k} \cdot \delta(t - \tau_k)$$

- Note that the delays, and thus any delay changes, are multiplied by the carrier frequency f_c to produce phase shifts.

Relating Delay Changes to Phase Changes

- Consequently, the phase change arising from the movement of the mobile is

$$\Delta\phi_k = -2\pi f_c/c|\vec{\Delta d}|\cos(\psi_k) = -2\pi|\vec{\Delta d}|/\lambda_c\cos(\psi_k),$$

  where
  - $\lambda_c = c/f_c$ - denotes the wave-length at the carrier frequency (e.g., at $f_c = 1$GHz, $\lambda_c \approx 0.3$m),
  - ψ_k - angle between direction of mobile and k-th arriving path.
- **Conclusion:** These phase changes are significant and lead to changes in the channel properties over short time-scales (fast fading).

Illustration

- To quantify these effects, compute the phase change over a time interval $\Delta T = 1$ms as a function of velocity.
  - Assume $\psi_k = 0$, and, thus, $\cos(\psi_k) = 1$.
  - $f_c = 1$GHz.

v (m/s)	$\vert\vec{\Delta d}\vert$ (mm)	$\Delta\phi$ (degrees)	Comment
1	1	1.2	Pedestrian; negligible phase change.
10	10	12	Residential area vehicle speed.
100	100	120	High-way speed; phase change significant.
1000	1000	1200	High-speed train or low-flying aircraft; receiver must track phase changes.

Doppler Shift and Doppler Spread

- If a mobile is moving at a constant velocity v, then the distance between an old position and the new position is a function of time, $|\vec{\Delta d}| = vt$.
- Consequently, the phase change for the k-th path is

$$\Delta\phi_k(t) = -2\pi v/\lambda_c \cos(\psi_k)t = -2\pi v/c \cdot f_c \cos(\psi_k)t.$$

  - The phase is a linear function of t.
  - Hence, along this path the signal experiences a frequency shift $f_{d,k} = v/c \cdot f_c \cdot \cos(\psi_k) = v/\lambda_c \cdot \cos(\psi_k)$.
  - This frequency shift is called Doppler shift.
- Each path experiences a different Doppler shift.
  - Angles of arrival θ_k are different.
  - Consequently, instead of a single Doppler shift a number of shifts create a Doppler Spectrum.

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Illustration: Time-Varying Frequency Response

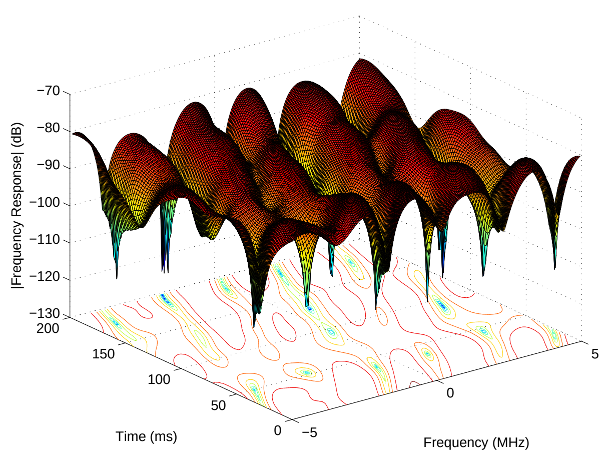

Figure: Time-varying Frequency Response for Ray-Tracing Data; velocity $v = 10$m/s, $f_c = 1$GHz, maximum Doppler frequency ≈ 33Hz.

Illustration: Time-varying Response to a Sinusoidal Input

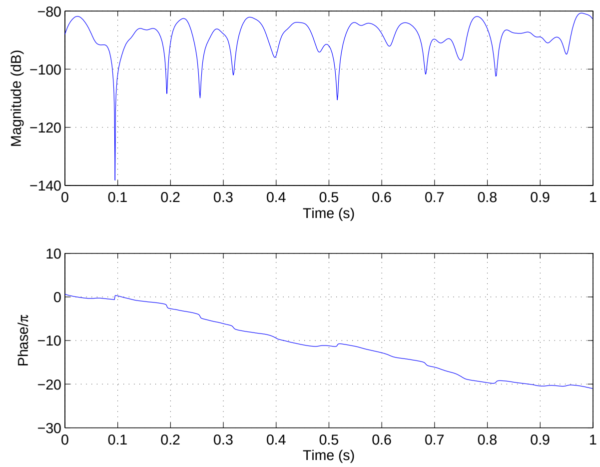

Figure: Response of channel to sinusoidal input signal; base-band equivalent input signal $s(t) = 1$, velocity $v = 10$m/s, $f_c = 1$GHz, maximum Doppler frequency ≈ 33Hz.

Doppler Spread and Coherence Time

- The time over which the channel remains approximately constant is called the coherence time of the channel.
- Coherence time and Doppler spectrum are dual characterizations of the time-varying channel.
  - **Doppler spectrum** provides frequency-domain interpretation:
    - It indicates the range of frequency shifts induced by the time-varying channel.
    - Frequency shifts due to Doppler range from $-f_d$ to f_d, where $f_d = v/c \cdot f_c$.
  - The **coherence time** T_c of the channel provides a time-domain characterization:
    - It indicates how long the channel can be assumed to be approximately constant.
- Maximum Doppler shift f_d and coherence time T_c are related to each through an inverse relationship $T_c \approx 1/f_d$.

System Considerations

- The time-varying nature of the channel must be accounted for in the design of the system.
- **Transmissions are shorter than the coherence time:**
  - Many systems are designed to use frames that are shorter than the coherence time.
  - Example: GSM TDMA structure employs time-slots of duration 4.6ms.
  - **Consequence:** During each time-slot, channel may be treated as constant.
  - From one time-slot to the next, channel varies significantly; this provides opportunities for diversity.
- **Transmission are longer than the coherence time:**
  - Channel variations must be tracked by receiver.
  - Example: use recent symbol decisions to estimate current channel impulse response.

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Illustration: Time-varying Channel and TDMA

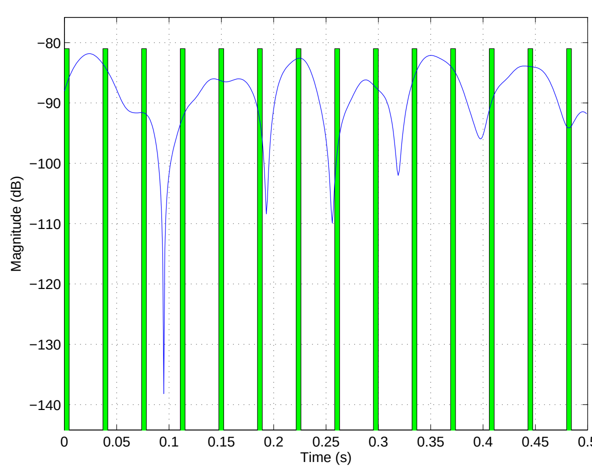


Figure: Time varying channel response and TDMA time-slots; time-slot duration 4.6ms, 8 TDMA users, velocity $v = 10$m/s, $f_c = 1$GHz, maximum Doppler frequency ≈ 33Hz.

Summary

- Illustrated by means of a concrete example the two main impairments from a mobile, wireless channel.
  - Multi-path propagation,
  - Doppler spread due to time-varying channel.
- **Multi-path** propagation induces ISI if the symbol duration exceeds the delay spread of the channel.
  - In frequency-domain terms, frequency-selective fading occurs if the signal bandwidth exceeds the coherence band-width of the channel.
- **Doppler Spreading** results from time-variations of the channel due to mobility.
  - The maximum Doppler shift $f_d = v/c \cdot f_c$ is proportional to the speed of the mobile.
  - In time-domain terms, the channel remains approximately constant over the coherence-time of the channel.

Outline

Statistical Characterization of Channel

- We have looked at the characterization of a concrete realization of a mobile, wire-less channel.
- For different locations, the properties of the channel will likely be very different.
- **Objective:** develop statistical models that capture the salient features of the wireless channel for areas of interest.
  - Models must capture multi-path and time-varying nature of channel.
- **Approach:** Models reflect correlations of the time-varying channel impulse response or frequency response.
  - Time-varying descriptions of channel are functions of two parameters:
    - Time t when channel is measured,
    - Frequency f or delay τ.

Power Delay Profile

- The impulse response of a wireless channel is time-varying, $h(t, \tau)$.
  - The parameter t indicates when the channel is used,
  - The parameter τ reflects time since the input was applied (delay).
  - Time-varying convolution: $r(t) = \int h(t, \tau) \cdot s(t - \tau) d\tau$.
- The power-delay profile measures the average power in the impulse response over delay τ.
  - **Thought experiment:** Send impulse through channel at time t_0 and measure response $h(t_0, \tau)$.
  - Repeat K times, measuring $h(t_k, \tau)$.
  - Power delay profile:

$$\Psi_h(\tau) = \frac{1}{K+1} \sum_{k=0}^{K} |h(t_k, \tau)|^2.$$

Power Delay Profile

- The power delay profile captures the statistics of the multi-path effects of the channel.
- The underlying, physical model assumes a large number of propagation paths:
  - each path has a an associated delay τ,
  - the gain for a path is modeled as a complex Gaussian random variable with second moment equal to $\Psi_h(\tau)$.
    - If the mean of the path loss is zero, the path is said to be Rayleigh fading.
    - Otherwise, it is Ricean.
  - The channel gains associated with different delays are assumed to be uncorrelated.

Example

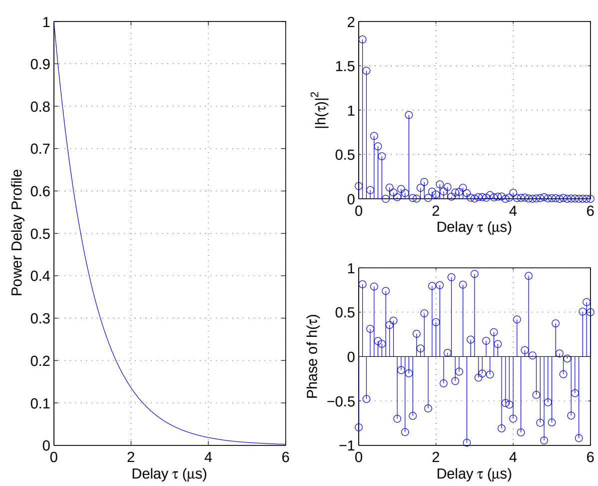


Figure: Power Delay Profile and Channel Impulse Response; the power delay profile (left) equals $\Psi_h(\tau) = \exp(-\tau/T_h)$ with $T_h = 1\mu$s; realization of magnitude and phase of impulse response (left).

RMS Delay Spread

- From a systems perspective, the extent (spread) of the delays is most significant.
  - The length of the impulse response of the channel determines how much ISI will be introduced by the channel.
- The spread of delays is measured concisely by the RMS delay spread T_d:

$$T_d^2 = \int_0^\infty \Psi_h^{(n)}(\tau)\tau^2 d\tau - (\int_0^\infty \Psi_h^{(n)}(\tau)\tau d\tau)^2,$$

  where

$$\Psi_h^{(n)} = \Psi_h / \int_0^\infty \Psi_h(\tau) d\tau.$$

- **Example:** For $\Psi_h(\tau) = \exp(-\tau/T_h)$, RMS delay spread equals T_h.
  - In urban environments, typical delay spreads are a few μs.

Frequency Coherence Function

- The Fourier transform of the Power Delay Spread $\Psi_h(\tau)$ is called the Frequency Coherence Function $\Psi_H(\Delta f)$

$$\Psi_h(\tau) \leftrightarrow \Psi_H(\Delta f).$$

- The frequency coherence function measures the correlation of the channel's frequency response.
  - **Thought Experiment:** Transmit two sinusoidal signal of frequencies f_1 and f_2, such that $f_1 - f_2 = \Delta f$.
  - The gain each of these signals experiences is $H(t, f_1)$ and $H(t, f_2)$, respectively.
  - Repeat the experiment many times and average the products $H(t, f_1) \cdot H^*(t, f_2)$.
  - $\Psi_H(\Delta f)$ indicates how similar the gain is that two sinusoids separated by Δf experience.

Coherence Bandwidth

- The width of the main lobe of the frequency coherence function is the coherence bandwidth B_c of the channel.
  - Two signals with frequencies separated by less than the coherence bandwidth will experience very similar gains.
- Because of the Fourier transform relationship between the power delay profile and the frequency coherence function:

$$B_c \approx \frac{1}{T_d}.$$

- **Example:** Fourier transform of $\Psi_h(\tau) = \exp(-\tau/T_h)$

$$\Psi_H(\Delta f) = \frac{T_h}{1 + j2\pi\Delta f T_h};$$

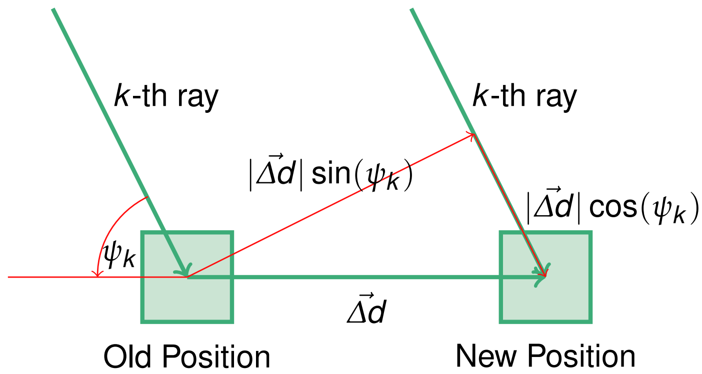
the 3-dB bandwidth of $\Psi_H(\Delta f)$ is $B_c = 1/(2\pi \cdot T_h)$.
  - For urban channels, coherence bandwidth is a few 100KHz.

Time Coherence

- The time-coherence function $\Psi_H(\Delta t)$ captures the time-varying nature of the channel.
  - **Thought experiment:** Transmit a sinusoidal signal of frequency f through the channel and measure the output at times t_1 and $t_1 + \Delta t$.
  - The gains the signal experiences are $H(t_1, f)$ and $H(t_1 + \Delta t, f)$, respectively.
  - Repeat experiment and average the products $H(t_k, f) \cdot H^*(t_k + \Delta t, f)$.
- Time coherence function measures, how quickly the gain of the channel varies.
  - The width of the time coherence function is called the coherence-time T_c of the channel.
  - The channel remains approximately constant over the coherence time of the channel.

Example: Isotropic Scatterer

- Old location: $H(t_1, f=0) = a_k \cdot \exp(-j2\pi f_c \tau_k)$.
- At new location: the gain a_k is unchanged; phase changes by $f_d \cos(\psi_k)\Delta t$:
  $H(t_1 + \Delta t, f=0) = a_k \cdot \exp(-j2\pi(f_c\tau_k + f_d\cos(\psi_k)\Delta t))$.



Example: Isotropic Scatterer

- The average of $H(t_1, 0) \cdot H^*(t_1 + \Delta t, 0)$ yields the time-coherence function.
- Assume that the angle of arrival ψ_k is uniformly distributed.
  - This allows computation of the average (isotropic scatterer assumption:

$$\Psi_H(\Delta t) = |a_k|^2 \cdot J_0(2\pi f_d \Delta t)$$

Time-Coherence Function for Isotropic Scatterer

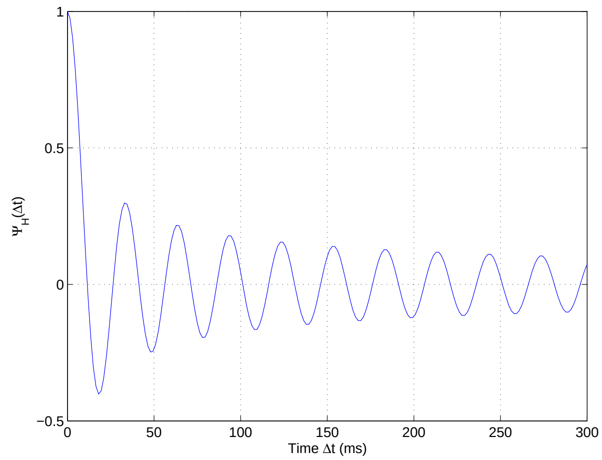


Figure: Time-Coherence Function for Isotropic Scatterer; velocity $v = 10$m/s, $f_c = 1$GHz, maximum Doppler frequency $f_d \approx 33$Hz. First zero at $\Delta t \approx 0.4/f_d$.

Doppler Spread Function

- The Fourier transform of the time coherence function $\Psi_H(\Delta t)$ is the Doppler Spread Function $\Psi_d(f_d)$

$$\Psi_H(\Delta t) \leftrightarrow \Psi_d(f_d).$$

- The Doppler spread function indicates the range of frequencies observed at the output of the channel when the input is a sinusoidal signal.
  - Maximum Doppler shift $f_{d,max} = v/c \cdot f_c$.
- **Thought experiment:**
  - Send a sinusoidal signal of
  - The PSD of the received signal is the Doppler spread function.

Doppler Spread Function for Isotropic Scatterer

- **Example:** The Doppler spread function for the isotropic scatterer is

$$\Psi_d(f_d) = \frac{|a_k|^2}{4\pi f_d} \frac{1}{\sqrt{1 - (f/f_d)^2}} \text{ for } |f| < f_d.$$

Doppler Spread Function for Isotropic Scatterer

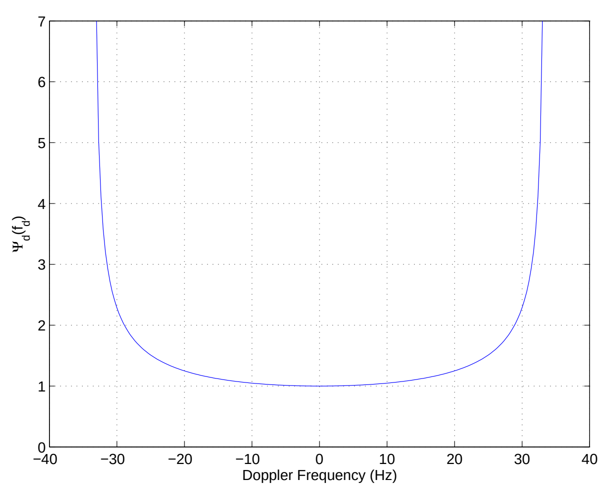


Figure: Doppler Spread Function for Isotropic Scatterer; velocity $v = 10$m/s, $f_c = 1$GHz, maximum Doppler frequency $f_d \approx 33$Hz. First zero at $\Delta t \approx 0.4/f_d$.

Simulation of Multi-Path Fading Channels

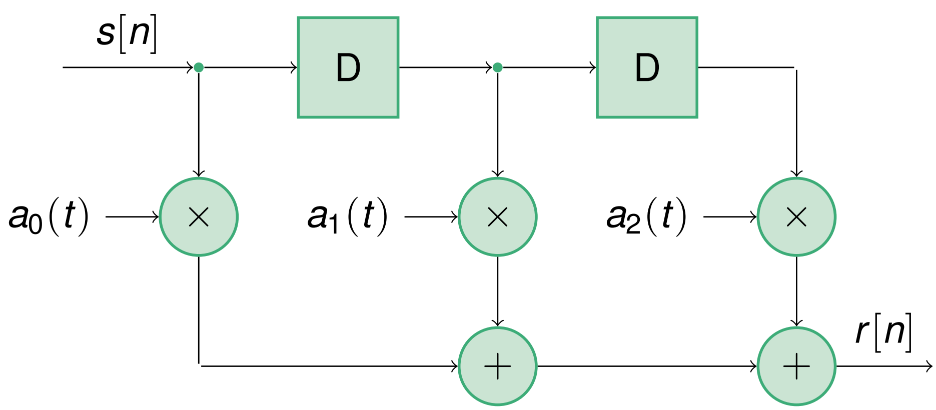
- We would like to be able to simulate the effects of time-varying, multi-path channels.
- **Approach:**
  - The simulator operates in discrete-time; the sampling rate is given by the sampling rate for the input signal.
  - The **multi-path** effects can be well modeled by an FIR (tapped delay-line)filter.
    - The number of taps for the filter is given by the product of delay spread and sampling rate.
    - Example: With a delay spread of 2μs and a sampling rate of 2MHz, four taps are required.
    - The taps should be random with a Gaussian distribution.
    - The magnitude of the tap weights should reflect the power-delay profile.

Simulation of Multi-Path Fading Channels

- **Approach (cont'd):**
  - The **time-varying** nature of the channel can be captured by allowing the taps to be time-varying.
    - The time-variations should reflect the Doppler Spectrum.

Simulation of Multi-Path Fading Channels

- The taps are modeled as
  - Gaussian random processes
  - with variances given by the power delay profile, and
  - power spectral density given by the Doppler spectrum.



Channel Model Parameters

- Concrete parameters for models of the above form have been proposed by various standards bodies.
  - For example, the following table is an excerpt from a document produced by the COST 259 study group.

Tap number	**Relative Time (μs)**	**Relative Power (dB)**	Doppler Spectrum
1	0	-5.7	`Class`
2	0.217	-7.6	`Class`
3	0.512	-10.1	`Class`
⋮	⋮	⋮	⋮
20	2.140	-24.3	`Class`

Channel Model Parameters

- The table provides a concise, statistical description of a time-varying multi-path environment.
- Each row corresponds to a path and is characterized by
  - the delay beyond the delay for the shortest path,
  - the average power of this path;
    - this parameter provides the variance of the Gaussian path gain.
  - the Doppler spectrum for this path;
    - The notation `Class` denotes the classical Doppler spectrum for the isotropic scatterer.
- The delay and power column specify the power-delay profile.
- The Doppler spectrum is given directly.
  - The Doppler frequency f_d is an additional parameter.

Toolbox Function `SimulateCOSTChannel`

- The result of our efforts will be a toolbox function for simulating time-varying multi-path channels:

```
function OutSig = SimulateCOSTChannel( InSig, ChannelParams, fs)
```

- Its input arguments are

```
% Inputs:
%   InSig          - baseband equivalent input signal
%   ChannelParams  - structure ChannelParams must have fields
%                     Delay   - relative delay
%                     Power   - relative power in dB
%                     Doppler - type of Dopller spectrum
%                     fd      - max. Doppler shift
%   fs             - sampling rate
```

Discrete-Time Considerations

- The delays in the above table assume a continuous time axis; our time-varying FIR will operate in discrete time.
- To convert the model to discrete-time:
  - Continuous-time is divided into consecutive “bins” of width equal to the sampling period, $1/fs$.
  - For all paths arriving in same “bin,” powers are added.
    - This approach reflects that paths arriving closer together than the sampling period cannot be resolved;
    - their effect is combined in the receiver front-end.
  - The result is a reduced description of the multi-path channel:
    - Power for each tap reflects the combined power of paths arriving in the corresponding “bin”.
    - This power will be used to set the variance of the random process for the corresponding tap.

Converting to a Discrete-Time Model in MATLAB

```
%% convert powers to linear scale
Power_lin = dB2lin( ChannelParams.Power);

%% Bin the delays according to the sample rate
QDelay = floor( ChannelParams.Delay*fs );

% set surrogate delay for each bin, then sum up the power in each bin
Delays = ( ( 0:QDelay(end) ) + 0.5 ) / fs;
Powers = zeros( size(Delays) );
for kk = 1:length(Delays)
    Powers( kk ) = sum( Power_lin( QDelay == kk-1 ) );
end
```

Generating Time-Varying Filter Taps

- The time-varying taps of the FIR filter must be Gaussian random processes with specified variance and power spectral density.
- To accomplish this, we proceed in two steps:
  1. Create a filter to shape the power spectral density of the random processes for the tap weights.
  2. Create the random processes for the tap weights by passing complex, white Gaussian noise through the filter.
     - Variance is adjusted in this step.
- Generating the spectrum shaping filter:

```
% desired frequency response of filter:
HH   = sqrt( ClassDoppler( ff, ChannelParams.fd ) );
% design filter with desired frequency response
hh = Persistent_firpm( NH-1, 0:1/(NH-1):1, HH );
hh = hh/norm(hh);    % ensure filter has unit norm
```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Generating Time-Varying Filter Taps

- The spectrum shaping filter is used to filter a complex white noise process.
  - Care is taken to avoid transients at the beginning of the output signal.
  - Also, filtering is performed at a lower rate with subsequent interpolation to avoid numerical problems.
    - Recall that f_d is quite small relative to f_s.

```
% generate a white Gaussian random process
ww = sqrt( Powers( kk )/2)*...
     ( randn( 1, NSamples) + j*randn( 1, NSamples) );
% filter so that spectrum equals Doppler spectrum
ww = conv( ww, hh );
ww = ww( length( hh )+1:NSamples ).';
% interpolate to a higher sampling rate
% ww = interp( ww, Down );
ww = interpft(ww, Down*length(ww));
% store time-varying filter taps for later use
```

Time-Varying Filtering

- The final step in the simulator is filtering the input signal with the time-varying filter taps.
  - MATLAB's filtering functions **`conv`** or **`filter`** cannot be used (directly) for this purpose.
- The simulator breaks the input signal into short segments for which the channel is nearly constant.
  - Each segment is filtered with a slightly different set of taps.

```
while ( Start < length(InSig) )
    EndIn  = min( Start+QDeltaH, length(InSig) );
    EndOut = EndIn + length(Powers)-1;
    OutSig(Start:EndOut) = OutSig(Start:EndOut) + ...
        conv( Taps(kk,:), InSig(Start:EndIn) );

    kk = kk+1;
    Start = EndIn+1;
```


Testing `SimulateCOSTChannel`

- A simple test for the channel simulator consists of "transmitting" a baseband equivalent sinusoid.

```
%% Initialization
ChannelParameters    = tux();       % COST model parameters
ChannelParameters.fd = 10;          % Doppler frequency

fs                   = 1e5;         % sampling rate
SigDur               = 1;           % duration of signal

%% generate input signal and simulate channel
tt         = 0:1/fs:SigDur;     % time axis
Sig        = ones( size(tt) ); % baseband-equivalent carrier

Received   =  SimulateCOSTChannel(Sig, ChannelParameters, fs);
```

Testing `SimulateCOSTChannel`

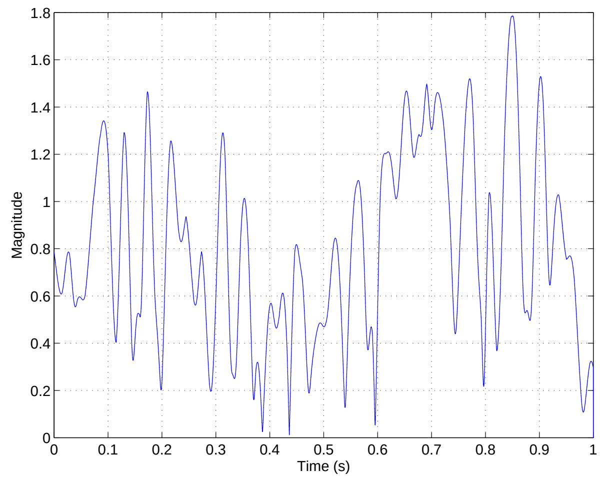

Figure: Simulated Response to a Sinusoidal Signal; $f_d = 10$Hz, baseband equivalent frequency $f = 0$.

Pathloss and Link Budget From Physical Propagation to Multi-Path Fading Statistical Characterization of Channels

Summary

- Highlighted unique aspects of mobile, wireless channels:
  - time-varying, multi-path channels.
- Statistical characterization of channels via
  - power-delay profile (RMS delay spread),
  - frequency coherence function (coherence bandwidth),
  - time coherence function (coherence time), and
  - Doppler spread function (Doppler spread).
- Relating channel parameters to system parameters:
  - signal bandwidth and coherence bandwidth,
  - frame duration and coherence time.
- Channel simulator in MATLAB.

Where we are ...

- Having characterized the nature of mobile, wireless channels, we can now look for ways to overcome the detrimental effects of the channel.
  - The importance of diversity to overcome fading.
  - Sources of diversity:
    - Time,
    - Frequency,
    - Space.
- Equalizers for overcoming frequency-selective fading.
  - Equalizers also exploit freqeuncy diversity.