5. Capacity of Wireless Channels

Information Theory

- So far we have only looked at specific communication schemes.
- Information theory provides a fundamental limit to (coded) performance.
- It succinctly identifies the impact of channel resources on performance as well as suggests new and cool ways to communicate over the wireless channel.
- It provides the basis for the modern development of wireless communication.

Capacity of AWGN Channel

Capacity of AWGN channel

$$C_{\text{awgn}} = \log(1 + \text{SNR}) \quad \text{bits/s/Hz}$$
$$= W \log(1 + \text{SNR}) \quad \text{bits/s}$$

If average transmit power constraint is $\bar{P}$ watts and noise psd is N_0 watts/Hz,

$$C_{\text{awgn}} = W \log\left(1 + \frac{\bar{P}}{N_0 W}\right) \quad \text{bits/s.}$$

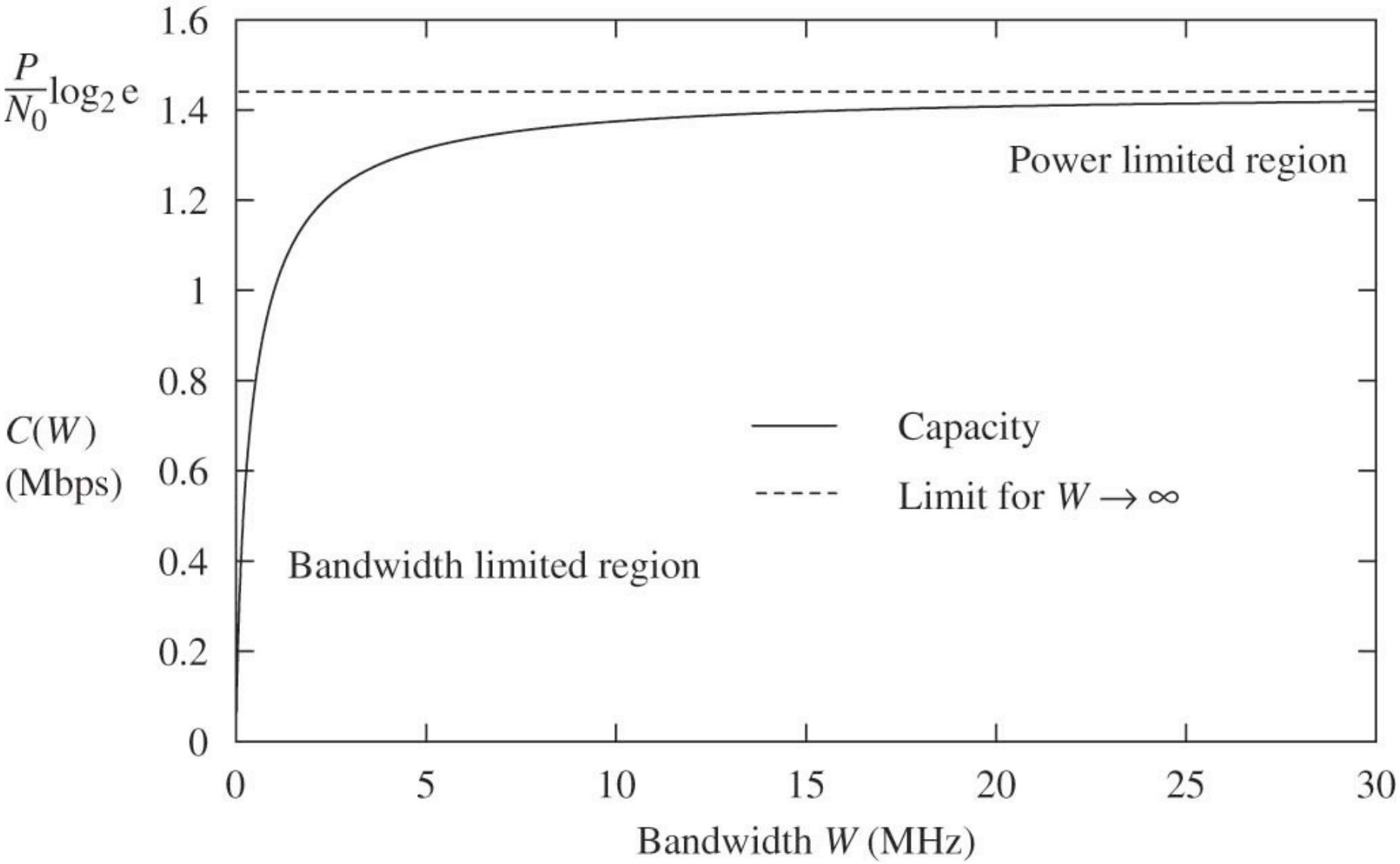
Power and Bandwidth Limited Regimes

$$C_{\text{awgn}} = W \log\left(1 + \frac{\bar{P}}{N_0 W}\right)$$

$$\text{SNR} = \frac{\bar{P}}{N_0 W}$$

Bandwidth limited regime $\text{SNR} \gg 1$: capacity logarithmic in power, approximately linear in bandwidth.

Power limited regime $\text{SNR} \ll 1$: capacity linear in power, insensitive to bandwidth.

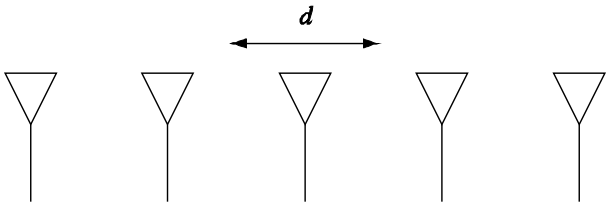

1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
$\frac{P}{N_0}\log_2 e$
$C(W)$
(Mbps)
Power limited region
Capacity
Limit for $W \to \infty$
Bandwidth limited region
0
5
10
15
20
25
30
Bandwidth W (MHz)

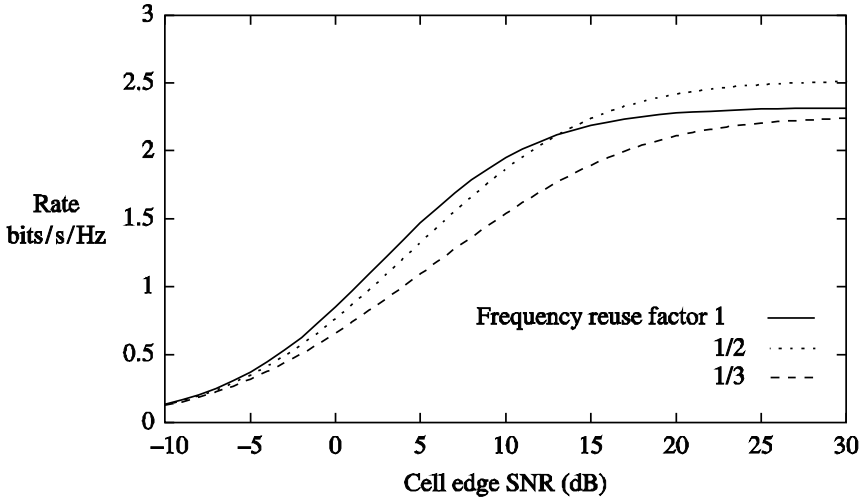
Example 1: Impact of Frequency Reuse

- Different degree of frequency reuse allows a tradeoff between SINR and degrees of freedom per user.
- Users in narrowband systems have high link SINR but small fraction of system bandwidth.
- Users in wideband systems have low link SINR but full system bandwidth.
- Capacity depends on both SINR and d.o.f. and can provide a guideline for optimal reuse.
- Optimal reuse depends on how the out-of-cell interference fraction $f(\rho)$ depends on the reuse factor ρ.

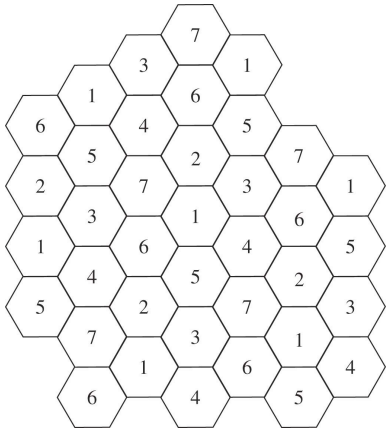
Numerical Examples

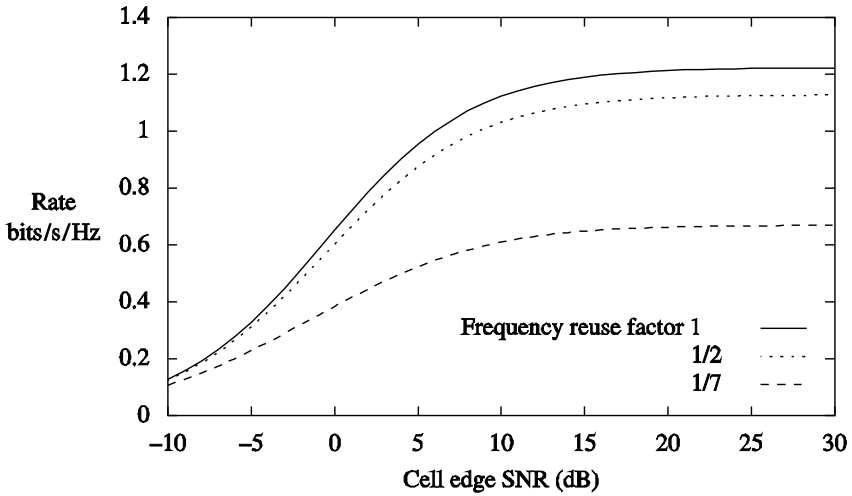
Linear cellular system





Hexagonal system





Example 2: CDMA Uplink Capacity

- Single cell with K users.

$$\text{SINR} = \frac{P}{N_0 + (K-1)P} \approx \frac{1}{K} \quad \text{(-15 dB for } K = 32\text{)}$$

- Capacity per user

$$= \log(1 + \text{SINR}) \approx \text{SINR} \log_2 e \quad \text{bits/s/Hz.}$$

- Cell capacity (interference-limited)

$$\approx K \cdot \text{SINR} \log_2 e \approx 1.442 \text{bits/s/Hz}$$

Example 2 (continued)

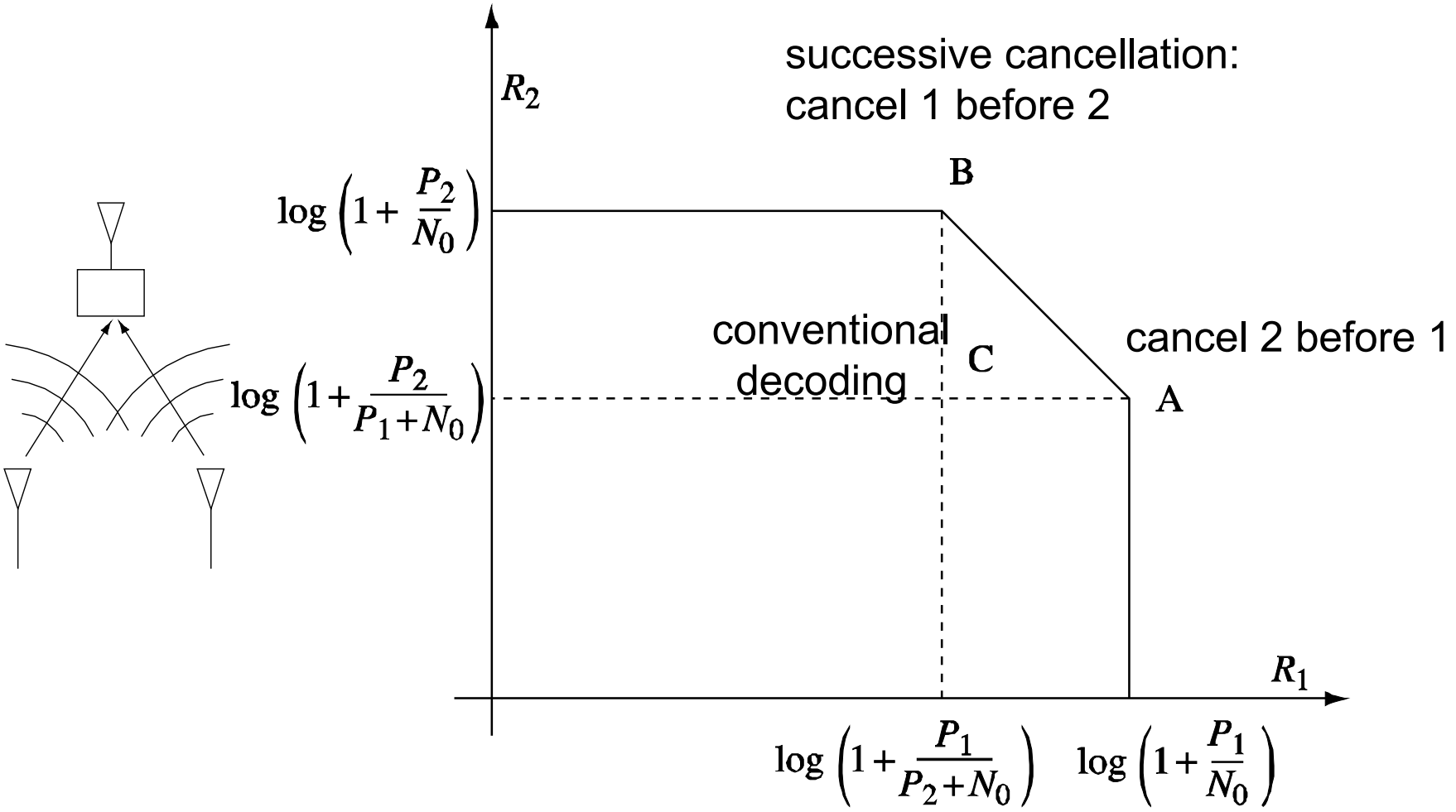
- If out-of-cell interference is a fraction f of in-cell interference:

$$C \approx \frac{1.442}{1+f} \quad \text{bits/s/Hz}$$

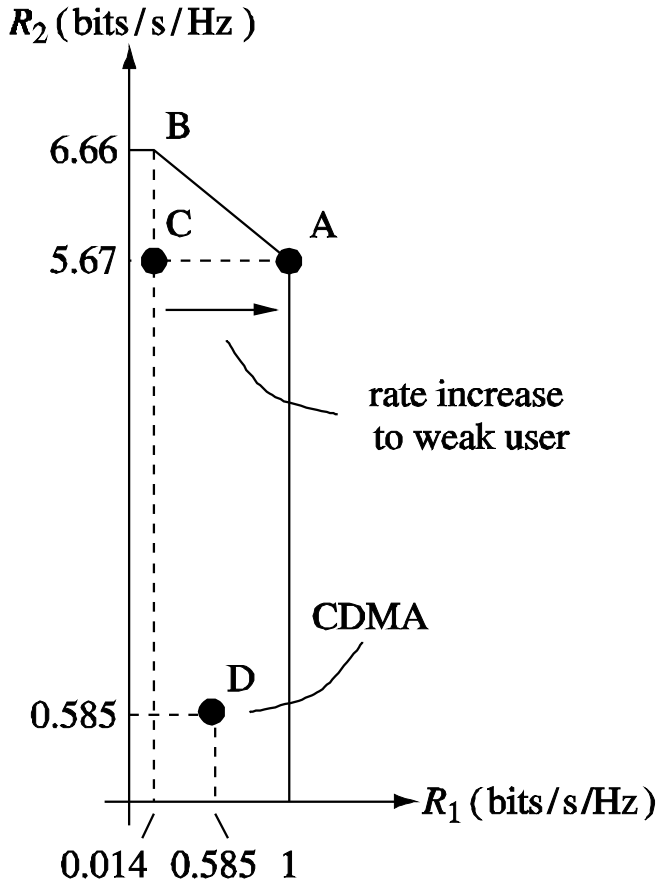
Uplink and Downlink Capacity

- CDMA and OFDM are specific multiple access schemes.
- But information theory tells us what is the capacity of the uplink and downlink channels and the optimal multiple access schemes.

Uplink AWGN Capacity



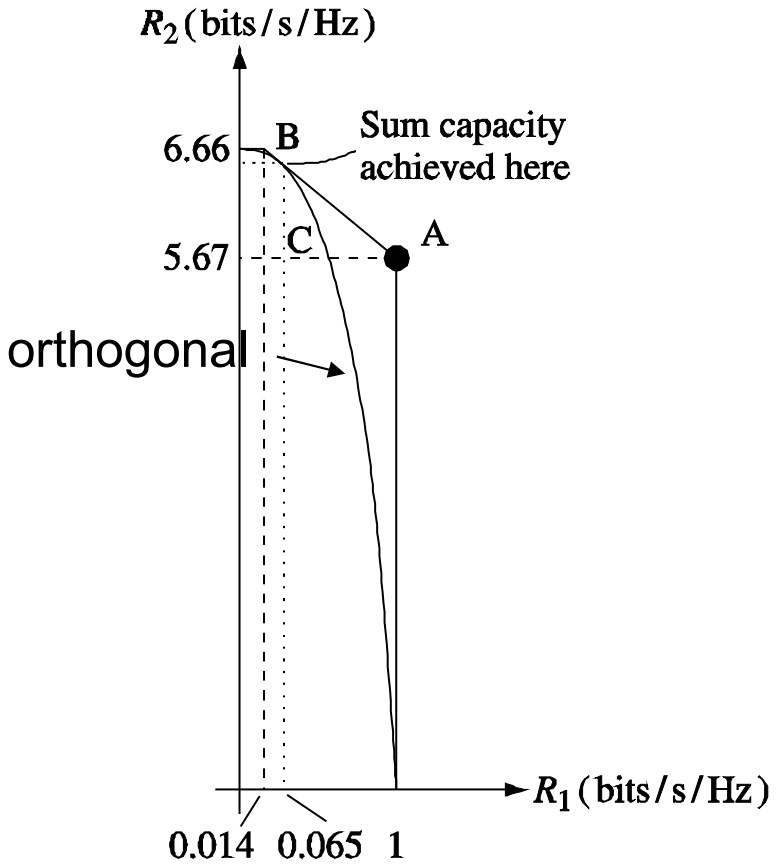
Conventional CDMA vs Capacity



20 dB power difference between 2 users

Successive cancellation allows the weak user to have a good rate without lowering the power of the strong user.

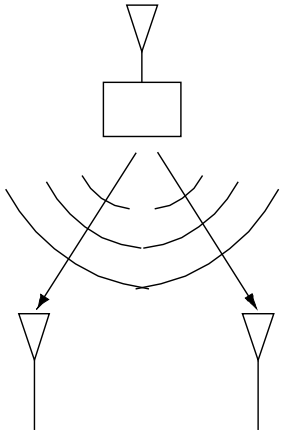
Orthogonal vs Capacity

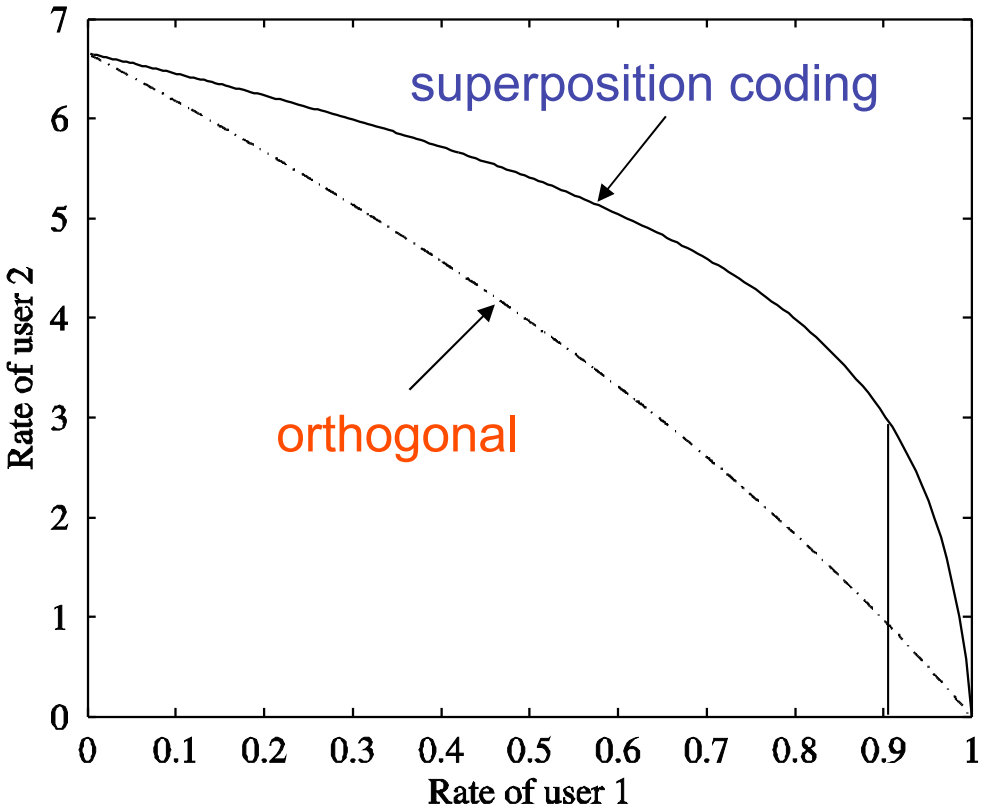


20 dB power difference between 2 users

Orthogonal achieves maximum throughput but may not be fair.

Downlink Capacity





20 dB gain difference between 2 users

Frequency-selective Channel

$$y[m] = \sum_{\ell} h_\ell x[m-\ell] + w[m]$$

h_l 's are time-invariant.

OFDM converts it into a *parallel channel*:

$$\tilde{y}_n = \tilde{h}_n \tilde{d}_n + \tilde{w}_n, \qquad n = 1, \ldots, N_c.$$

$$C_{N_c} = \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n^* |\tilde{h}_n|^2}{N_0}\right),$$

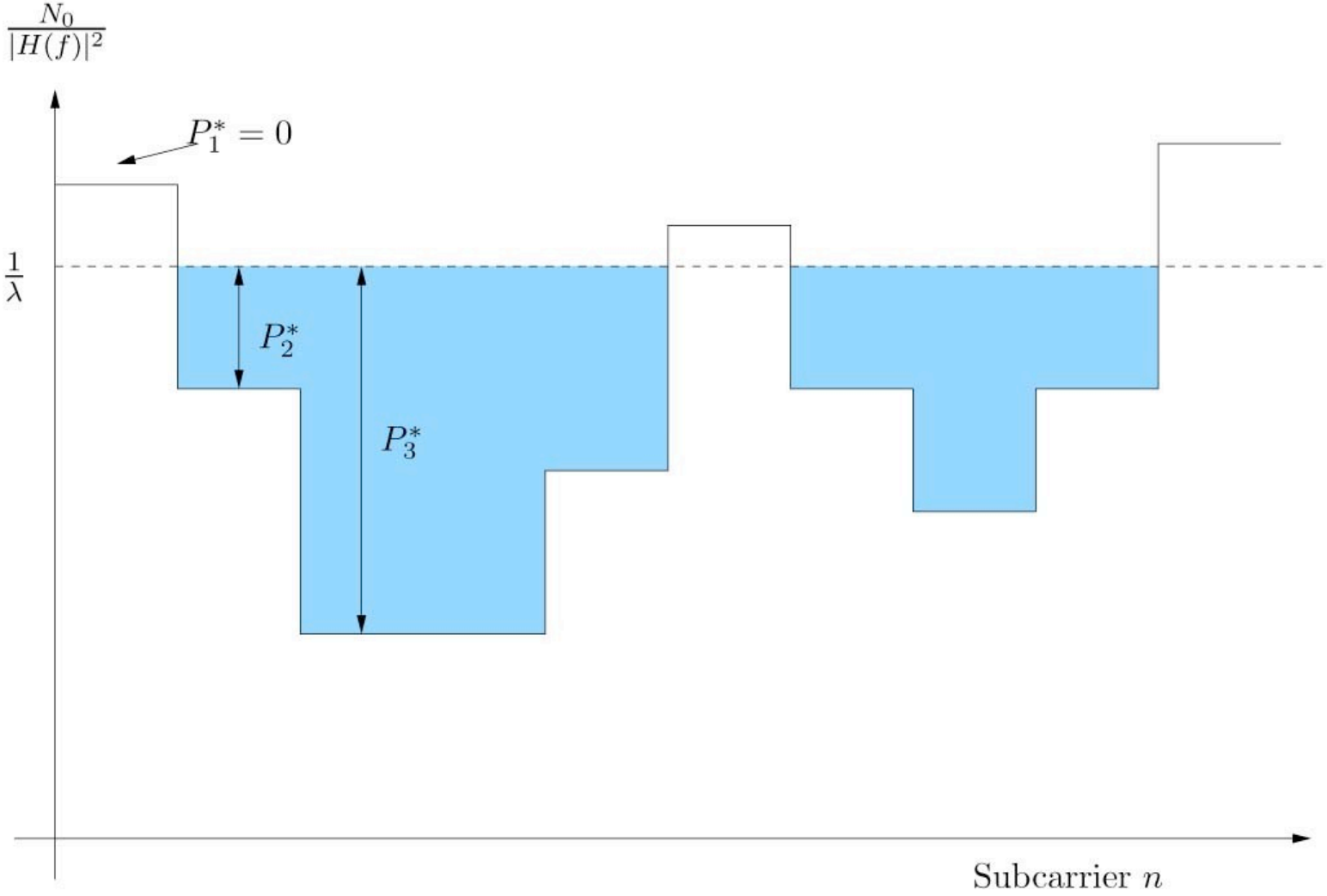
where P_n^* is the waterfilling allocation:

$$P_n^* = \left(\frac{1}{\lambda} - \frac{N_0}{|\tilde{h}_n|^2}\right)^+$$

with λ chosen to meet the power constraint.

Can be achieved with separate coding for each sub-carrier.

Waterfilling in Frequency Domain



Slow Fading Channel

$$y[m] = hx[m] + w[m]$$

h random.

There is no definite capacity.

Outage probability:

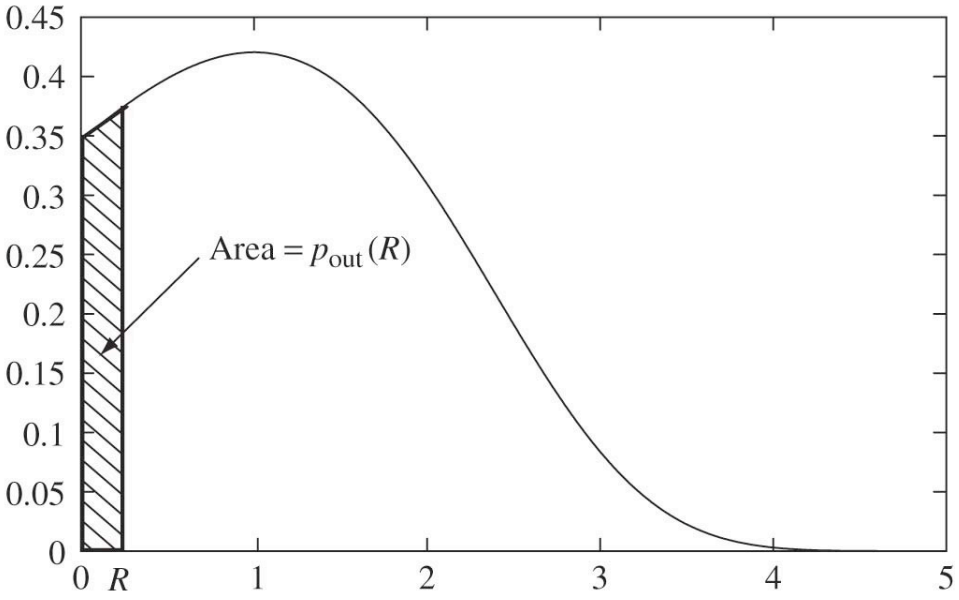
$$p_{\text{out}}(R) = \mathcal{P}\left\{\log(1 + |h|^2 \text{SNR}) < R\right\}$$

ϵ -outage capacity:

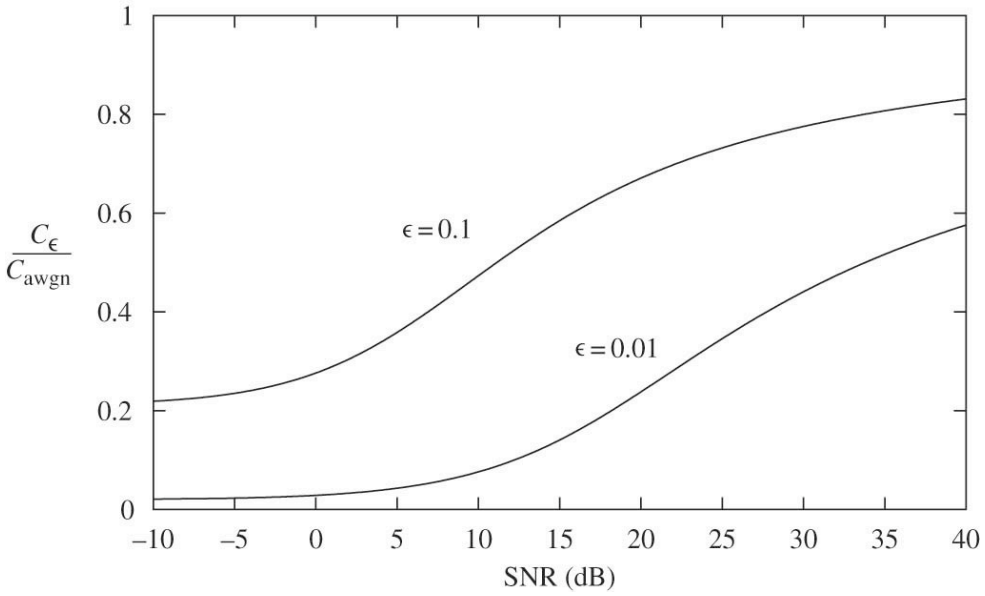
$$C_\epsilon = p_{\text{out}}^{-1}(\epsilon)$$

Outage for Rayleigh Channel

Pdf of log(1+|h|²SNR)



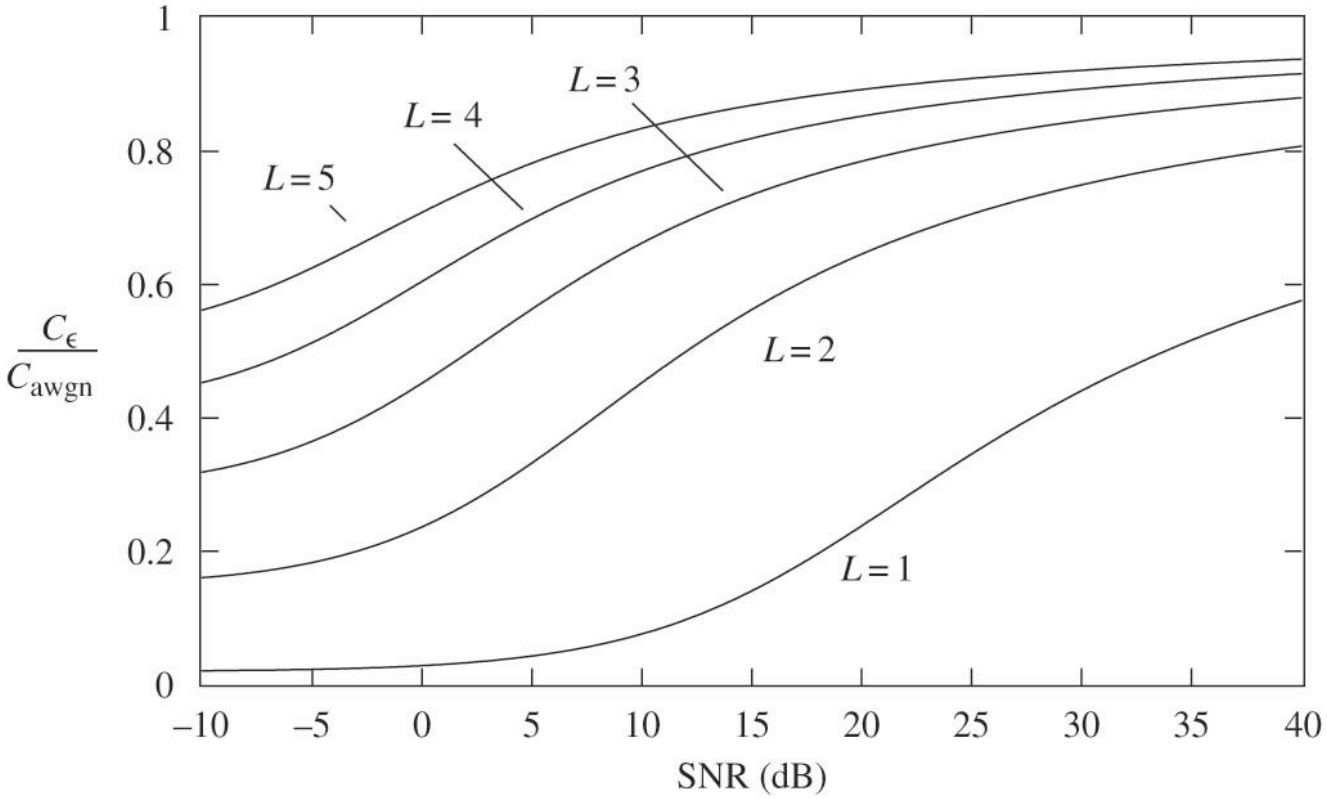
Outage cap. as fraction of AWGN cap.



$$p_{\text{out}}(R) \approx \frac{2^R - 1}{\text{SNR}}$$

Receive Diversity

$$p_{\text{out}}(R) = \mathcal{P}\left\{\log\left(1 + \|\mathbf{h}\|^2 \text{SNR}\right) < R\right\}$$



Diversity plus power gain.