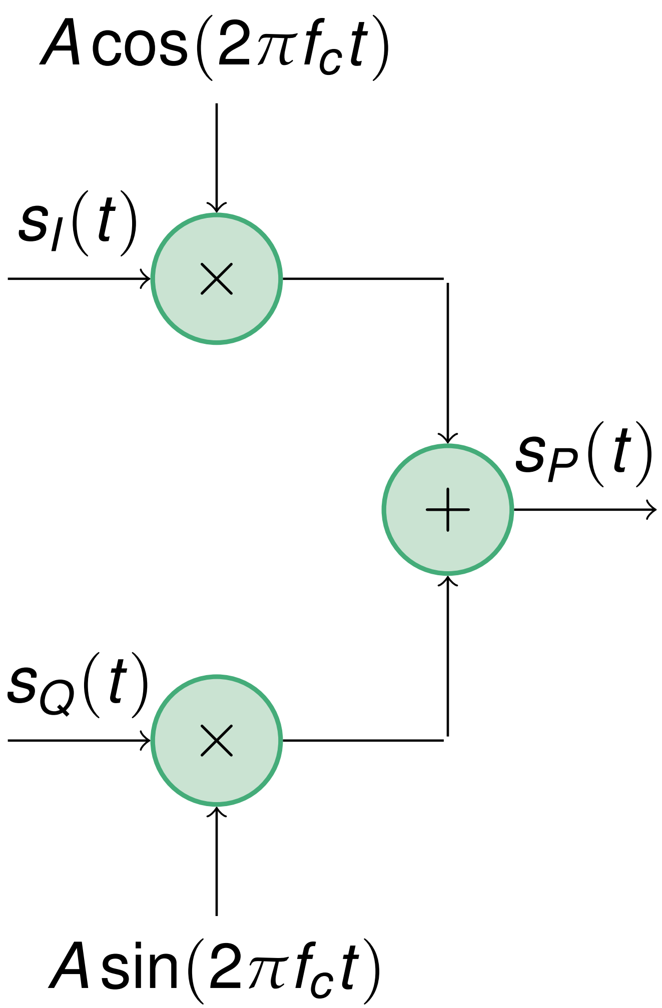
Passband Signals

- So far, all modulated signals we considered are baseband signals.
  - Baseband signals have frequency spectra concentrated near zero frequency.
- However, for wireless communications passband signals must be used.
  - Passband signals have frequency spectra concentrated around a carrier frequency f_c.
- Baseband signals can be converted to passband signals through up-conversion.
- Passband signals can be converted to baseband signals through down-conversion.

Up-Conversion



- The passband signal $s_P(t)$ is constructed from two (digitally modulated) baseband signals, $s_I(t)$ and $s_Q(t)$.
  - Note that two signals can be carried simultaneously!
  - This is a consequence of $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ being orthogonal.

Baseband Equivalent Signals

- The passband signal $s_P(t)$ can be written as

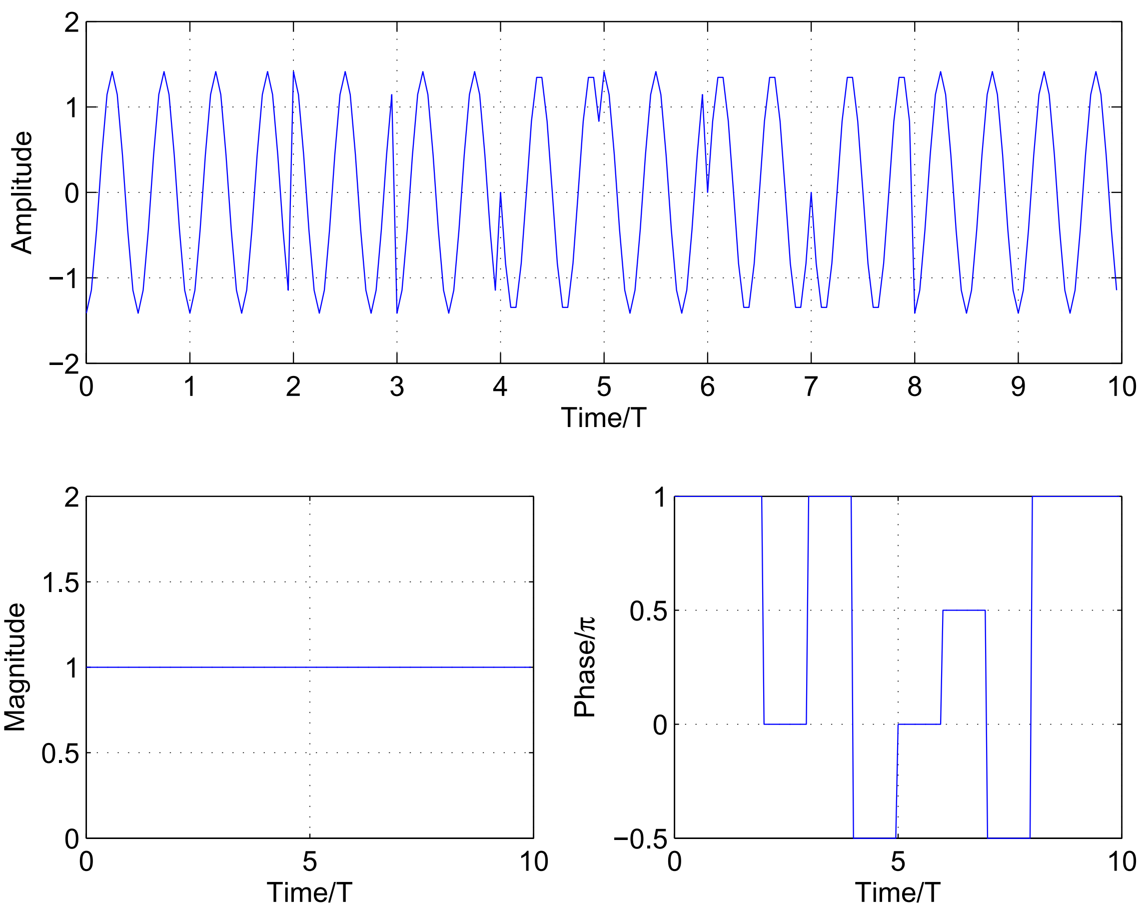
$$s_P(t) = \sqrt{2} \cdot A s_I(t) \cdot \cos(2\pi f_c t) + \sqrt{2} \cdot A s_Q(t) \cdot \sin(2\pi f_c t).$$

- If we define $s(t) = s_I(t) - j \cdot s_Q(t)$, then $s_P(t)$ can also be expressed as

$$s_P(t) = \sqrt{2} \cdot A \cdot \Re\{s(t) \cdot \exp(j2\pi f_c t)\}.$$

- The signal $s(t)$:
  - is called the baseband equivalent or the complex envelope of the passband signal $s_P(t)$.
  - It contains the same information as $s_P(t)$.
  - Note that $s(t)$ is complex-valued.

Illustration: QPSK with $f_c = 2/T$



- Passband signal (top): segments of sinusoids with different phases.
  - Phase changes occur at multiples of T.
- Baseband signal (bottom) is complex valued; magnitude and phase are plotted.
  - Magnitude is constant (rectangular pulses).

MATLAB Code for QPSK Illustration

Listing : plot_LinearModQPSK.m

```
%% Parameters:
fsT      = 20;
L        = 10;
fc       =  2;              % carrier frequency
Alphabet = [1, j, -j, -1];% QPSK
Priors   = 0.25*[1 1 1 1];
Pulse    = ones(1,fsT);     % rectangular pulse

%% symbols and Signal using our functions
Symbols    = RandomSymbols(10, Alphabet, Priors);
Signal     = LinearModulation(Symbols,Pulse,fsT);
%% passband signal
tt = (0 : length(Signal)-1 )/fsT;
Signal_PB = sqrt(2)*real( Signal .* exp(-j*2*pi*fc*tt) );
```

MATLAB Code for QPSK Illustration

Listing : plot_LinearModQPSK.m

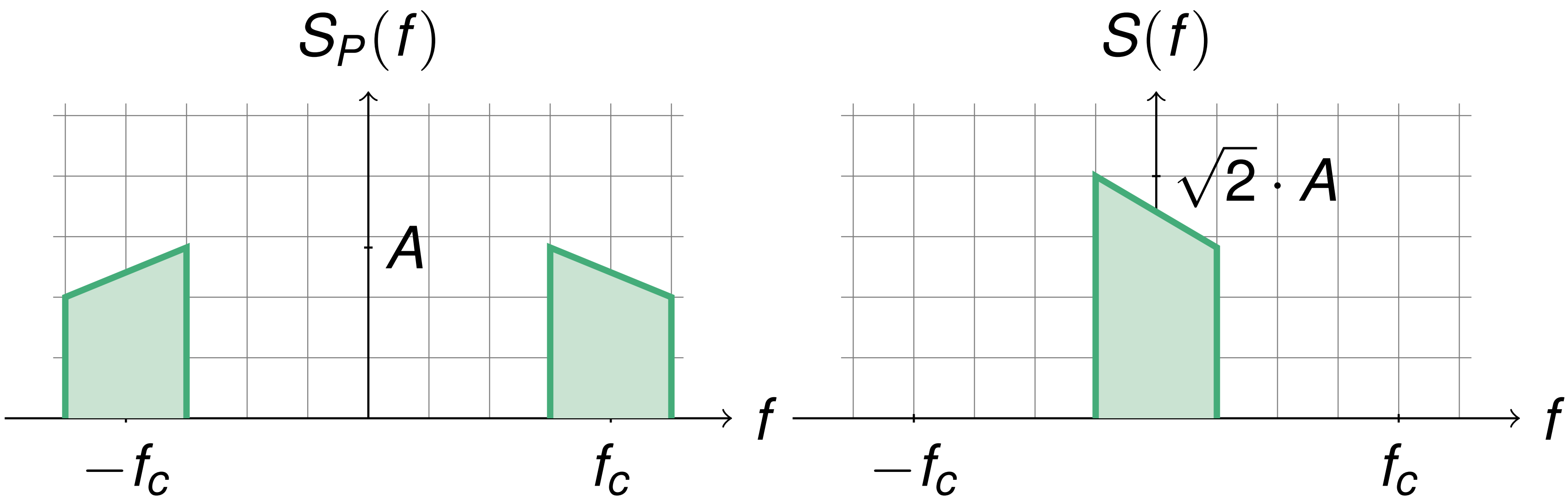
```
subplot(2,1,1)
plot( tt, Signal_PB )
grid
xlabel('Time/T')
ylabel('Amplitude')

subplot(2,2,3)
plot( tt, abs( Signal ) )
grid
xlabel('Time/T')
ylabel('Magnitude')

subplot(2,2,4)
plot( tt, angle( Signal )/pi )
grid
xlabel('Time/T')
ylabel('Phase/\pi')
```

Frequency Domain Perspective

- In the frequency domain:

$$S(f) = \begin{cases} \sqrt{2} \cdot S_P(f + f_c) & \text{for } f + f_c > 0 \\ 0 & \text{else.} \end{cases}$$

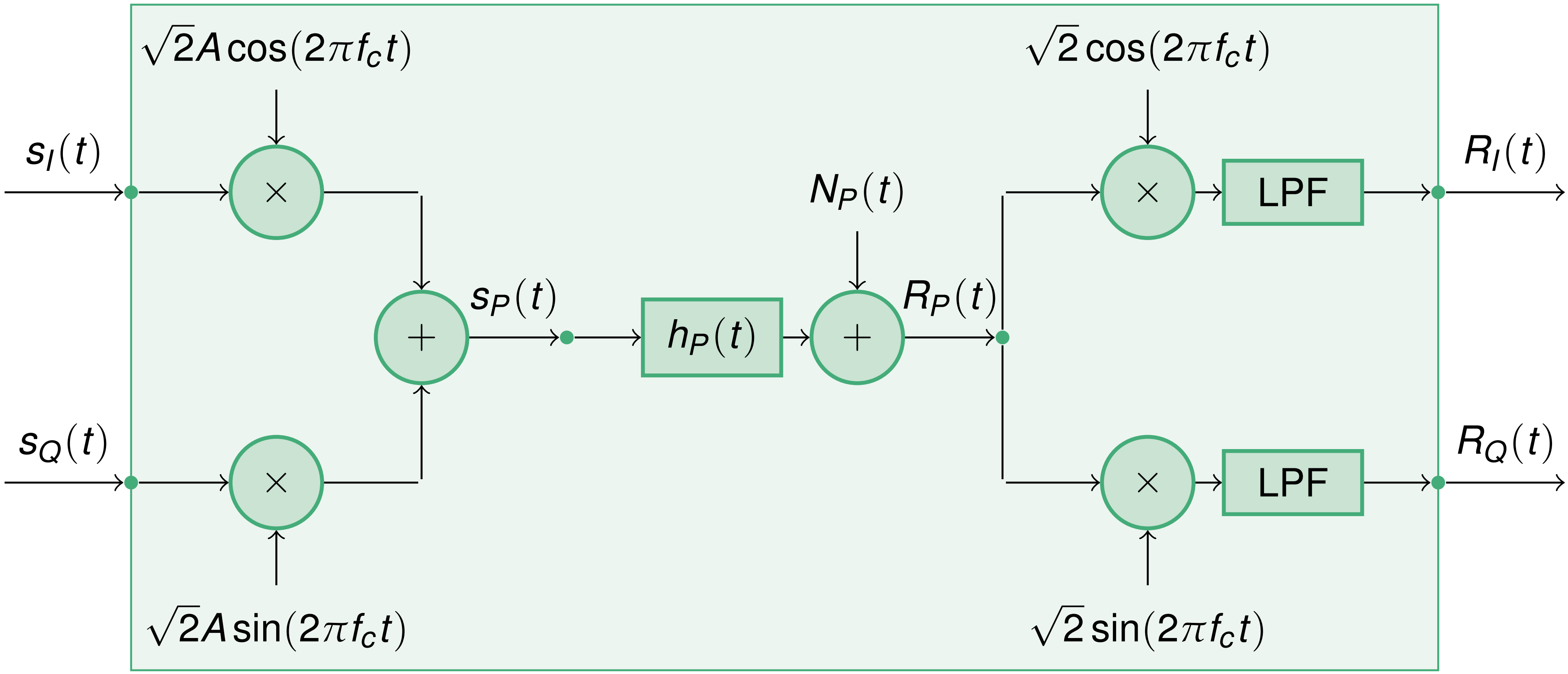
  - Factor $\sqrt{2}$ ensures both signals have the same power.



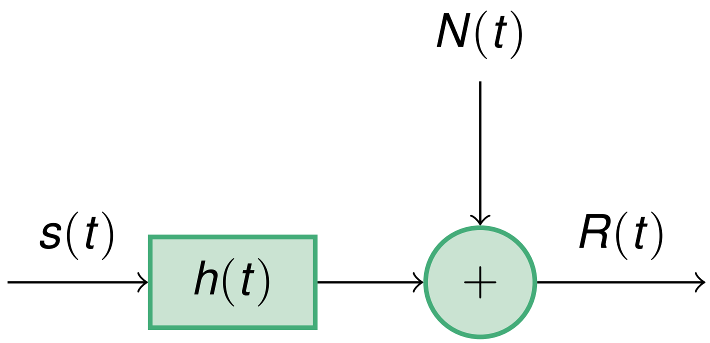
Baseband Equivalent System

- The baseband description of the transmitted signal is very convenient:
  - it is more compact than the passband signal as it does not include the carrier component,
  - while retaining all relevant information.
- However, we are also concerned what happens to the signal as it propagates to the receiver.
  - **Question:** Do baseband techniques extend to other parts of a passband communications system?

Passband System



Baseband Equivalent System



- The passband system can be interpreted as follows to yield an equivalent system that employs only baseband signals:
  - baseband equivalent transmitted signal: $s(t) = s_I(t) - j \cdot s_Q(t)$.
  - baseband equivalent channel with complex valued impulse response: $h(t)$.
  - baseband equivalent received signal: $R(t) = R_I(t) - j \cdot R_Q(t)$.
  - complex valued, additive Gaussian noise: $N(t)$

Baseband Equivalent Channel

- The baseband equivalent channel is defined by the entire shaded box in the block diagram for the passband system (excluding additive noise).
- The relationship between the passband and baseband equivalent channel is

$$h_P(t) = \Re\{h(t) \cdot \exp(j2\pi f_c t)\}$$

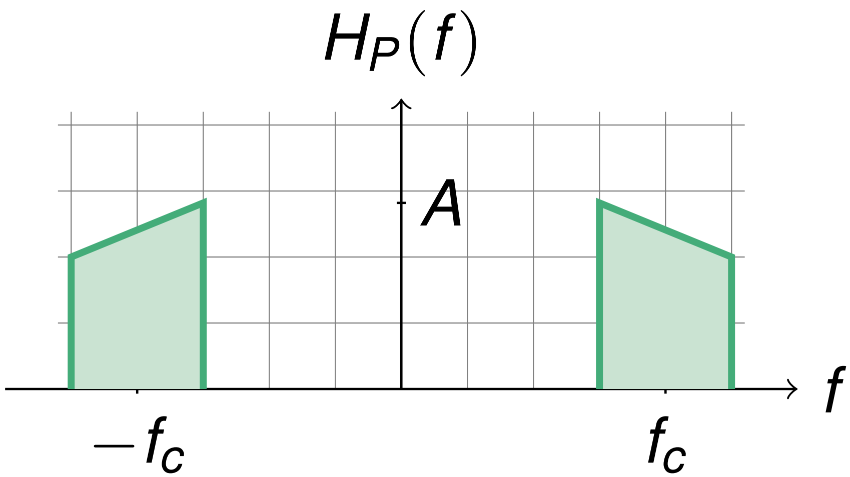
  in the time domain.
- **Example:**

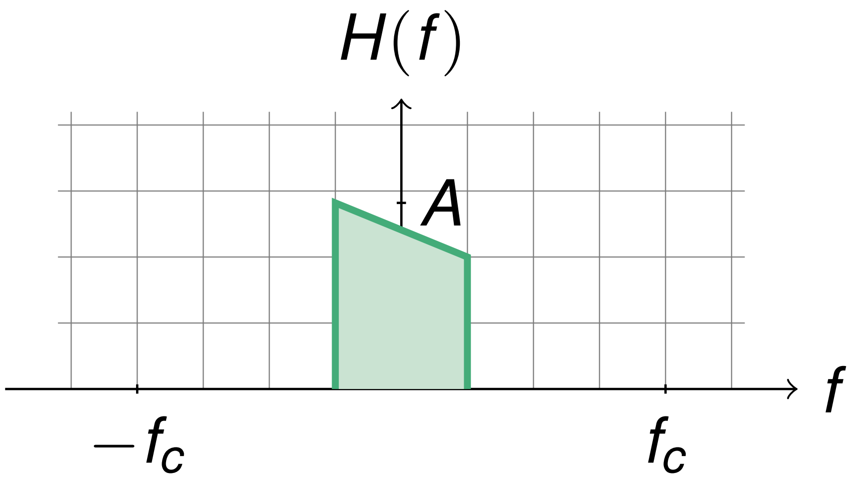
$$h_P(t) = \sum_k a_k \cdot \delta(t - \tau_k) \Longrightarrow h(t) = \sum_k a_k \cdot e^{-j2\pi f_c \tau_k} \cdot \delta(t - \tau_k).$$

Baseband Equivalent Channel

- In the frequency domain

$$H(f) = \begin{cases} H_P(f + f_c) & \text{for } f + f_c > 0 \\ 0 & \text{else.} \end{cases}$$





Summary

- The baseband equivalent channel is much simpler than the passband model.
  - Up and down conversion are eliminated.
  - Expressions for signals do not contain carrier terms.
- The baseband equivalent signals are easier to represent for simulation.
  - Since they are low-pass signals, they are easily sampled.
- No information is lost when using baseband equivalent signals, instead of passband signals.
- Standard, linear system equations hold:

$$R(t) = s(t) * h(t) + n(t) \text{ and } R(f) = S(f) \cdot H(f) + N(f).$$

- **Conclusion:** Use baseband equivalent signals and systems.