Path Loss

- Path loss L_P relates the received signal power P_r to the transmitted signal power P_t:

$$P_r = P_t \cdot \frac{G_r \cdot G_t}{L_P},$$

where G_t and G_r are antenna gains.

- Path loss is very important for cell and frequency planning or range predictions.
  - Not needed when designing signal sets, receiver, etc.

Received Signal Power

- Received Signal Power:

$$P_r = P_t \cdot \frac{G_r \cdot G_t}{L_P \cdot L_R},$$

where L_R is implementation loss, typically 2–3 dB.

Noise Power

- (Thermal) Noise Power:

$$P_N = kT_0 \cdot B_W \cdot F, \text{ where}$$

  - k — Boltzmann's constant ($1.38 \cdot 10^{-23}$ Ws/K),
  - T_0 — temperature in K (typical room temperature, $T_0 = 290$ K),
  - $\Rightarrow kT_0 = 4 \cdot 10^{-21}$ W/Hz $= 4 \cdot 10^{-18}$ mW/Hz $=$ -174 dBm/Hz,
- B_W — signal bandwidth,
- F — noise figure, figure of merit for receiver (typical value: 5dB).

Signal-to-Noise Ratio

- The ratio of received signal power and noise power is denoted by SNR.
- From the above, SNR equals:

$$\text{SNR} = \frac{P_r}{P_N} = \frac{P_t G_r \cdot G_t}{kT_0 \cdot B_W \cdot F \cdot L_P \cdot L_R}.$$

  - SNR increases with transmitted power P_t and antenna gains.
  - SNR decreases with bandwidth B_W, noise figure F, and path loss L_P.

E_s / N_0

- For the symbol error rate performance of communications system the ratio of signal energy E_s and noise power spectral density N_0 is more relevant than SNR.
- Since $E_s = P_r \cdot T_s = \frac{P_r}{R_s}$ and $N_0 = kT_0 \cdot F = P_N / B_W$, it follows that
$$\frac{E_s}{N_0} = \text{SNR} \cdot \frac{B_W}{R_s},$$
where T_s and R_s denote the symbol period and symbol rate, respectively.
- The ratio $\frac{R_s}{B_W}$ is called the bandwidth efficiency; it is a property of the signaling scheme.

E_s / N_0

- Thus, E_s / N_0 is given by:

$$\frac{E_s}{N_0} = \frac{P_t G_r \cdot G_t}{kT_0 \cdot R_s \cdot F \cdot L_P \cdot L_R}.$$

- in dB:

$$\begin{aligned} \left(\frac{E_s}{N_0}\right)_{(dB)} = \quad & P_{t(dBm)} + G_{t(dB)} + G_{r(dB)} \\ & -(kT_0)_{(dBm/Hz)} - R_{s(dBHz)} - F_{(dB)} - L_{R(dB)}. \end{aligned}$$

Receiver Sensitivity

- All receiver-related terms are combined into *receiver sensitivity*, S_R:

$$S_R = \frac{E_s}{N_0} \cdot kT_0 \cdot R_s \cdot F \cdot L_R.$$

- in dB:

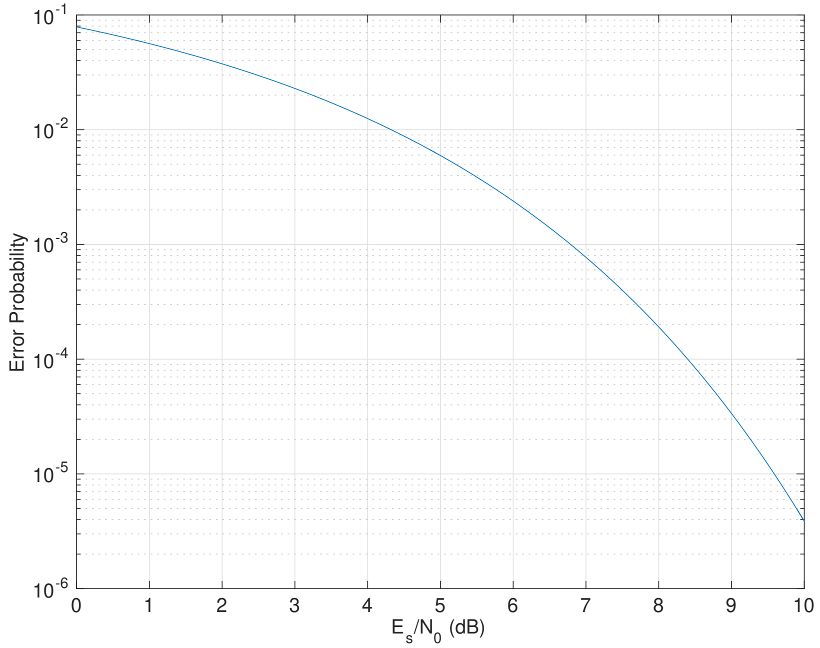
$$\begin{aligned} S_{R(dBm)} = \; & (\frac{E_s}{N_0})_{(dB)} \\ & + (kT_0)_{(dBm/Hz)} + R_{s(dBHz)} + F_{(dB)} + L_{R(dB)}. \end{aligned}$$

- Receiver sensitivity indicates the minimum required received power to close the link.

Exercise: Receiver Sensitivity

Find the sensitivity of a receiver with the following specifications:

- Modulation: BPSK
- bit error rate: 10^{-4}
- data rate: $R_s = 1\,\text{Mb/s}$
- noise figure: $F = 5\,\text{dB}$
- receiver loss: $L_R = 3\,\text{dB}$



Bit error probability for BPSK in AWGN

Exercise: Maximum Permissible Pathloss

- A communication system has the following specifications:
  - Transmit power: $P_t = 1\,\text{W}$
  - Antenna gains: $G_t = 3\,\text{dB}$ and $G_R = 0\,\text{dB}$
  - Receiver sensitivity: $S_R = -98\,\text{dBm}$
- What is the maximum pathloss that this system can tolerate?

Path Loss

- Path loss modeling may be "more an art than a science."
  - Typical approach: fit model to empirical data.
  - Parameters of model:
    - d - distance between transmitter and receiver,
    - f_c - carrier frequency,
    - h_b, h_m - antenna heights,
    - Terrain type, building density, ...
- Examples that admit closed form expression: free space propagation, two-ray model

Example: Free Space Propagation

- In free space, path loss L_P is given by Friis's formula:

$$L_P = \left(\frac{4\pi d}{\lambda_c}\right)^2 = \left(\frac{4\pi f_c d}{c}\right)^2.$$

  - Path loss increases proportional to the square of distance d and frequency f_c.

- In dB:

$$L_{P(dB)} = -20\log_{10}(\frac{c}{4\pi}) + 20\log_{10}(f_c) + 20\log_{10}(d).$$

  - Example: $f_c = 1\,\text{GHz}$ and $d = 1\,\text{km}$

$$L_{P(dB)} = -146\,\text{dB} + 180\,\text{dB} + 60\,\text{dB} = 94\,\text{dB}.$$

Example: Two-Ray Channel

- Antenna heights: h_b and h_m.
- Two propagation paths:
  1. direct path, free space propagation,
  2. reflected path, free space with perfect reflection.
- Depending on distance d, the signals received along the two paths will add constructively or destructively.

Example: Two-Ray Channel

- For the two-ray channel, path loss is approximately:

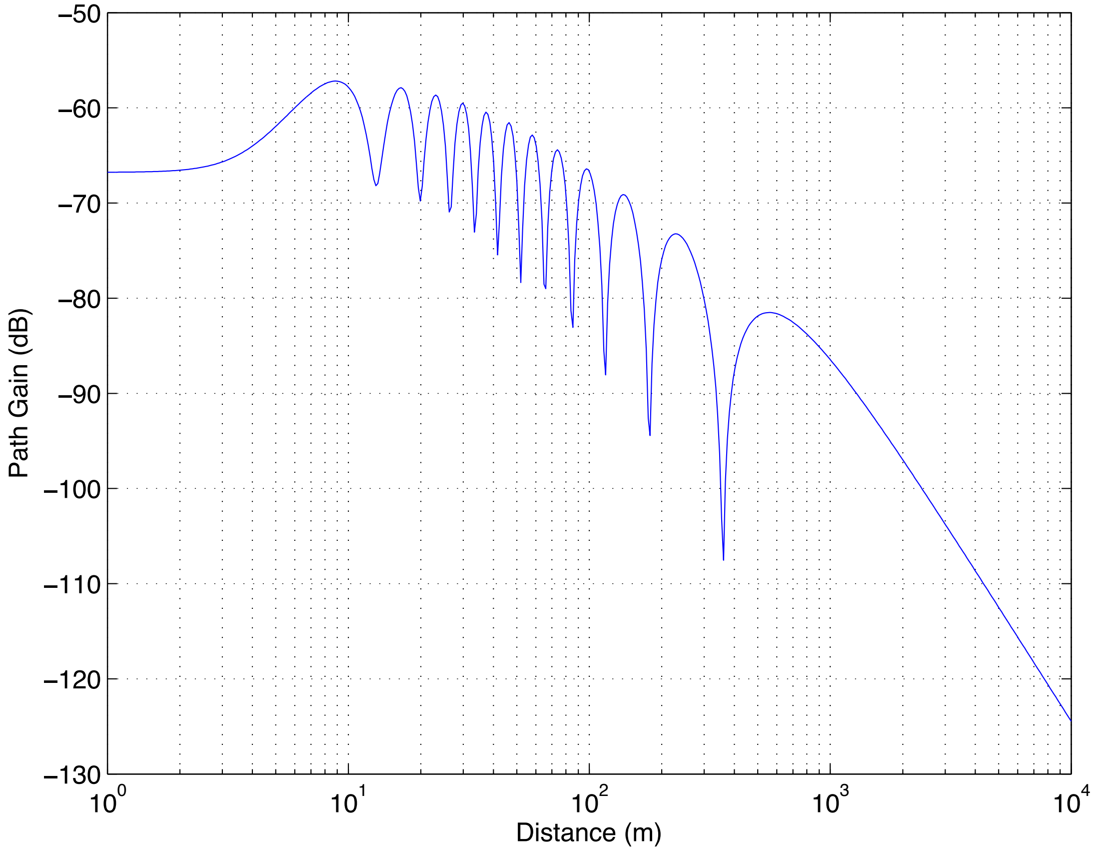
$$L_P = \frac{1}{4} \cdot \left(\frac{4\pi f_c d}{c}\right)^2 \cdot \left(\frac{1}{\sin(\frac{2\pi f_c h_b h_m}{cd})}\right)^2.$$

- For $\lambda d \gg h_b h_m$, path loss is further approximated by:

$$L_P \approx \left(\frac{d^2}{h_b h_m}\right)^2$$

  - Path loss proportional to d^4 is typical for urban environment.

Example: Two-Ray Channel



Exercise: Maximum Communications range

- Path loss models allow translating between path loss P_L and range d.
- A communication system can tolerate a maximum path loss of 131 dB.
- What is the maximum distance between transmitter and receiver if path loss is according to the free-space model.
- How does your answer change when path loss is modeled by the two-ray model and $h_m = 1\,\text{m}$, $h_b = 10\,\text{m}$.

Okumura-Hata Model for Urban Area

- Okumura and Hata derived empirical path loss models from extensive path loss measurements.
  - Models differ between urban, suburban, and open areas, large, medium, and small cities, etc.
- Illustrative example: Model for Urban area (small or medium city)

$$L_{P(dB)} = A + B \log_{10}(d),$$

where

$$\begin{aligned} A &= 69.55 + 26.16 \log_{10}(f_c) - 13.82 \log_{10}(h_b) - a(h_m) \\ B &= 44.9 - 6.55 \log_{10}(h_b) \\ a(h_m) &= (1.1 \log_{10}(f_c) - 0.7) \cdot h_m - (1.56 \log_{10}(f_c) - 0.8) \end{aligned}$$

Simplified Model

- Often a simpler path loss model that emphasizes the dependence on distance suffices.
- Simplified path loss model:

$$L_P = K \cdot \left(\frac{d}{d_0}\right)^{\gamma}$$

in dB:

$$L_{P(dB)} = 10\log_{10}(K) + 10\gamma\log_{10}(\frac{d}{d_0}).$$

  - Frequency dependence, antenna gains, and geometry are absorbed in K.
  - d_0 is a reference distance, typically 10m - 100m; model is valid only for $d > d_0$.
  - Path loss exponent γ is usually between 3 and 5.
- Model is easy to calibrate from measurements.

Shadowing

- Shadowing or shadow fading describes random fluctuations of the path loss.
  - due to small scale propagation effects, e.g., blockage from small obstructions.
- Path loss becomes a random variable Ψ_{dB}.
- Commonly used model: log-normal shadowing; path loss Ψ_{dB} in dB is modeled as a Gaussian random variable with:
  - mean: $P_{L(dB)}(d)$ - deterministic part of path loss
  - standard deviation: σ_Ψ - describes variation around $P_{L(dB)}$; common value 4dB – 10dB.
- When fitting measurements to an empirical model, σ_Ψ captures the model error (residuals).

Outage Probability

- As discussed earlier, the received power must exceed a minimum level P_{min} so that communications is possible; we called that level the receiver sensitivity S_R.
- Since path loss Ψ_{dB} is random, it cannot be guaranteed that a link covering distance d can be closed.
- The probability that the received power $P_{r(dB)}(d)$ falls below the required minimum is given by:

$$\Pr(P_{r(dB)}(d) \leq S_R) = Q(\frac{P_t + G_t + G_R - P_{L(dB)}(d) - S_R}{\sigma_\Psi}).$$

- The quantitity $P_t + G_t + G_R - P_{L(dB)}(d) - S_R$ is called the fade margin.

Exercise: Outage Probability

- Assume that a communication system is characterized by:
  - Transmit power: $P_t = 1\,\text{W}$
  - Antenna gains: $G_t = 3\,\text{dB}$ and $G_R = 0\,\text{dB}$
  - Receiver sensitivity: $S_R = -98\,\text{dBm}$
  - Path loss according to the two-ray model with $h_m = 1\,\text{m}$, $h_b = 10\,\text{m}$.
  - Communications range: $d = 1\,\text{km}$

  Querstion: What is the outage probability of the system when the shadowing standard deviation $\sigma_\Psi = 6\,\text{dB}$?
- **Question:** For a channel with $\sigma_\Psi = 6\,\text{dB}$, how much fade margin is required to achieve an outage probability of 10^{-3}?

Cell Coverage Area

- Expected percentage of cell area where received power is above S_R.
- For a circular cell of radius R, cell coverage area is computed as:

$$C = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^{R} Q\left(\frac{S_R - (P_t - P_{L(dB)}(r))}{\sigma_\Psi}\right) dr d\theta.$$

Cell Coverage Area

- For the simplified (range only) path loss model $L_P = K \cdot \left(\frac{d}{d_0}\right)^{\gamma}$ this can be computed in closed form:

$$C = Q(a) + \exp\left(\frac{2 - 2ab}{b^2}\right) \cdot Q\left(\frac{2 - ab}{b}\right)$$

where:

$$a = \frac{S_R - (P_t - 10\log_{10}(K) - 10\gamma\log_{10}(R/d_0))}{\sigma_\Psi}$$

and

$$b = \frac{10\gamma\log_{10}(e)}{\sigma_\Psi}.$$