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Gaussian Random Processes

Def: A signal $X(t)$ is called a Gaussian random process if the pdf is Gaussian for any sample $X(t_0)$ at any time-instant t_0.

White Gaussian Noise (model for noise in system)

A signal $N(t)$ is called white Gaussian noise (WGN) if:

i) $N(t)$ is a Gaussian random process

ii) $E[N(t)] = 0$

iii) Autocorrelation function

$$R_N(\tau) = E[N(t) \cdot N(t+\tau)]$$
$$= \sigma^2 \cdot \delta(\tau)$$

i.e., two distinct samples are not correlated

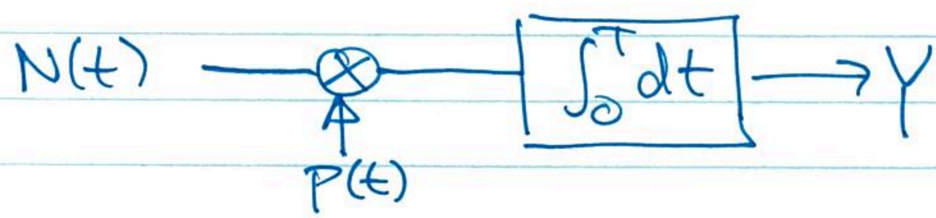
PSD of WGN:

- The PSD of a random process is the Fourier transform of the autocorrelation function

$$S_N(f) \longleftrightarrow R_N(\tau)$$

$$\Rightarrow S_N(f) = \sigma^2 \text{ for all } f$$

Integrating White Noise



Y must be a Gaussian random variable.

- linear transformation of Gaussian input

$\Rightarrow$ find mean & variance

Mean:

$$E[Y] = E\left[\int_0^T N(t)\cdot p(t)\,dt\right]$$

$E[a\cdot X] = a\cdot E[X]$

$$= \int_0^T \overset{=0}{E[N(t)]}\cdot p(t)\,dt = 0$$

Variance: since $E[Y]=0 \Rightarrow Var[Y] = E[Y^2]$

$$E[Y^2] = E\left[\left(\int_0^T N(t)\cdot p(t)\,dt\right)^2\right]$$

$$= E\left[\int_0^T N(t)\,p(t)\,dt \cdot \int_0^T N(u)\,p(u)\,du\right]$$

$$= \int_0^T\int_0^T \underbrace{E[N(t)\cdot N(u)]}_{=\sigma^2\cdot\delta(t-u)}\,p(t)\cdot p(u)\,dt\,du$$