10/17/18

Spectra of digitally modulated signals

Recall:

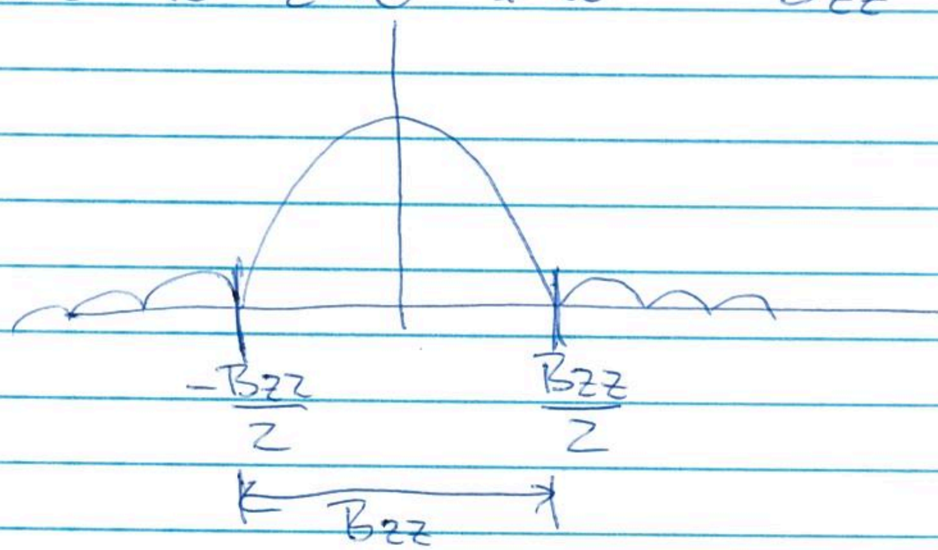
The power spectral density of a linearly, digitally modulated signal

$$S_m(f) = \frac{\sigma_b^2}{T} \cdot |P(f)|^2$$

But: The "width" of $|P(f)|^2$ is approximately equal to $\frac{2}{T}$

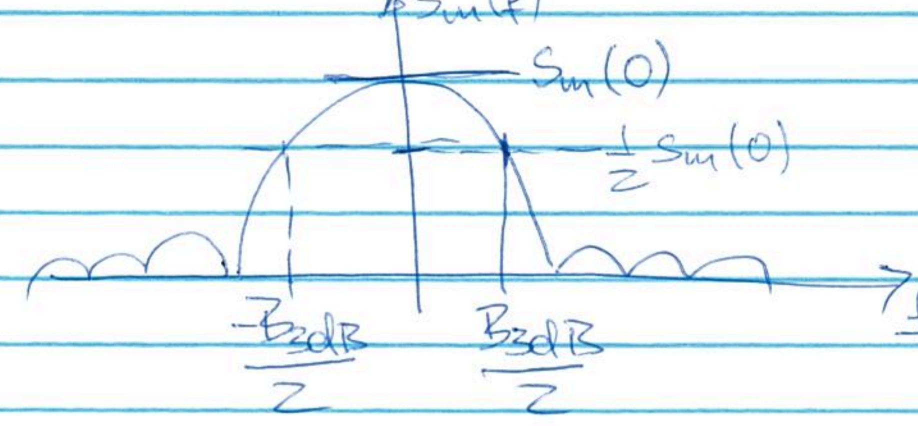
Measures of bandwidth

1. zero-to-zero BW B_{zz}



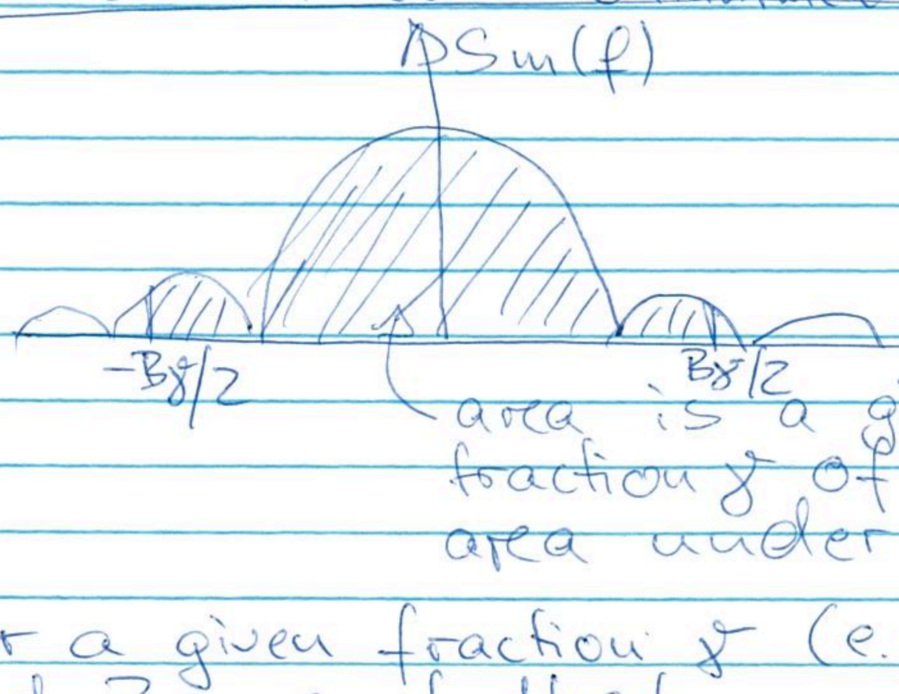
- Easy to measure
- Does not consider side-lobes
- Not all pulses have zeros in spectrum

2. 3dB - bandwidth



- also easy to measure
- also does not consider sidelobes

3. Fractional Power Containment BW



area is a given fraction γ of entire area under $S_m(f)$

Def: For a given fraction γ (e.g. $\gamma = 0.99$) find B_γ such that

$$\int_{-\frac{B_\gamma}{2}}^{B_\gamma/2} S_m(f)\,df = \gamma \cdot \int_{-\infty}^{\infty} S_m(f)\,df$$

	B_{ZZ}	B_{3dB}	$B_{0.99}$	$B_{0.98}$
Rect. Pulse	$2/T$	$0.88/T$	$1.7/T$	$1.55/T$
Half-sine Pulse	$3/T$	$1.20/T$	$20.4/T$	$2.4/T$

real problem

Partial Response Signaling

So far, considered pulses of duration T

$\longrightarrow$ full-response signaling

Good: power associated with each symbol is confined to a single symbol period T

$\Longrightarrow$ no interference b/w symbol

Example: Half-sine pulse

Not so good: Limited pulse duration implies infinite bandwidth

Idea: consider pulses that are wider than a symbol period.

Goal: truly band-limited PSD

$\rightarrow$ partial-response signaling

Example: Gaussian pulse:

$$p(t) = \exp\left(-\frac{1}{2}\left(\frac{t}{\sigma \cdot T}\right)^2\right)$$

Good: PSD is very narrow:

99% containment BW $\approx \frac{0.58}{\sigma \cdot T}$

BAD: power associated with a given symbol ($b_n \cdot p(t-nT)$) "spills" into adjacent symbol periods

$\Rightarrow$ signal does not go through symbols any more

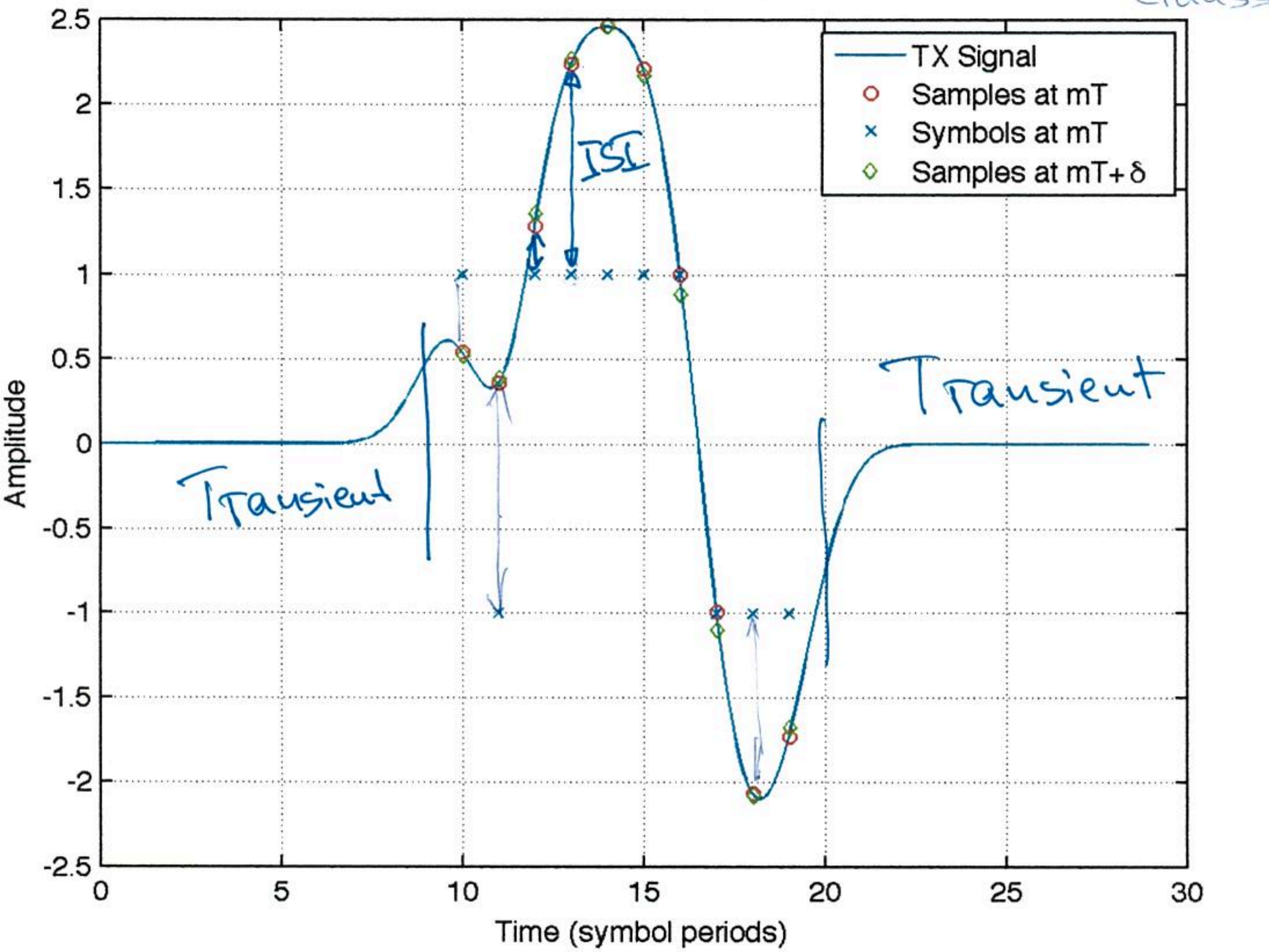
This is called intersymbol interference (ISI)

VERY BAD

Gaussian Pulse: $\sigma = 1$

$\Rightarrow$ severe ISI

Gauss



Question: Is it possible to simultaneously

(1) use long pulses with good (finite?) BW

(2) avoid ISI

Recall:

$$s_m(t) = \sum_n b_m \cdot p(t - nT)$$

Want: for no ISI

$$s_m(mT) = b_m$$

no ISI at "sampling" instants $t = mT$

$$\Rightarrow s_m(mT) = \sum_n b_n \cdot p(mT - nT)$$

$$= \sum_n b_n \cdot p((m-n)\cdot T) \overset{!}{=} b_m$$

$$\Rightarrow \text{We need: } \quad p(mT) = \begin{cases} 1 & \text{for } m = 0 \\ 0 & \text{for } m \neq 0 \end{cases}$$

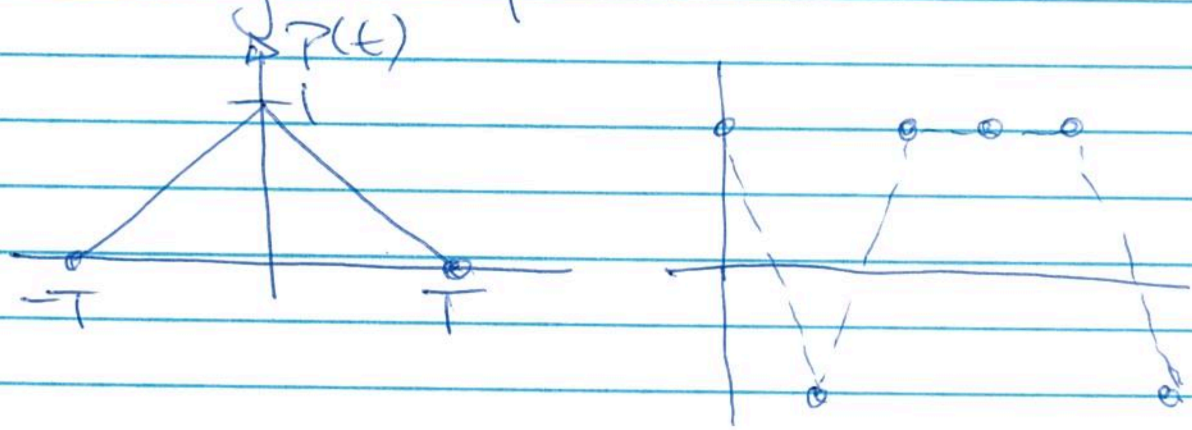
Nyquist criterion
(Time-domain)

Equivalent condition in frequency domain:

$$\frac{1}{T}\sum_{k} P\left(f+\frac{k}{T}\right) = 1$$

Examples:

- Triangular pulse:



- sinc-pulse $p(t) = \mathrm{sinc}(t/T)$