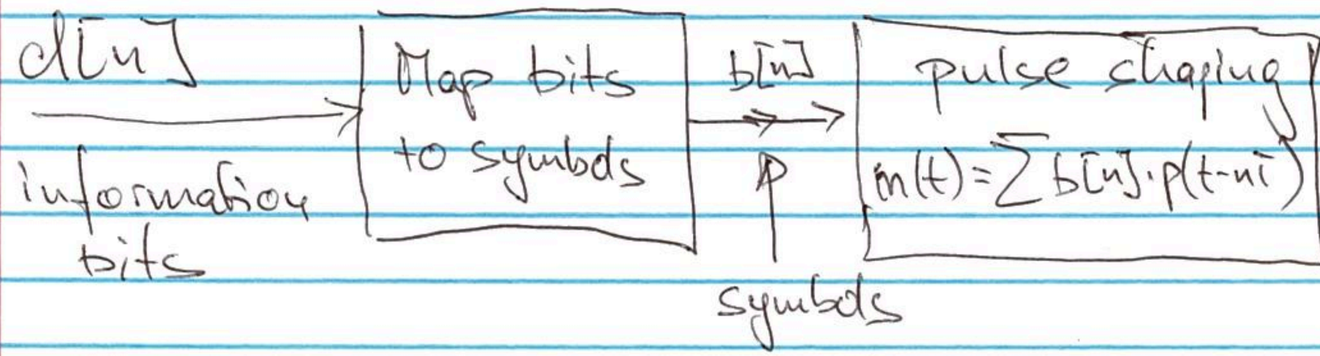
10/15/18

Digital Modulation

Recall from last time:



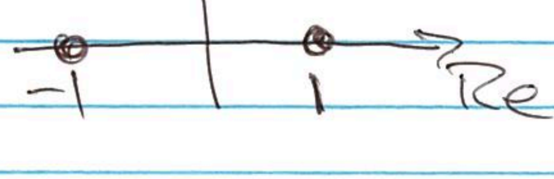
This is called linear, digital modulation

Signal Constellations

The symbols that constitute the elements ~~of a~~ b[n] of a linearly modulated signal are called the signal constellation

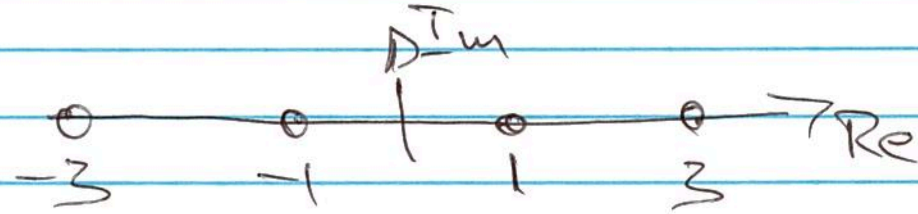
- signal constellations are often shown as points in the complex plane

BPSK



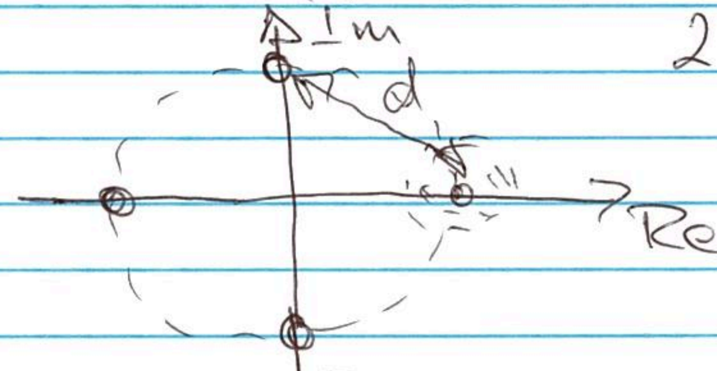
4-PAM:

2 bits per T



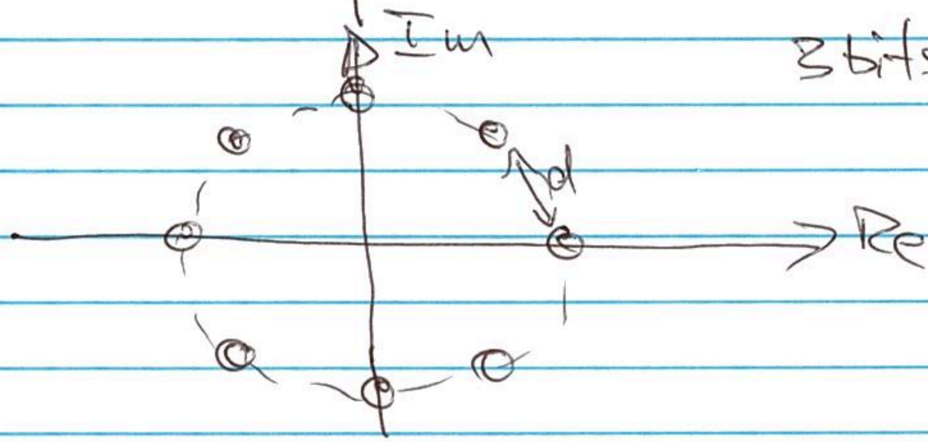
QPSK:

2 bits per T



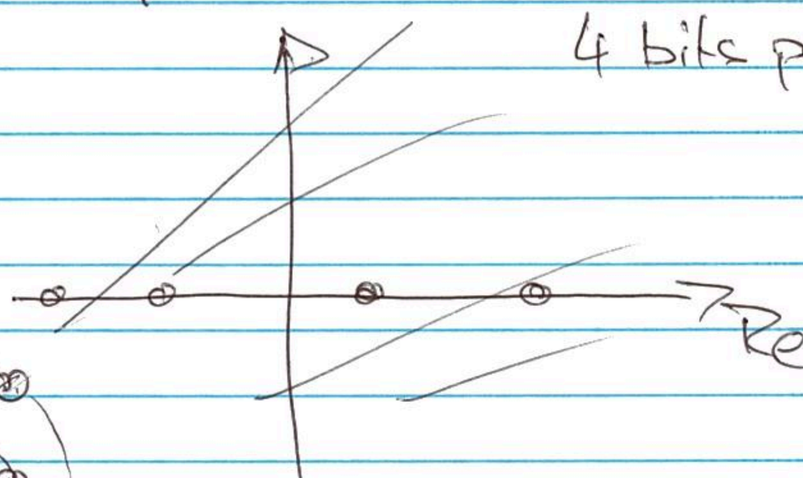
8PSK

3 bits per T

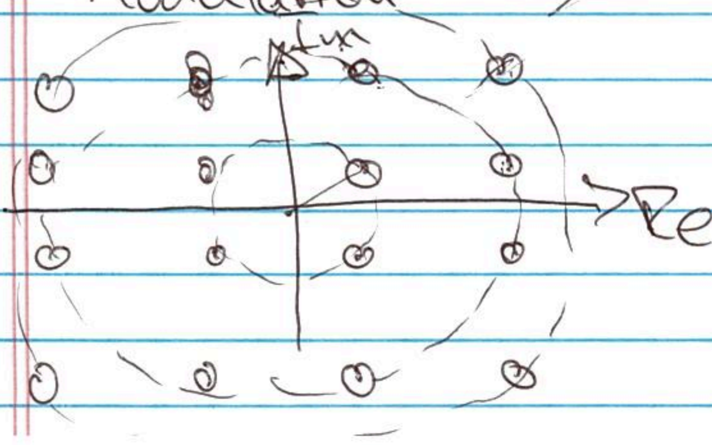


16 QAM

4 bits per T



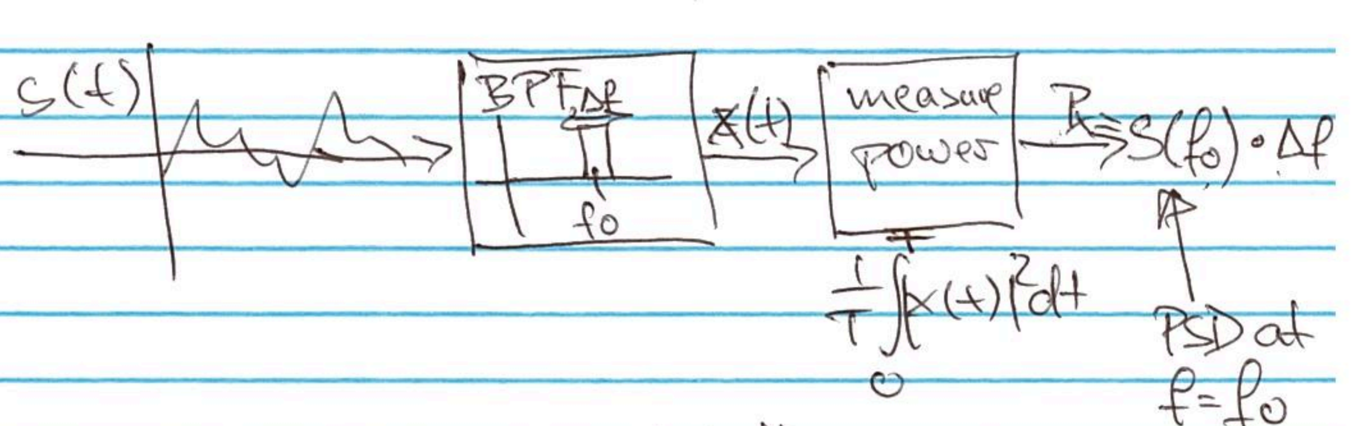
Quadrature
Amplitude
Modulation



Bandwidth of digitally modulated signals

Bandwidth of "random" signal measured by the power-spectral density (PSD)

Think about PSD like this:



- PSD has units $\frac{\text{Watts}}{\text{Hertz}}$
- PSD measures distribution of power over frequency

more formally:

$$\text{let } x_{T_0}(t) = s(t) \cdot I_{[0, T_0]}(t)$$

Fourier transform:

$$X_{T_0}(f) = \mathcal{F}\{x_{T_0}(t)\}$$

Fourier transform

PSD: $S_x(f) = \lim_{T_0 \to \infty} \frac{|X_{T_0}(f)|^2}{T_0}$

PSD of linearly modulated signals

$$m(t) = \sum b[n] \cdot p(t-nT)$$ (complex envelope)

where $p(t)$ is pulse shape

Result:

The PSD of a linearly modulated signal is given by:

$$S_m(f) = \frac{\sigma_b^2}{T} \cdot |P(f)|^2$$

where

- $P(f)$ is the Fourier transform of pulse $p(t)$
- T is the symbol period
- σ_b^2 is the average symbol power

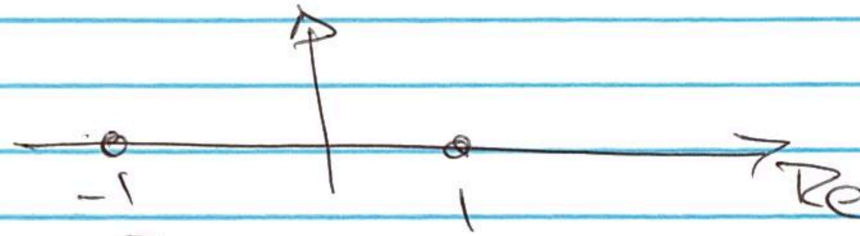
$$\sigma_b = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} |b[n]|^2$$

- this depends on constellation.

Examples:

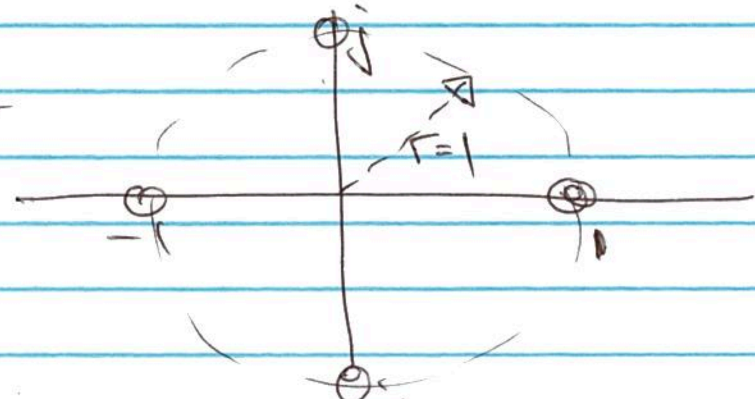
- all examples assume $b[n]$ are uncorrelated
- mean $b[n] = 0$

BPSK:



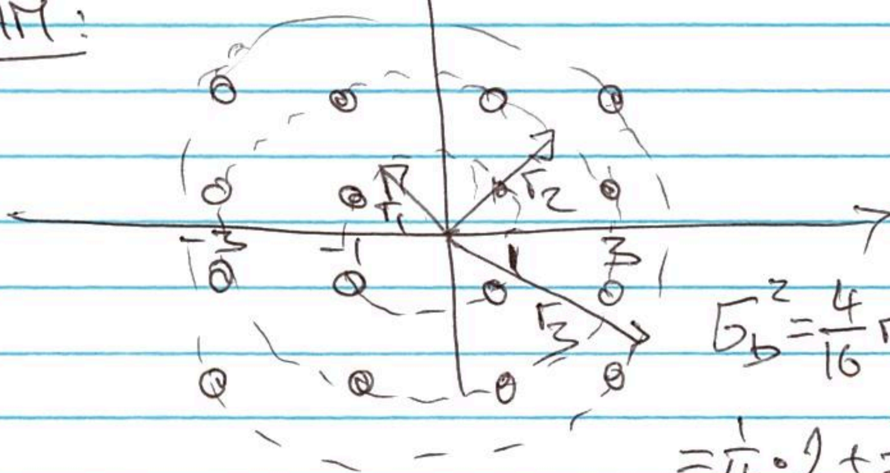
$$\sigma_b^2 = \frac{1}{2} \cdot 1^2 + \frac{1}{2} \cdot (-1)^2 = 1$$

QPSK:



$$\sigma_b^2 = \frac{1}{4} \cdot \left[1^2 + |j|^2 + (-1)^2 + |-j|^2 \right] = 1$$

16QAM:



$r_1 = \sqrt{2}$

$r_2 = \sqrt{10}$

$r_3 = \sqrt{18}$

$$\sigma_b^2 = \frac{4}{16} r_1^2 + \frac{8}{16} r_2^2 + \frac{4}{16} r_3^2$$

$$= \frac{1}{4} \cdot 2 + \frac{1}{2} \cdot 10 + \frac{1}{4} \cdot 18$$

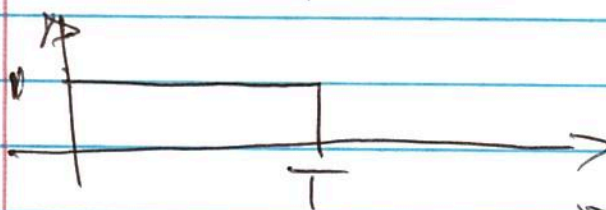
$$= 10$$

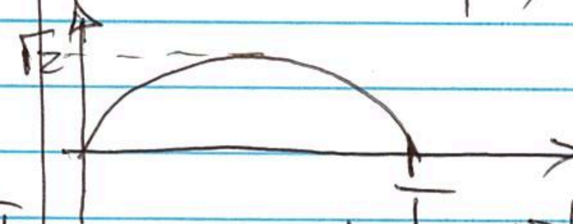
Note: The shape of the pulse $p(t)$ determines the PSD.

- constellation (σ_b^2) ~~and~~ ~~symbol pe~~ affects amplitude
- symbol period T affects amplitude and width

Examples:

Rectangular	Half-sine
$p(t) = I_{[0,T]}(t)$	$p(t) = \sqrt{2} \cdot \sin\left(\frac{\pi t}{T}\right) \cdot I_{[0,T]}(t)$





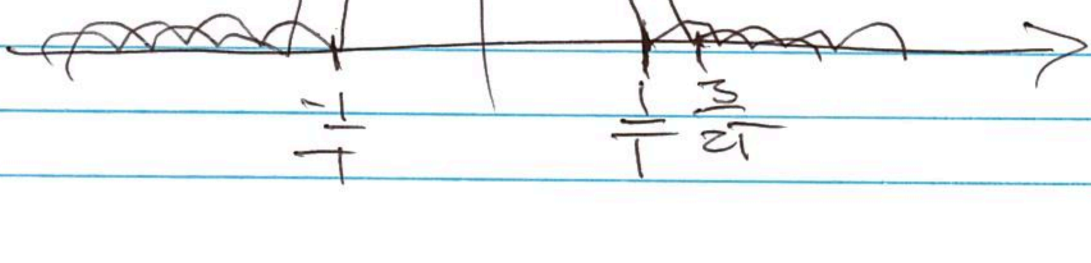
$$P_R(f) = \text{sinc}(fT) \cdot e^{-j\pi f T}$$

$$\Rightarrow |P_R(f)|^2 = \text{sinc}^2(fT)$$

$$\frac{2\cos(\pi f T)}{\pi \cdot (1 - 4(fT)^2)} \cdot e^{-j\pi f T} = P_S(f)$$

$$|P_S(f)|^2 = \frac{4\cos^2(\pi f T)}{\pi^2 (1 - 4(fT)^2)^2}$$

First zero:
Rect: $\frac{1}{T}$
Halfsine: $\frac{3}{2T}$



Measures of bandwidth:

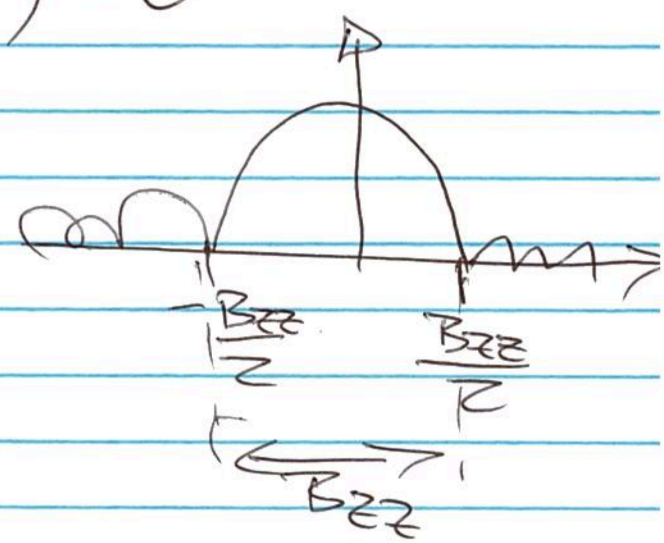
a) zero-to-zero bandwidth B_{ZZ}

B_{ZZ} is the smallest Bandwidth such that

$$S_m\left(\frac{B_{ZZ}}{2}\right) = 0$$

Rect: $\frac{2}{T}$

Half sine: $\frac{3}{T}$



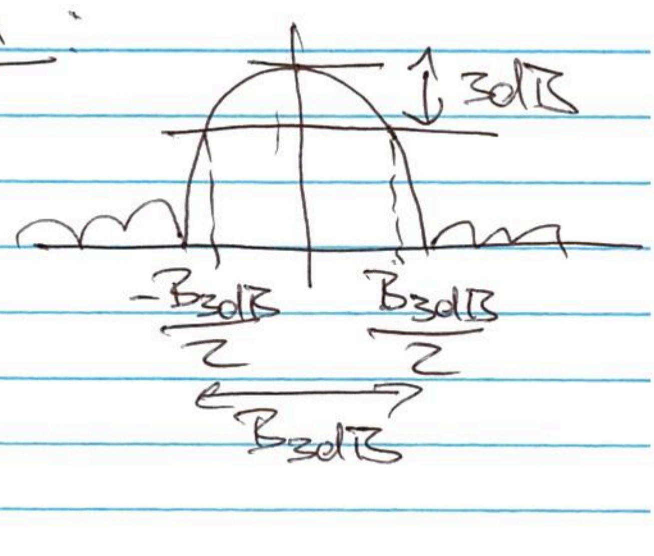
Problem:

some pulses don't have zeros in their PSD

b) 3dB bandwidth:

defined by

$$S_m\left(\frac{B_{3dB}}{2}\right) = \frac{1}{2} S_m(0)$$



Rect: $0.833 \cdot \frac{1}{T}$

Half sine: $1.196 \cdot \frac{1}{T}$