10/10/18

Addendum to Angle Modulation

- FM is a non-linear modulation

→ Fourier transforms of modulated signals are very difficult to compute

E.g.: $m(t) = A_m \cdot \cos(2\pi f_m t)$

$$\Rightarrow \Theta(t) = 2\pi K_f \cdot \int_{-\infty}^{t} m(\tau)\, d\tau$$

$$= \frac{2\pi K_f}{f_m} \cdot A_m \cdot \sin(2\pi f_m t$$

$$\Rightarrow s_p(t) = A_c \cdot \cos\left(2\pi f_c t - 2\pi \frac{K_f A_m}{f_m} \sin(2\pi f_m t)\right)$$

Fourier transform is <u>not</u> easy to compute

Carson's rule:

is a useful rule-of-thumb to estimate the bandwidth of an FM signal

$$BW_{FM} \approx 2 \cdot B + 2 \cdot \Delta f_{max} = 2B \cdot (1 + \beta)$$

B: signal bandwidth

Δf_{max}: $K_f \cdot \max(|m(t)|)$

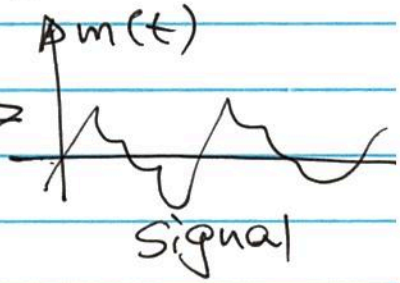
β: modulation index

Digital Modulation

Objective: Transmit digital information, i.e., a sequence of bits

Leverage: Know how to transmit analog (baseband) signal

Question: Can we turn a sequence of bits into a (complex baseband) analog signal?

E.g.: $\{0,1,1,1,0,0\}$ $\xrightarrow{\text{map}}$ 

Idea:

1.) Map bits to (complex) amplitud
- these are called "symbols"

E.g.: bits $d[n]$ $\xrightarrow{\text{map}}$ symbols $s[n]$

0 → 1

1 → −1

2) select a pulse shape $p(t)$

e.g. $p(t) = I_{[0,T)}(t)$ ← rectangular pulse

where: T – pulse duration or symbol period

$R = \frac{1}{T}$ – symbol rate or baud rate

3.) Construct the analog message signal:

$$m(t) = \sum_{n=-\infty}^{\infty} b[n] \cdot p(t - nT)$$

4.) (optional) If a passband signal is needed:

– upconvert $m(t)$ using DSB AM – SC

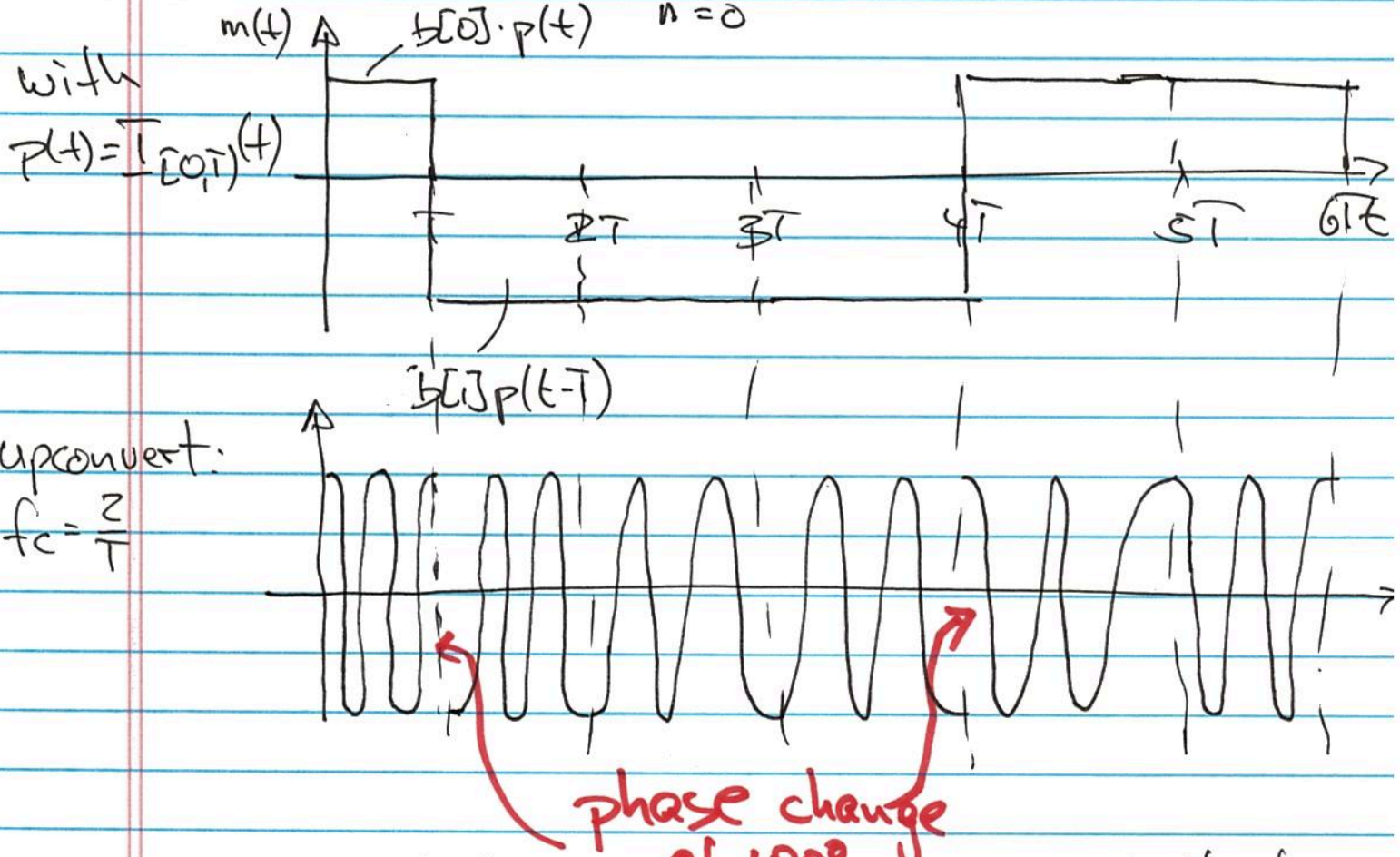
Example:

1.) bits to symbols

$$\{0,1,1,1,0,0\} \longrightarrow \{1,-1,-1,-1,1,1\}$$

3.) construct baseband message signal:

$$m(t) = \sum_{n=0}^{5} b[n] \cdot p(t - n\bar{T})$$



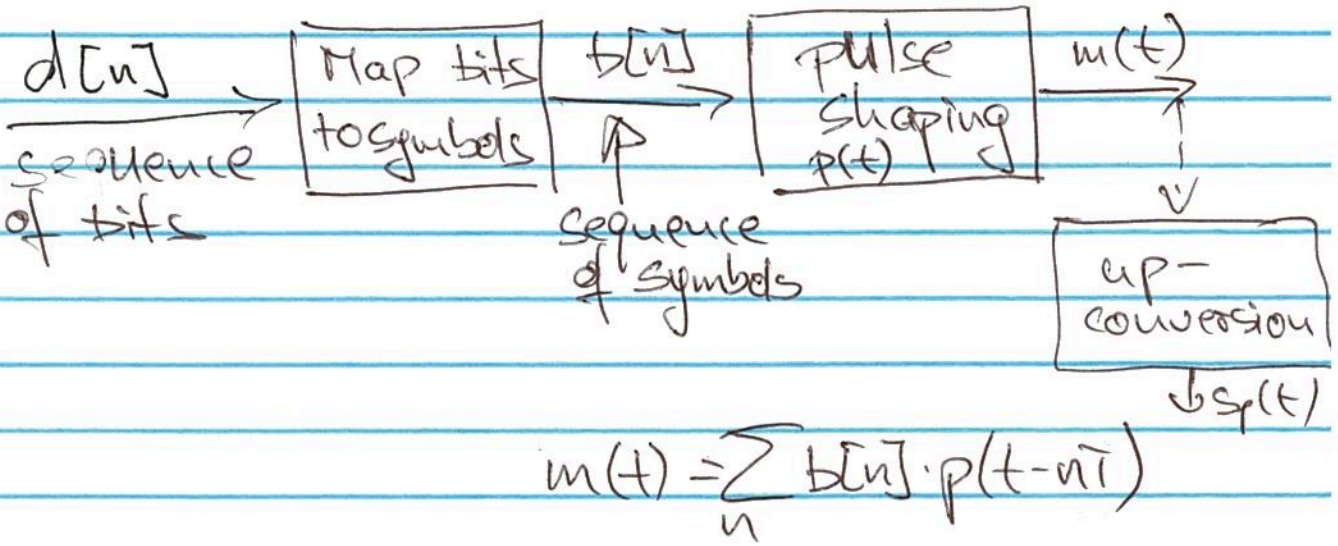
This digital modulation is called

Passband: BPSK – binary phase shift keying

Baseband: 2PAM – binary pulse amplitude mod

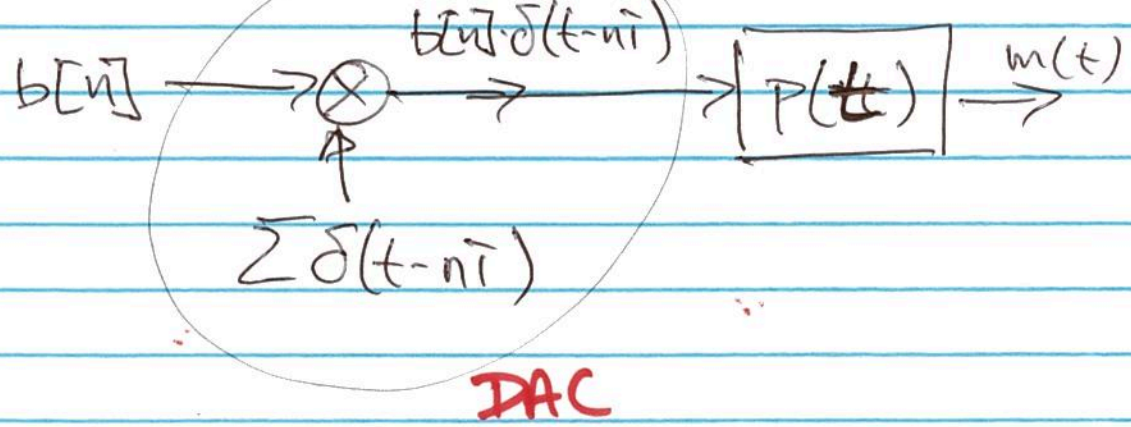
either : binary, antipodal modulation

In general:



$$m(t) = \sum_n b[n] \cdot p(t - nT)$$

Pulse shaping is often shown like this:



Observation:

We are not restricted to binary modulation!

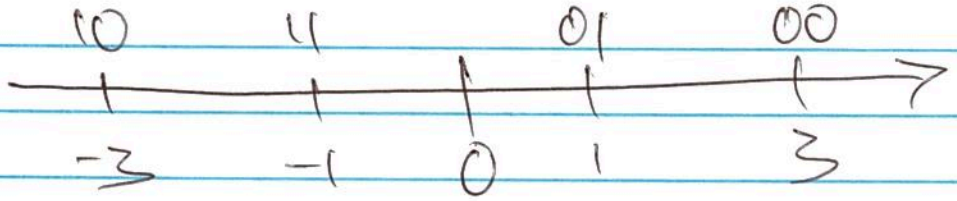
i.e., we can use more than two symbols

$\Rightarrow$ we can transmit multiple bits per symbol

E.g.: 4PAM

bit pairs	Symbols
00	3
01	1
11	-1
10	-3

Gray coding



E.g.: QPSK ↑ quaternary

bit pairs	Symbols
00	$(1+j)/\sqrt{2}$
01	$(-1+j)/\sqrt{2}$
10	$(-1-j)/\sqrt{2}$
11	$(1-j)/\sqrt{2}$