10/9/18

Angle Modulation

Recall: in amplitude modulation (AM) the amplitude of carrier signal varies proportional to message signal $m(t)$:

$$S_p(t) = \left(A_c + m(t)\right) \cdot \cos(2\pi f_c t)$$

or $$S_B(t) = A_c + m(t)$$

For angle modulation:

$$s_p(t) = A_c \cdot \cos(2\pi f_c t + \theta(t))$$

Two options:

1.) phase modulation

$$\theta(t) = K_p \cdot m(t)$$

2.) frequency modulation

$$\theta(t) = \theta(0) + 2\pi K_f \cdot \int_{-\infty}^{t} m(\tau)\, d\tau$$

E.g. for $m(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{else} \end{cases}$

$$\theta(t) = 2\pi \cdot K_f \cdot t$$

$$\Rightarrow S_p(t) = A_c \cdot \cos(2\pi f_c t + 2\pi K_f t)$$ Freq: $f_c + K_f$

Recall: Instantaneous frequency $f_i(t)$

For signals of form: $s(t) = \cos(\psi(t))$

$$f_i(t) = \frac{1}{2\pi} \cdot \frac{d\psi(t)}{dt}$$

For FM signal:

$$s_p(t) = A_c \cdot \cos\Big(2\pi f_c t + \underbrace{\theta(0) + \int_{-\infty}^{t} m(\tau)\,d\tau}_{=\theta(t)}\Big)$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left(2\pi f_c t + \theta(0) + 2\pi K_f \int_{-\infty}^{t} m(\tau)\,d\tau\right)$$

$$\boxed{f_i(t) = f_c + 0 + K_f \cdot m(t)}$$

Baseband equivalent:

$$s_p(t) = A_c \cdot \cos(2\pi f_c t + \theta(t))$$

$$= \mathrm{Re}\left\{A_c\, e^{j(2\pi f_c t + \theta(t))}\right\}$$

$$= \mathrm{Re}\Big\{\underbrace{A_c \cdot e^{j\theta(t)}}_{\text{complex envelope}} \cdot e^{j2\pi f_c t}\Big\}$$

$$s_B(t) = A_c \cdot e^{j\theta(t)}$$

Definitions

K_f - frequency deviation constant

Maximum frequency deviation from carrier frequency:

$$\Delta f_{,max} = K_f \cdot \max|m(t)|$$

Modulation Index:

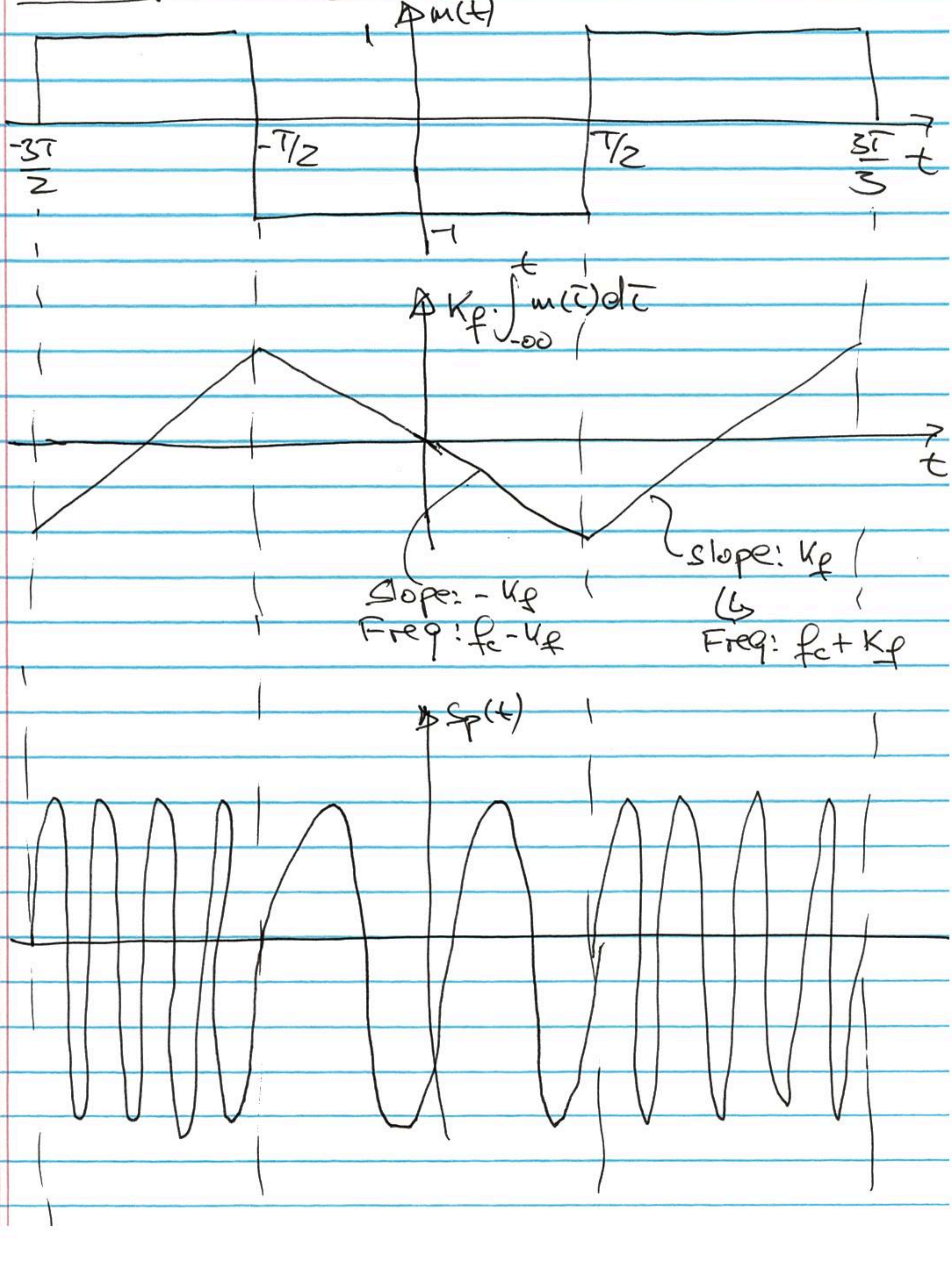
$$\beta = \frac{\Delta f_{,max}}{B}$$

where B is the bandwidth of $m(t)$

If $\beta < 1$: narrow-band FM

If $\beta > 1$: wide-band FM

Example (FM)



m(t)
1
-1
$-\frac{3T}{2}$
$-T/2$
$T/2$
$\frac{3T}{3}$
t
$K_f \cdot \int_{-\infty}^{t} m(\tau)\,d\tau$
t
slope: $-K_f$
Freq: $f_c - K_f$
slope: K_f
Freq: $f_c + K_f$
$s_p(t)$

Analog FM Modulator:

voltage-controlled oscillator (VCO):

$$m(t) \longrightarrow \boxed{\text{VCO}} \longrightarrow A_V \cdot \cos\left(2\pi f_c t + 2\pi K_V \cdot \int_{-\infty}^{t} m(\tau)\,d\tau\right)$$

$$\boxed{\text{A VCO is an FM modulator}}$$

VCO parameters:

f_c – quiescent frequency
(freq. when $m(t) = 0$)

K_V – frequency deviation constant

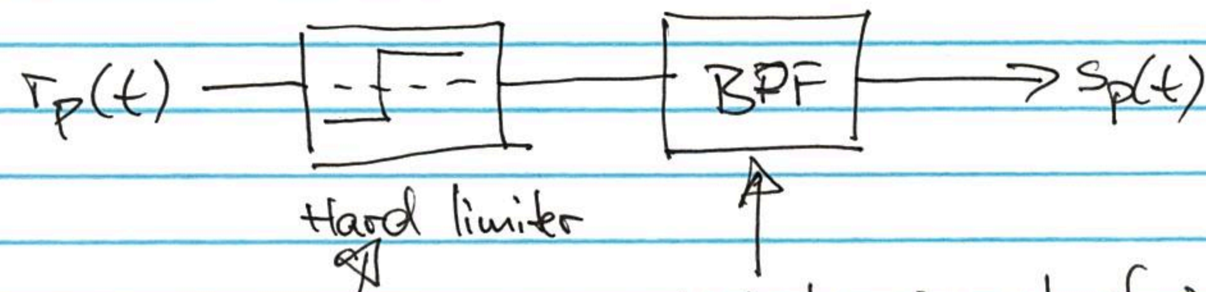
FM demodulation:

Two classes of FM demodulators:

- Discriminator (non-coherent)
- Phase-locked loop (PLL)
coherent

FM demodulation with a discriminator

Discriminator frontend:



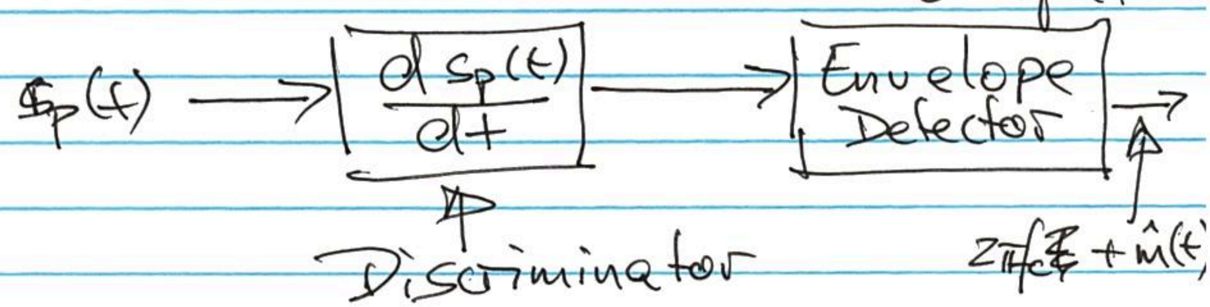
helps to reduce noise

- isolate signal of interest
- band select filter

The discriminator itself:

(1) takes passband signal as input

(2) produces the derivative as the output



Q: What does the discriminator do?

$$s_p(t) = A_c \cdot \cos(2\pi f_c t + \theta(t))$$

$$\Rightarrow \frac{d}{dt} s_p(t) = A_c \cdot (-\sin(2\pi f_c t + \theta(t))) (2\pi f_c + \frac{d\theta(t)}{dt})$$

$$= A_c \cdot \left(2\pi f_c + \frac{d\theta(t)}{dt}\right) \cdot \cos\left(2\pi f_c t + \theta(t) + \frac{\pi}{2}\right)$$

with $\theta(t) = 2\pi K_f \cdot \int_{-\infty}^{t} m(\tau)\, d\tau$

$$\frac{d\theta(t)}{dt} = 2\pi K_f \cdot m(t)$$

$$\Rightarrow \frac{d}{dt} s_p(t) = A_c \cdot \underbrace{(2\pi f_c + 2\pi K_f \cdot m(t))}_{\text{Like an AM signal}} \cdot \cos(2\pi f_c t + \theta(t))$$

Discriminator converts FM signal to an AM signal

Since $f_c \gg K_f \cdot m(t) \Rightarrow$ well suited for envelope detection

Summary:

- discriminator (derivative) turns FM into AM
- demod AM signal with envelope detector

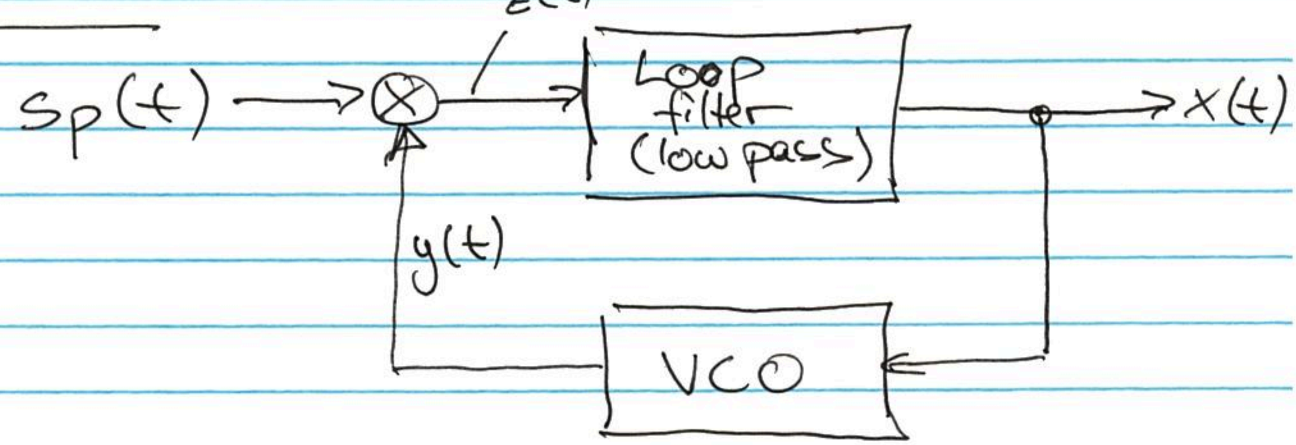
Demodulation with a PLL

PLL: phase-locked loop

PLL is a feedback control system
- similar conceptually to the discrete-time phase and frequency control we discussed.

PLL works in continuous-time (analog)

PLL:



$$s_p(t) = A_c \cdot \cos(2\pi f_c t + \theta_i(t))$$

$$y(t) = A_v \cdot \overset{-\sin}{\cancel{\cos}}(2\pi f_c t + \theta_o(t))$$

where $\theta_o(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$\theta_i(t) = \int_{-\infty}^{t} m(\tau)\, d\tau$

$$\Rightarrow z(t) = s_p(t) \cdot y(t)$$

$$= -A_c \cdot A_v \cdot \cos(2\pi f_c t + \theta_i) \cdot \sin(2\pi f_c t + \theta_o)$$

$$= \frac{A_c \cdot A_v}{2} \cdot \sin(\theta_i - \theta_o) + \underbrace{\text{double freq.}}_{\text{rejected by loop filter}}$$

$\sin(\theta_i - \theta_o) \approx \theta_i - \theta_o$ if $|\theta_i - \theta_o|$ is small

If $\theta_i - \theta_o > 0$: θ_o increases (VCO)

$\Rightarrow$ difference $\theta_i - \theta_o$ decrease

If $\theta_i - \theta < 0$: θ_o decreases

$\Rightarrow \theta_i - \theta_o$ increases (towards zero)

$\Rightarrow$ In steady state $\theta_i \approx \theta_o$

$$\theta_i = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau \approx 2\pi K_v \int_{-\infty}^{\tau} x(\tau)\, d\tau = \theta_o$$

$$\Rightarrow \frac{d\theta_i}{dt} = 2\pi K_f \cdot m(t) \approx 2\pi K_v \cdot x(t) \approx \frac{d\theta_o}{dt}$$

$$\Rightarrow \boxed{x(t) = m(t) \cdot \frac{K_f}{K_v}}$$