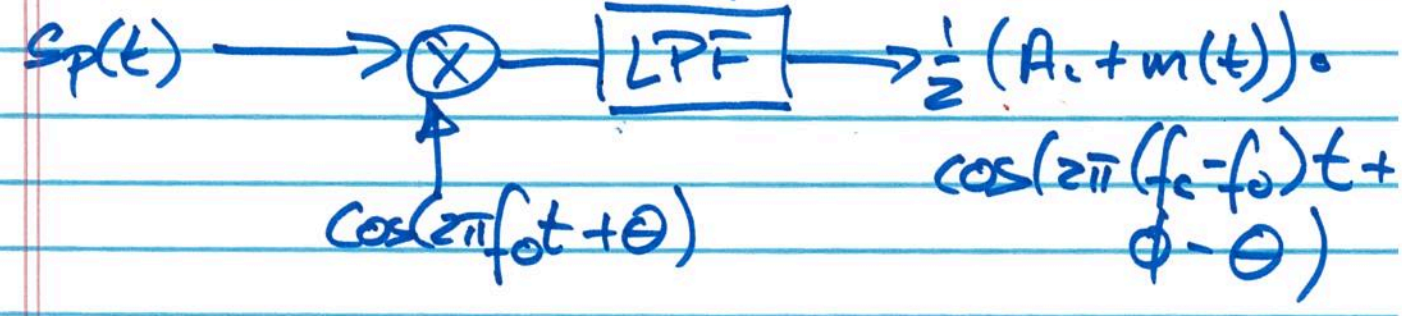
10/01

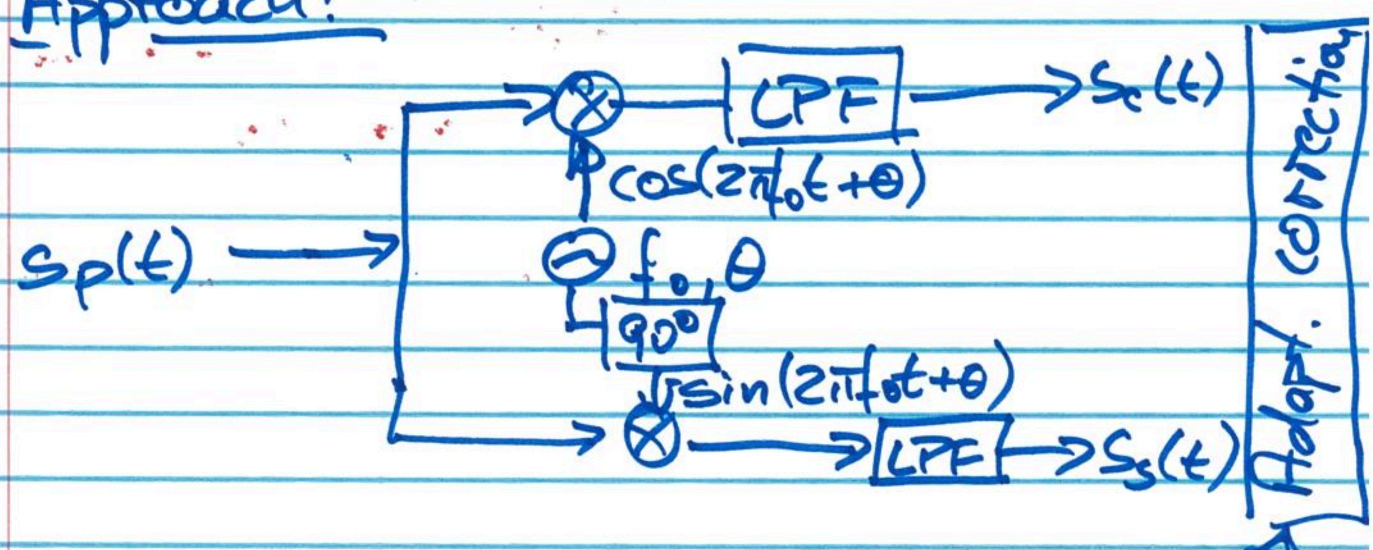
Adaptive Phase and Frequency Synchroniza

Problem:

$$s_p(t) = (A_c + m(t)) \cdot \cos(2\pi f_c t + \phi)$$



Approach:



This block is responsible for adaptively correcting phase, frequency error

How to build an adaptive System?

1.) Define a goal — • a performance metric
• objective function

- often: squared error
"power" or "energy" of error

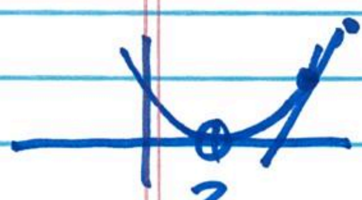
2.) Find a method/algorithm for finding the minimum (sometimes the maximum) of the goal.

3.) Test and verify algorithm
- analysis and/or Simulatio

Example:

Assume objective function is given:

$$J(x) = x^2 - 4x + 4$$



Clearly, $J(x)$ is minimized for $x=2$ ($J'(x) = 2x - 4 \Rightarrow J'(2) = 0$)

Alternative: Iterative solution

Adaptive method: steepest descent

0. Initial guess:

1. Find direction of steepest
   $\Rightarrow$ needs gradient

Iterate/repeat

2: update: adjust x by a small step in direction of steepest descent

Key Insight: direction of steepest descent is given by negative gradient

Basic idea for adaptive algorithm

$$x[n+1] = x[n] - \mu \cdot \frac{dJ(x)}{dx}\bigg|_{x=x[n]}$$

step size
(small, positive, real)
e.g.: 0.05

For example:

$$J(x) = x^2 - 4x + 4$$

$$J'(x) = \frac{dJ(x)}{dx} = 2x - 4$$

Iterative, adaptive algorithm:

$$x[n+1] = x[n] - \mu \cdot (2 \cdot x[n] - 4)$$

$$= x[n] \cdot (1 - 2\mu) + 4\mu$$

Sanity Check:

Steady State: $n = \infty$

$$x[\infty] = x[\infty] \cdot (1 - 2\mu) + 4\mu$$

$$\Rightarrow x[\infty] \cdot 2\mu = 4\mu$$

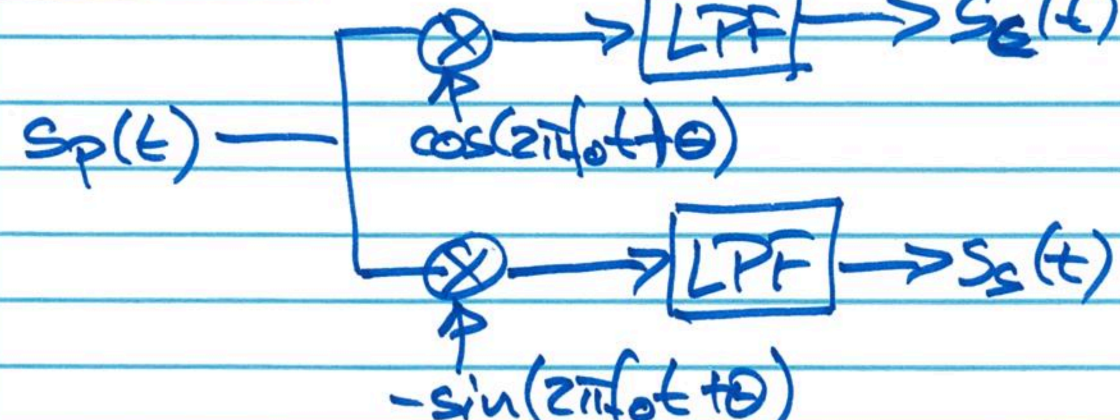
$$\Rightarrow x[\infty] = 2$$

Q: How can we apply these ideas to phase / frequency synchronization

Initially, focus on carrier only:

$s_p(t) = A_c \cos(2\pi f_c t + \phi)$ | transmitted:

Receiver:



=> complex envelope: $s_R(t) = s_c(t) + j s_s(t)$ equals:

$$s_R(t) = A_c e^{j2\pi(f_c - f_0)t} \cdot e^{j(\phi - \theta)}$$

Want:

$$s_R(t) \longrightarrow \boxed{\text{Adaptive System}} \longrightarrow A_c \cdot e^{j\theta}$$

carrier w/o freq. and phase error

Assume, freq error is zero; only phase error

$$\Rightarrow s_R(t) = A_c \cdot e^{j(\phi-\theta)} \longrightarrow \otimes \longrightarrow A_c$$

$$e^{-j(\phi-\theta)}$$

Q: Can we define an objective function s.t. it has a minimum at the correct phase correction?

$$s_R(t) \longrightarrow \otimes \longrightarrow r(t)$$

g

control: adapt this

want this equal to A_c

Define: error:

$r(t) - A_c$

actual desired

objective function:

$$J(g) = |r(t) - A_c|^2 \quad \text{squared error}$$

<u>Goal:</u> figure out how to adapt g so that we converge to minimum of $J(g)$.

$$J(g) = |s_R(t)\cdot g - A_c|^2$$

but, g is complex! How do you take derivativ

with $g = g_c + j\,g_s$

$$s_R = s_c + j\,s_s$$

$$A_c = A_c + j\cdot 0$$

$$\Rightarrow J(g_c, g_s) = \left(\mathrm{Re}\{s_R(t)\cdot g - A_c\}\right)^2 + \left(\mathrm{Im}\{s_R\cdot g - A_c\}\right)^2$$

$$= (s_c\cdot g_c - s_s\cdot g_s - A_c)^2 + (s_c\cdot g_s + s_s\cdot g_c)^2$$

<u>Derivatives:</u>

$$\frac{dJ}{dg_c} = 2\cdot s_c\cdot(s_c g_c - s_s g_s - A_c) + 2\cdot s_s\cdot(s_c g_s + s_s g_c)$$

$$\frac{dJ}{dg_s} = -2 s_s\,(s_c g_c - s_s g_s - A_c) + 2 s_c\,(s_c g_s + s_s g_c)$$

Equivalent to: $\boxed{s_R^* \cdot (s_R\cdot g - A_c)}$

Just computed: Gradient

$$\nabla J(g) = s_R^* \cdot (s_R \cdot g - A_c)$$
$$= s_R^* \cdot \text{error}(g)$$

=> Adaptive, iterative solution

$$\boxed{\cancel{g[n+1] = g[n] - \mu \cdot s_R^* \cdot (s_R \cdot g - A_c)}}$$

$$g[n+1] = g[n] - \mu \cdot s_R^*[n] \cdot (s_R[n] \cdot g[n] - A_c)$$