9/24/18

Reminders: - complex envelope
- complex baseband
- baseband equivalent signal

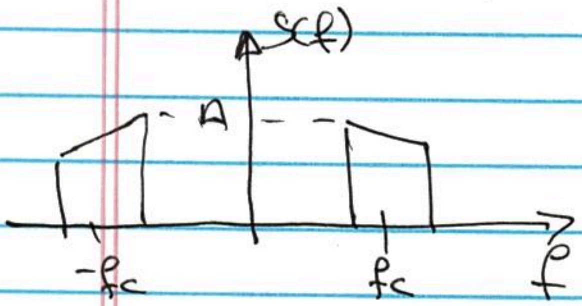
Time domain:

$$s(t) = \mathrm{Re}\{\overbrace{\underbrace{u(t)}_{\text{complex envelope}} \cdot e^{j2\pi f_c t}}^{\text{complex passband } c(t)}\}$$

$s(t)$: passband

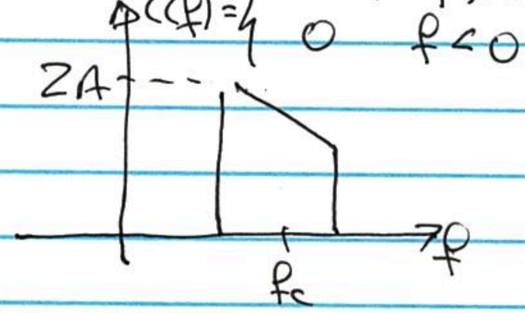
$$= \underbrace{u_c(t)}_{\text{I-signal}} \cdot \cos(2\pi f_c t) - \underbrace{u_s(t)}_{\text{Q-signal}} \cdot \sin(2\pi f_c t)$$

Frequency Domain

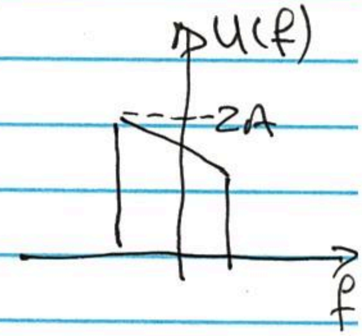


Passband

$$C(f) = \begin{cases} 2 \cdot S(f) & f \geq 0 \\ 0 & f < 0 \end{cases}$$



Complex Passband



Complex Baseband

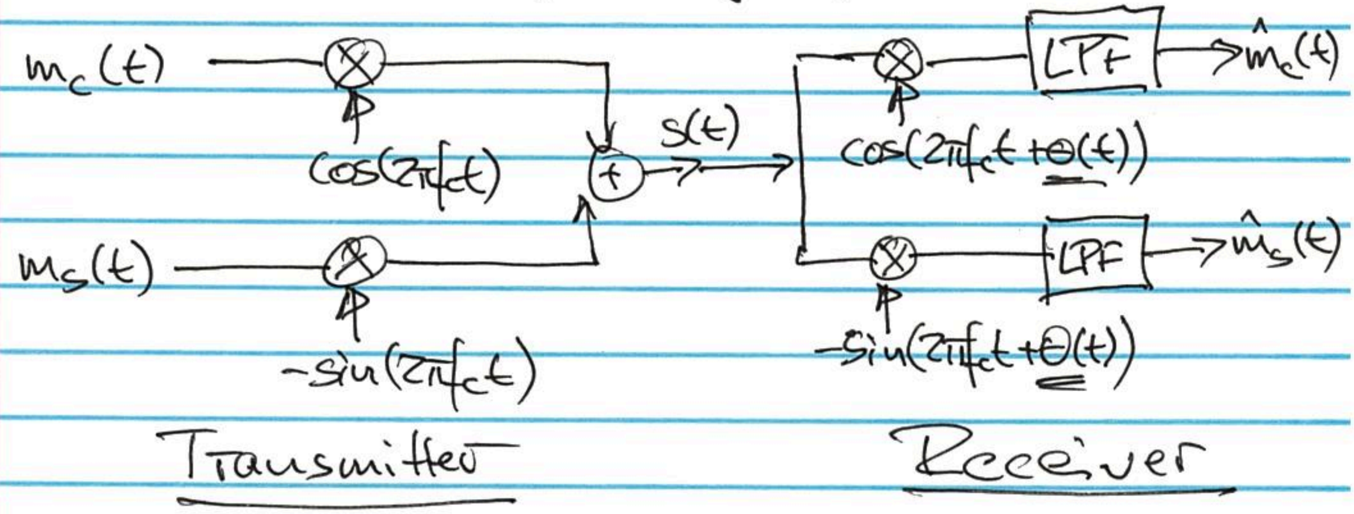
Today:

Two applications for complex envelope

1.) Phase + Frequency offset

2.) Passband Filtering

Phase and Frequency Offset



- Transmitter and receiver oscillators differ by $\theta(t)$

- Complex envelope techniques simplify analysis.

"Trick" or idea: choose a carrier reference that is relevant for the system under analysis.

Here:

Transmitter: reference $e^{j2\pi f_c t}$

Receiver : reference $e^{j(2\pi f_c t + \theta(t))}$

Transmitter: transmitted signal

$$s(t) = \mathrm{Re}\{\underbrace{u(t)}_{\text{complex envelope}} \cdot \underbrace{e^{j2\pi f_c t}}_{\text{reference}}\}$$

$$u(t) = m_c(t) + j m_s(t)$$

Receiver:

$$s(t) = \mathrm{Re}\{\underbrace{u(t) \cdot e^{-j\theta(t)}}_{\text{complex envelope}} \cdot \underbrace{e^{j(2\pi f_c t + \theta(t))}}_{\text{reference at receiver}}\}$$

I and Q signals at receiver

I: $\hat{m}_c(t) = \mathrm{Re}\{u(t) \cdot e^{-j\theta(t)}\} = \mathrm{Re}\{\overbrace{(m_c(t) + j m_s(t))}^{=u(t)} \cdot e^{-j\theta(t)}\}$

$= m_c(t) \cdot \cos\theta(t) + m_s(t) \cdot \sin(\theta(t))$

Q: $\hat{m}_s(t) = \mathrm{Im}\{u(t) \cdot e^{-j\theta(t)}\}$

$= m_s(t) \cdot \cos(\theta(t)) - m_c(t) \cdot \sin(\theta(t))$

Examples

- $\theta(t) = 0$: $\hat{m}_c(t) = m_c(t)$, $\hat{m}_s(t) = m_s(t)$ } Ideal case: perfect synch

- $\theta(t) = \frac{\pi}{2}$: $\hat{m}_c(t) = m_s(t)$, $\hat{m}_s(t) = -m_c(t)$ } 90° phase error: channels "flip"

- $\theta(t) = 2\pi\Delta f t$: Frequency offset

$$\hat{m}_c(t) = m_c(t)\cdot\cos(2\pi\Delta f t) + m_s(t)\cdot\sin(2\pi\Delta f t)$$
$$\hat{m}_s(t) = m_s(t)\cdot\cos(2\pi\Delta f t) - m_c(t)\cdot\sin(2\pi\Delta f t)$$

→ Channels "roll" or "fade" in and out

Example:

$$s_p(t) = I_{[-1,1]}(t) \cdot \cos(2\pi f_c t)$$

$$h_p(t) = I_{[0,3]}(t) \cdot \sin(2\pi f_c t)$$

Then:

$$y_p(t) = s_p(t) * h_p(t)$$

$$= \int s_p(\tau) \cdot h_p(t-\tau)\, d\tau$$

$$= \int I_{[-1,1]} \cdot \cos(2\pi f_c \tau) \cdot I_{[0,3]}(t-\tau) \cdot \sin(2\pi f_c (t-\tau))\, d\tau$$

OUCH!

Q: Can we solve this problem more easily using complex envelope techniques?

Passband Filtering

Filtering of passband signals occurs:

- at transmitter: Frequeny shaping to ensure spectrum mask is met
- at receiver: channel selection
  ⇒ isolate signal of interest
- channel: e.g. multipath

Fundamentally, it is possible to describe/model filtering at passband

- all LTI relationships hold:

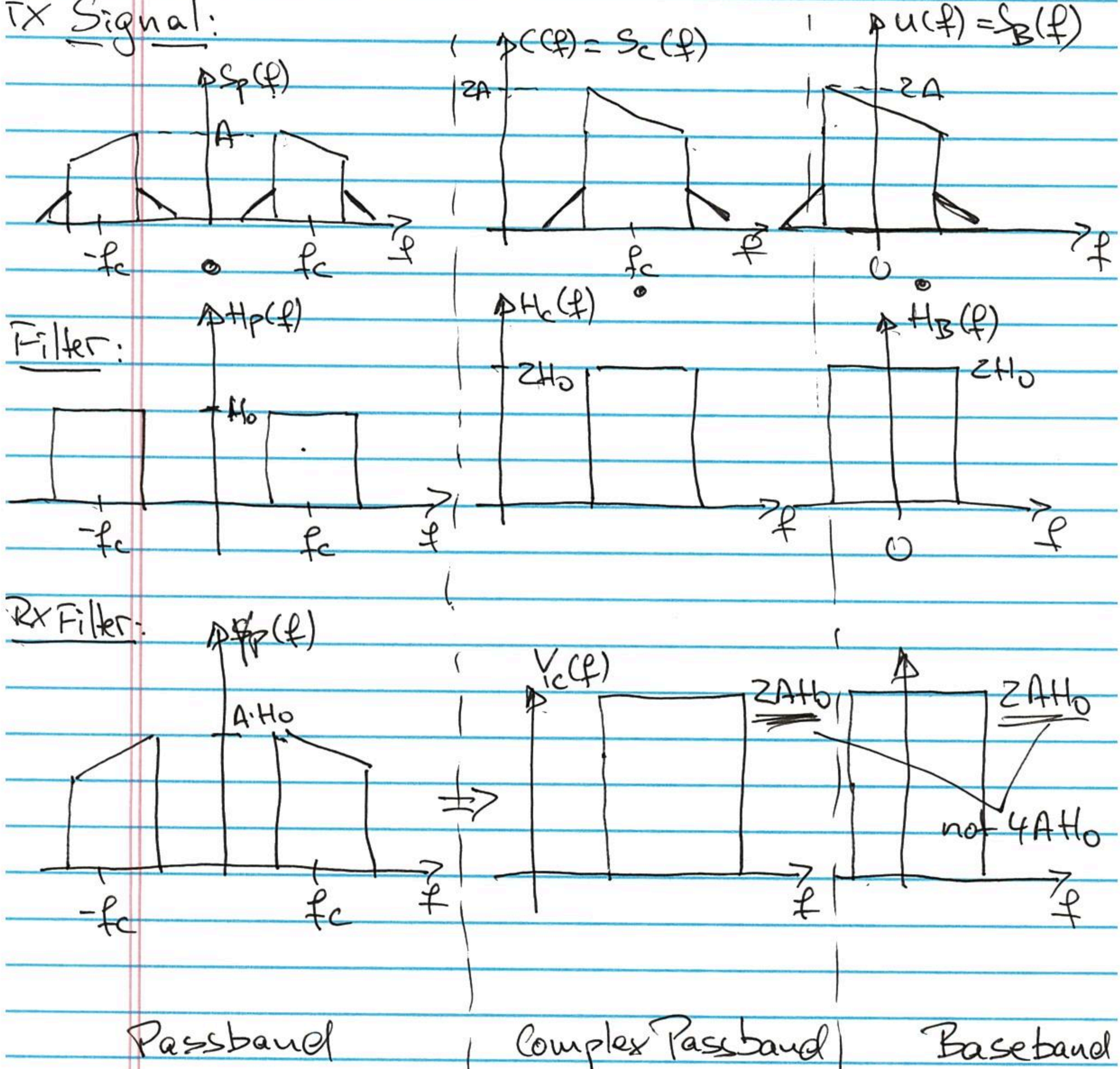
Freq: $Y_p(f) = S_p(f) \cdot H_p(f)$

received, transmitted, filter respon

Time: $y_p(t) = s_p(t) * h_p(t)$

Q: What's hard about this?

Q: How is filtering at passband related to filtering of complex envelopes?



From the above diagrams:

Passband:

$$Y_p(f) = S_p(f) \cdot H_p(f)$$
$$y_p(t) = s_p(t) * h_p(t)$$

Complex passband:

$$Y_c(f) = \frac{1}{2} S_c(f) \cdot H_c(f)$$
$$y_p(t) = \frac{1}{2} s_c(t) * h_c(t)$$

Compl. Baseband:

$$Y_B(f) = \frac{1}{2} S_B(f) \cdot H_B(f)$$
$$y_B(t) = \frac{1}{2} s_B(t) \cdot h_B(t)$$

The $\frac{1}{2}$ factor in the complex versions stems from up/down - conversion:

$$\cos^2(2\pi f_c t) = \frac{1}{2} + \frac{1}{2}\cos(4\pi f_c t)$$

Recipe for passband filtering:

1.) Convert passband signal and filter to baseband equivalent

2.) convolve/filter at baseband

3.) convert baseband to passband result.

Back to example:

$$s_p(t) = I_{[-1,1]}(t) \quad \cos(2\pi f_c t)$$

$$h_p(t) = I_{[0,3]}(t) \cdot \sin(2\pi f_c t)$$

1.) Find complex envelopes for $s_p(t)$ and $h_p(t)$:

In general:

$$x_p(t) = \mathrm{Re}\{x_b(t) \cdot e^{j2\pi f_c t}\}$$

$$= x_{b,c}(t) \cdot \cos(2\pi f_c t) - x_{b,s}(t) \cdot \sin(2\pi f_c t)$$

and: $x_b(t) = x_{b,c}(t) + j x_{b,s}(t)$

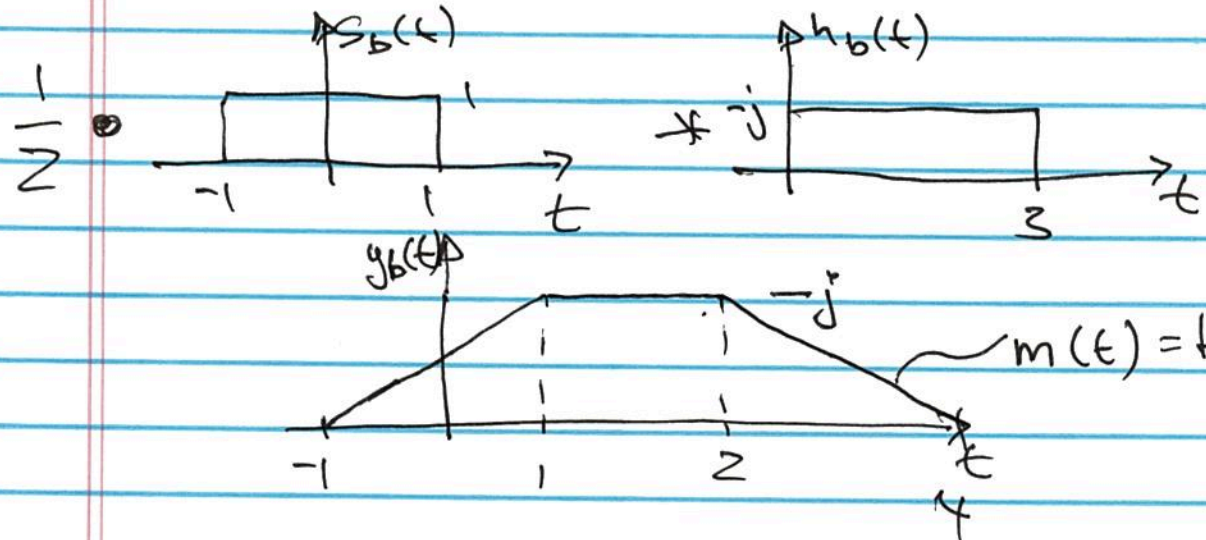
$$\Rightarrow s_b(t) = I_{[-1,1]}(t) \quad + j \cdot 0$$

$$h_b(t) = 0 \quad - j \cdot I_{[0,3]}(t)$$

2.) convolve:

$$s_b(t) = I_{[-1,1]}(t)$$

$$h_b(t) = -j \cdot I_{[0,3]}(t)$$



3.) Passband output signal:

$$y_p(t) = Re\{y_b(t) \cdot e^{j2\pi f_c t}\}$$

$$= Re\{(-j \cdot m(t)) \cdot (\cos(2\pi f_c t) + j\sin(2\pi f_c t))\}$$

$$= m(t) \cdot \sin(2\pi f_c t)$$