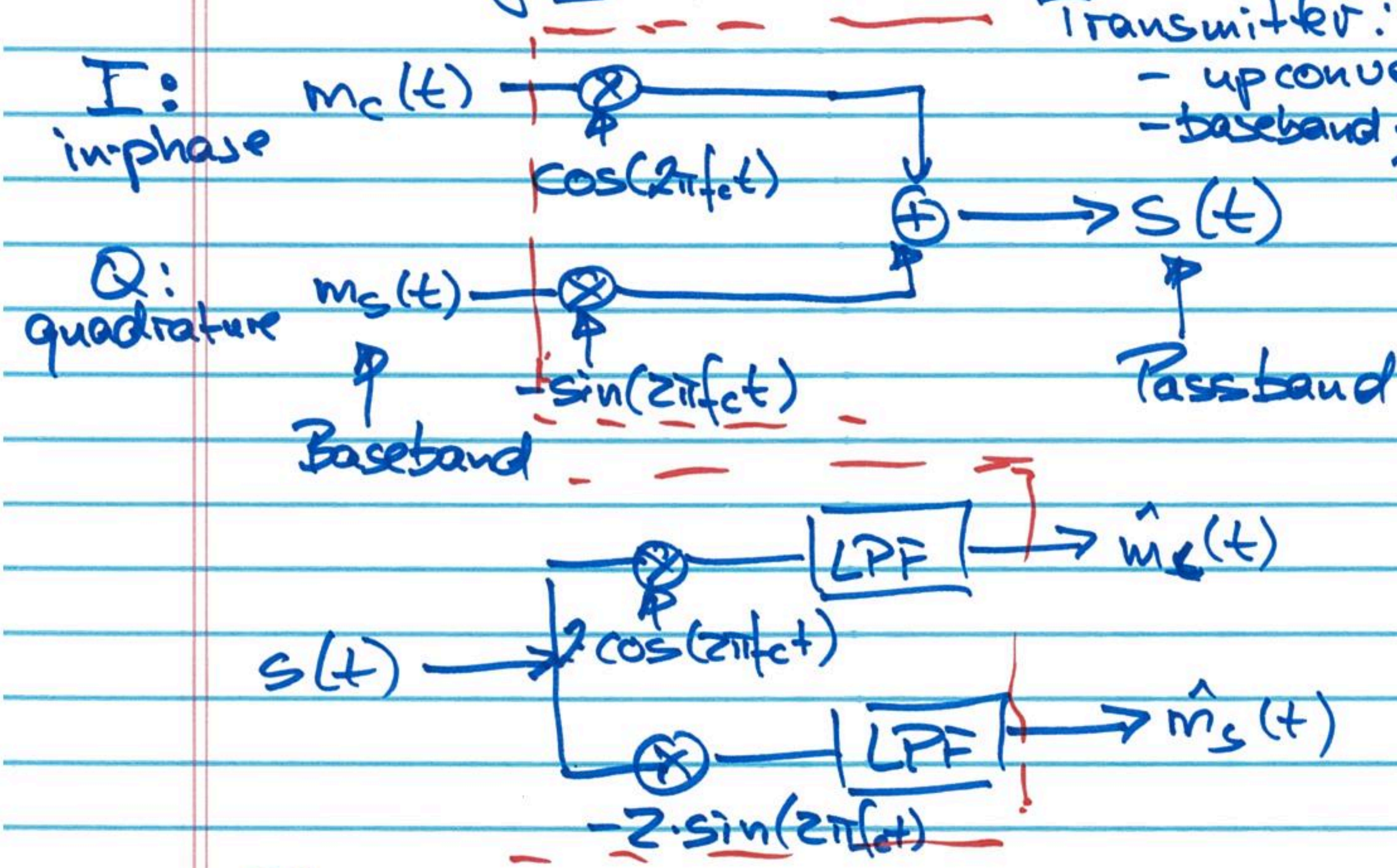
9/17/18

Complex Envelope

- also called:
  - baseband equivalent signal
  - complex baseband signal

QAM System:

Transmitter:
- upconversion
- baseband → passband



Receiver:
- down-conversion
- passband → baseband

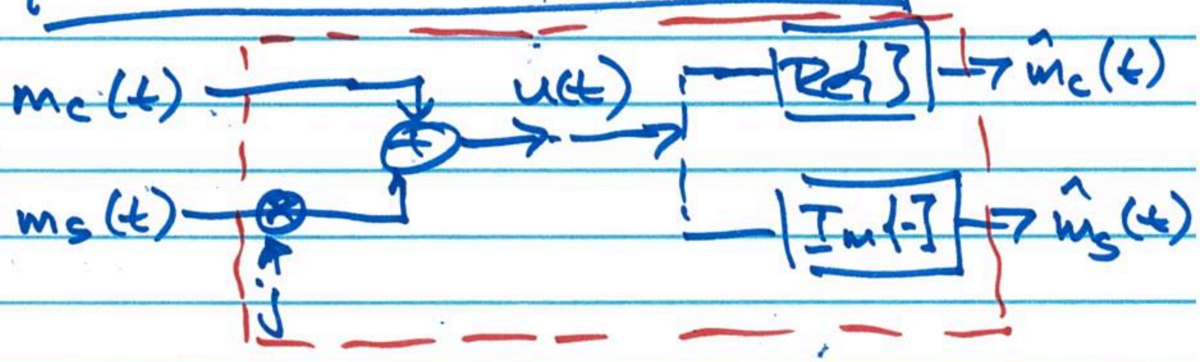
Big idea: Emphasize messages, de-emphasize carrier

Passband signal:

$$\boxed{s(t) = m_c(t) \cdot \cos(2\pi f_c t) - m_s(t) \cdot \sin(2\pi f_c t)}$$

Complex envelope $u(t)$ is defined as:

$$\boxed{u(t) = m_c(t) + j \cdot m_s(t)}$$



Q: Can we get the passband signal $s(t)$ from complex envelope $u(t)$?

A: Yes:

$$s(t) = \mathrm{Re}\{u(t) \cdot e^{j2\pi f_c t}\}$$

$\cancel{s(t)}$

$$c(t) = u(t) \cdot e^{j2\pi f_c t}$$

$$= \left(m_c(t) + j\, m_s(t)\right) \cdot \left(\cos(2\pi f_c t) + j \sin(2\pi f_c t)\right)$$

$$= m_c(t) \cdot \cos(2\pi f_c t) - m_s(t) \cdot \sin(2\pi f_c t)$$

$$+ j \cdot \left(m_c(t) \cdot \sin(2\pi f_c t) + m_s(t) \cdot \cos(2\pi f_c t)\right)$$

$$\Rightarrow s(t) = \mathrm{Re}\{c(t)\} = \mathrm{Re}\{u(t) \cdot e^{j2\pi f_c t}\}$$

$$= m_c(t) \cdot \cos(2\pi f_c t) - m_s(t) \cdot \sin(2\pi f_c t)$$

QED

$\Rightarrow$ no information lost when using $u(t)$
$\rightarrow$ just add carrier

<u>Note:</u> The complex envelope representation simplifies description of modulated signals

- complex envelope $u(t)$ contains all information about messages $m_c(t)$ and $m_s(t)$
- avoids trigonometry, replaces with complex algebra
- will see:
  - + support phase or frequency offset
  - + cleanly supports: filtering
  - [illegible]ce

Frequency Domain Relationships

Time Domain:

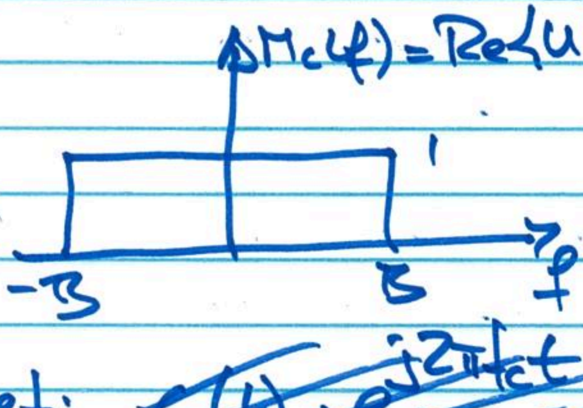
$$u(t) = m_c(t) + j\cdot m_s(t)$$

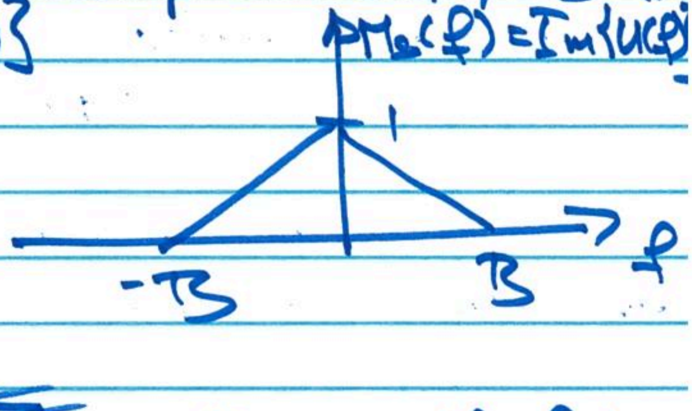
$$\updownarrow \qquad\qquad \updownarrow$$

$$U(f) = M_c(f) + j\cdot M_s(f)$$

Illustration:

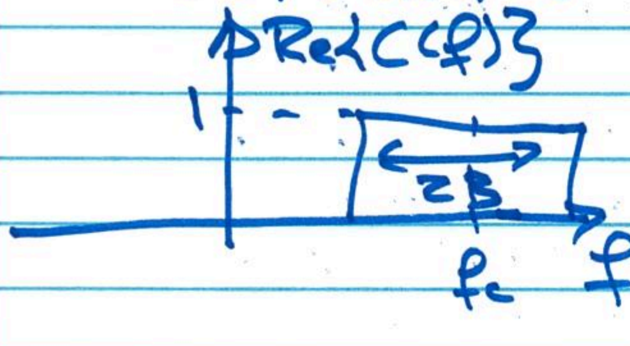
- Illustration assumes $M_c(f)$ and $M_s(f)$ are real
  - relationships all hold for complex $M_c(f)$, $M_s(f)$

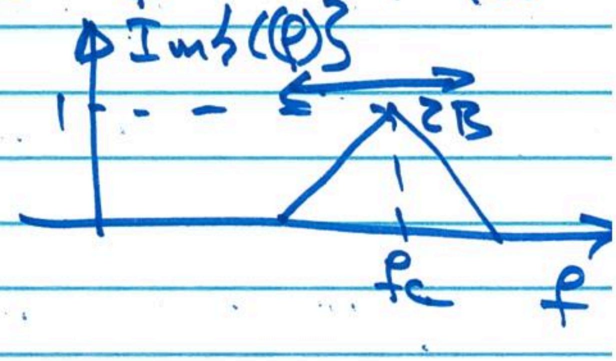




Let: $\cancel{c(t) = e^{j2\pi f_c t}}$

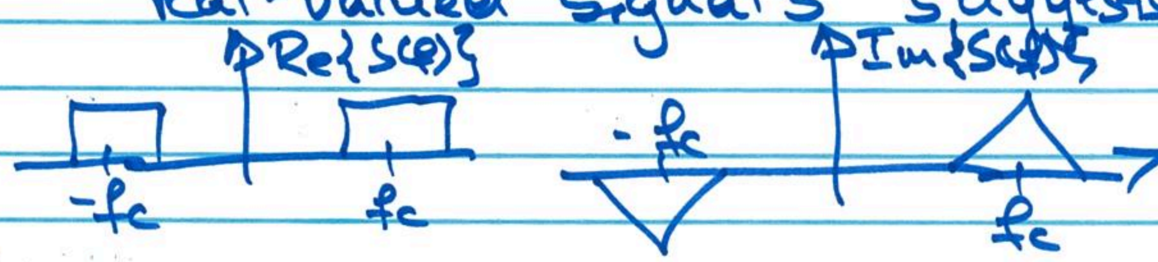
$$c(t) = u(t)\cdot e^{j2\pi f_c t} \Leftrightarrow C(f) = U(f-f_c)$$





Q: What is the spectrum of the (real-valued) passband signal $s(t)$?

Note: Intuition and symmetry properties of Fourier Transform for real-valued signals suggests:

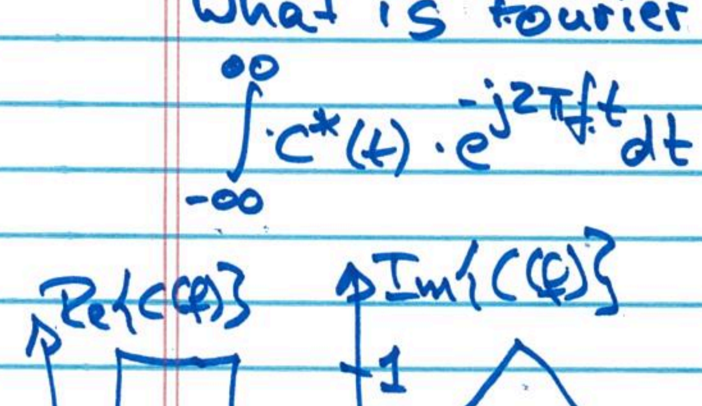


Verification:

$$s(t) = \mathrm{Re}\{c(t)\} = \frac{1}{2}c(t) + \frac{1}{2}c^*(t)$$

What is Fourier Transform of $s^*(t)$?

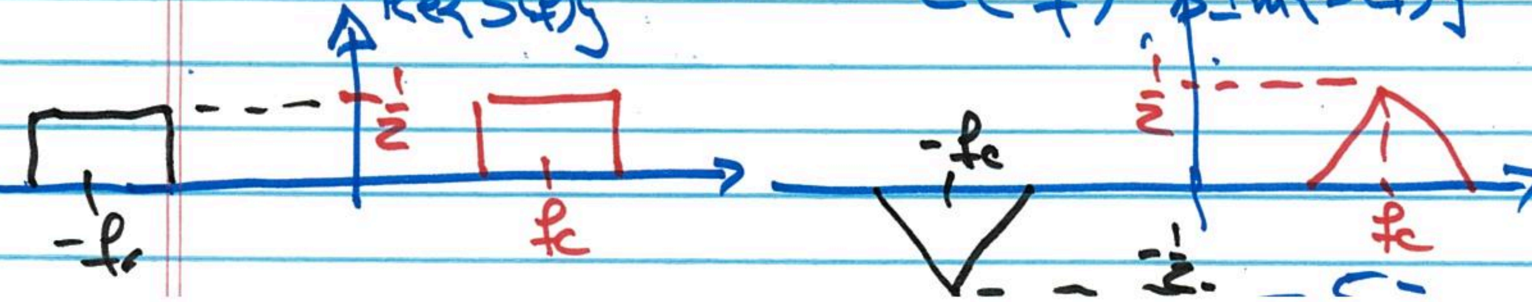
$$\int_{-\infty}^{\infty} c^*(t)\cdot e^{-j2\pi f t}\,dt = \int_{-\infty}^{\infty}\left(c(t)\cdot e^{j2\pi f t}\right)^* dt$$



$$= \int_{-\infty}^{\infty}\left(c(t)\cdot e^{-j2\pi(-f)t}\right)^* dt$$

$$= \left(\int_{-\infty}^{\infty} c(t)\cdot e^{-j2\pi(-f)t}\,dt\right)^*$$

$$= C^*(-f)$$



In Summary:

$$\boxed{\begin{aligned} S(f) &= \frac{1}{2}U(f-f_c) + \frac{1}{2}U^*(-(f-f_c)) \\ &= \frac{1}{2}\,C(f) + \frac{1}{2}\,C^*(-f) \end{aligned}}$$

Has the necessary symmetries for a real-valued signal

Also:

$$C(f) = \begin{cases} 2 \cdot S(f) & \text{for } f>0 \\ 0 & \text{for } f<0 \end{cases}$$

Recovering I and Q from $u(t)$:

I: $m_c(t) = \mathrm{Re}\{u(t)\} = \frac{1}{2}\left(u(t) + u^*(t)\right)$

Q: $m_s(t) = \mathrm{Im}\{u(t)\} = \frac{1}{2j}\left(u(t) - u^*(t)\right)$

In the freq. domain:

$$M_c(f) = \frac{1}{2}\left(U(f) + U^*(-f)\right)$$

$$M_s(f) = \frac{1}{2j}\left(U(f) - U^*(-f)\right)$$