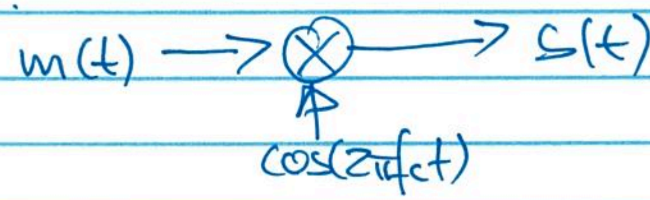
9/12/18

# QAM - Quadrature Amplitude Modulation

So far:



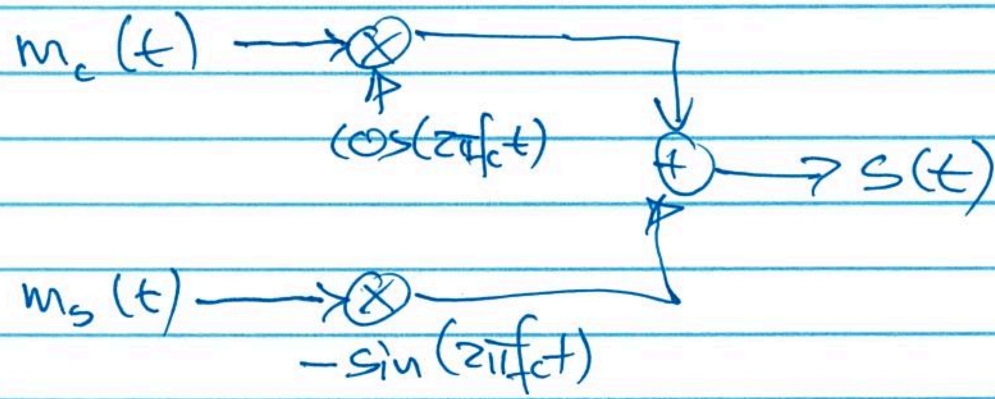

It is possible to transmit two messages
- at the same time
- in the same frequency band

E.g.: left and right channel of stereo

Principle:

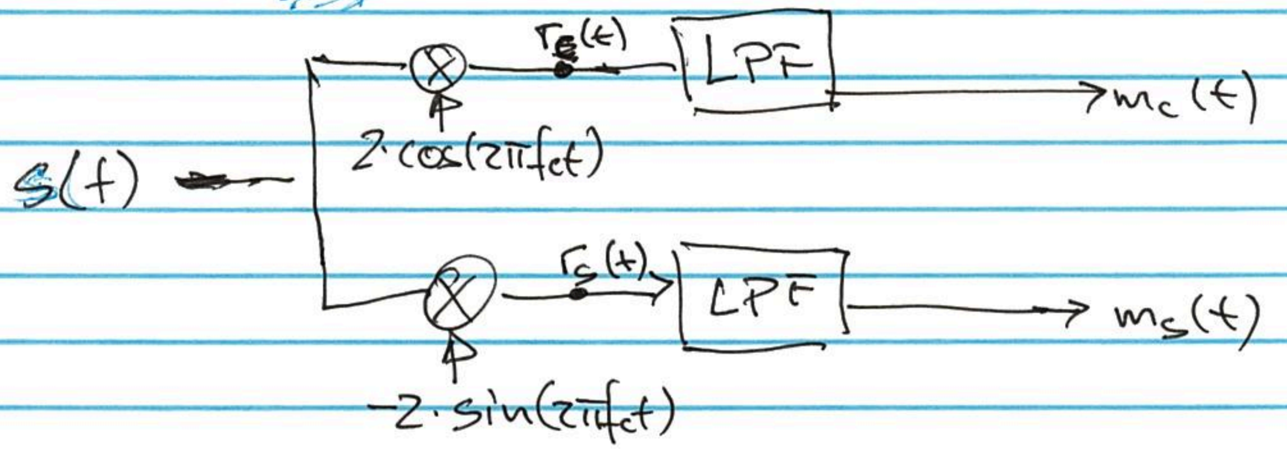
- modulate one signal w/ $\cos(2\pi f_c t)$
  - in-phase (I) channel
- modulate other signal w/ $-\sin(2\pi f_c t)$
  - quadrature (Q) channel

QAM Transmitter:



# QAM Demodulator:

- coherent demodulator
- assume perfect synchronization




consider signals after mixer: $(r_c(t), r_s(t))$

$$r_c(t) = s(t)\cdot 2\cos(2\pi f_c t)$$

$$= \underbrace{\left(m_c(t)\cdot\cos(2\pi f_c t) - m_s(t)\cdot\sin(2\pi f_c t)\right)}_{s(t)} 2\cos(2\pi f_c t)$$

$$= 2m_c(t)\cdot\cos^2(2\pi f_c t) - 2m_s(t)\cdot\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)$$

$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos 2x$

$$= 1\cdot m_c(t) + \underline{1\cdot m_c(t)\cdot\cos(4\pi f_c t)} - \underline{1\cdot m_s(t)\cdot\sin(4\pi f_c t)}$$

$\cos x\cdot\sin x = \frac{1}{2}\sin(2x)$

components at $2f_c$ will be rejected by LPF

$\Rightarrow$ I-branch (top) produces $\underline{\underline{m_c(t)}}$

## Q channel:

$$r_s(t) = s(t) \cdot \left(-2\sin(2\pi f_c t)\right)$$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$$= \left(m_c(t)\cdot\cos(2\pi f_c t) - m_s(t)\cdot\sin(2\pi f_c t)\right)\cdot\left(-2\sin(2\pi f_c t)\right)$$

$$= -2m_c(t)\cdot\cos(2\pi f_c t)\cdot\sin(2\pi f_c t) + 2m_s(t)\cdot\sin^2(2\pi f_c t)$$

$$= -m_c(t)\cdot\sin(4\pi f_c t) + \underline{m_s(t)} + m_s(t)\cdot\cos(4\pi f_c t)$$

rejected by LPF

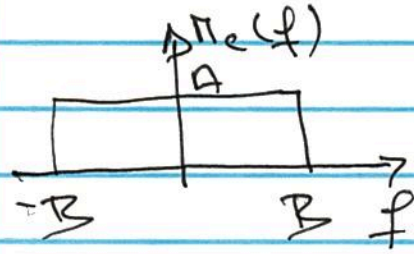
$\Rightarrow$ Q branch (bottom) produces $m_s(t)$

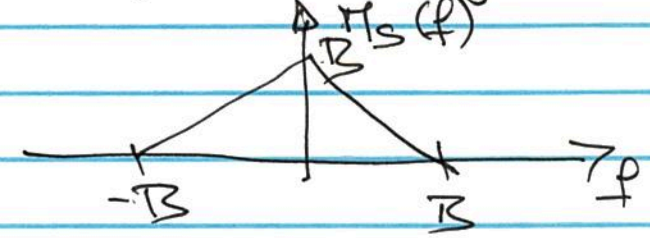
- Both signals are recovered
- no interference between signal

$\Rightarrow$ Two signals can be transmitted in the same band.

**But:** assumed perfect synchronization

## Fourier transform of QAM signal




Reminder:

$$\sin(2\pi f_c t) = \frac{1}{2j}\left(e^{j2\pi f_c t} - e^{-j2\pi f_c t}\right)$$

$$\frac{1}{j} = -j = e^{-j\pi/2} \qquad = \frac{1}{2} e^{-j\pi/2}\left(e^{j2\pi f_c t} - e^{-j2\pi f_c t}\right)$$

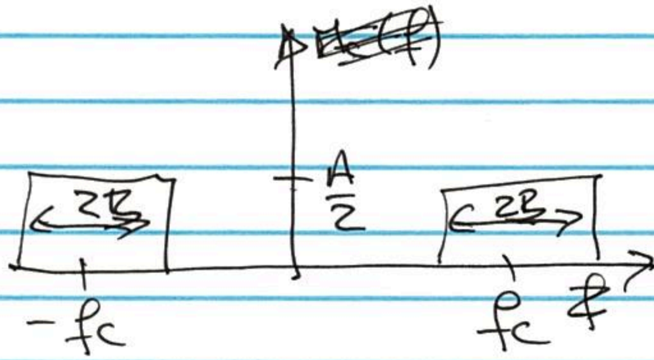
$$j = e^{j\pi/2} \qquad = \frac{1}{2} e^{-j\pi/2} e^{j2\pi f_c} + \frac{1}{2} e^{j\pi/2} e^{-j2\pi f_c t}$$

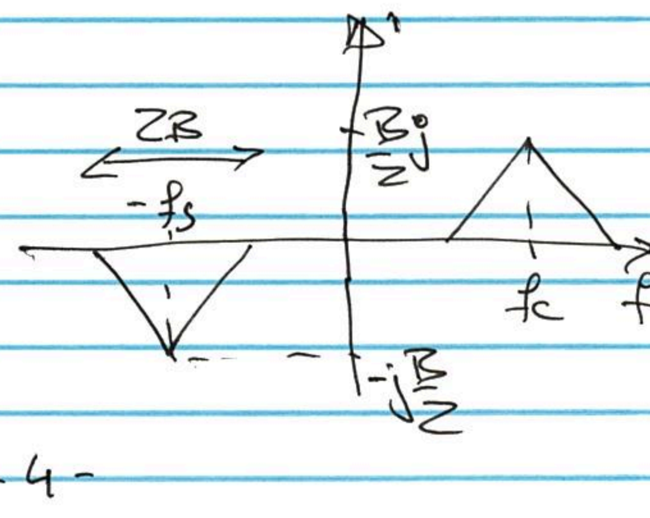
$$s(t) = \underbrace{m_c(t)\cdot\cos(2\pi f_c t)}_{\updownarrow} + m_s(t)\cdot\sin(2\pi f_c t)$$

$$\updownarrow$$

$$S(f) = \frac{1}{2}\left(M_c(f-f_c) + M_c(f+f_c)\right) - \frac{1}{2} e^{-j\pi/2} M_s(f-f_c) - \frac{1}{2} e^{j\pi/2} M_s(f+f_c)$$

$$= \frac{1}{2}\left(M_c(f-f_c) + M_c(f+f_c)\right) + \frac{j}{2}\cdot\left(M_s(f-f_c) - \frac{j}{2} M_s(f+f_c)\right)$$



# QAM Demodulator and phase error

Recall: phase and/or frequency offsets b/w transmitter and receiver are problems:

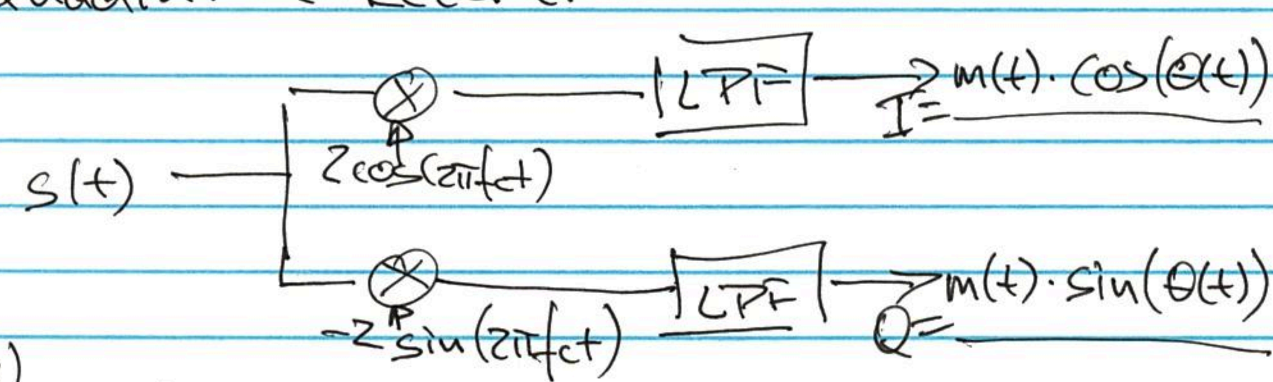
- drop out if $\Delta\phi = \frac{\pi}{2}$
- fade in and out with freq. error.

$$s(t) = m(t) \cdot \cos(2\pi f_c t + \theta(t))$$

$\theta(t)$: phase/freq offset

Receiver:


Quadrature Receiver




$\cos(x) \cdot \sin(y)$

$\frac{1}{2}\sin(x-y) + \frac{1}{2}\sin(x+y)$

m(t) can be recovered from I and Q channels: (assumes $\theta(t)$ is known)

$$\cancel{m(t)} \cdot I \cdot \cos(\theta(t)) + Q \cdot \sin(\theta(t))$$

$$= m(t) \quad \text{(Phase correction)}$$