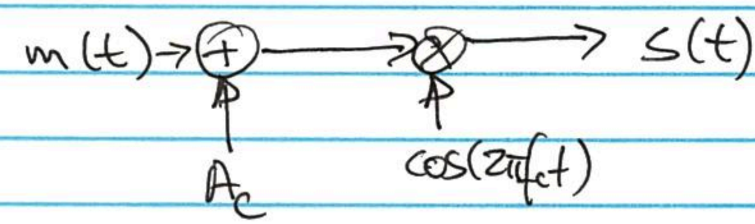
9/10/18

Conventional AM and non-coherent demod

Reminder: conventional AM



$$s(t) = (m(t) + A_c) \cdot \cos(2\pi f_c t)$$

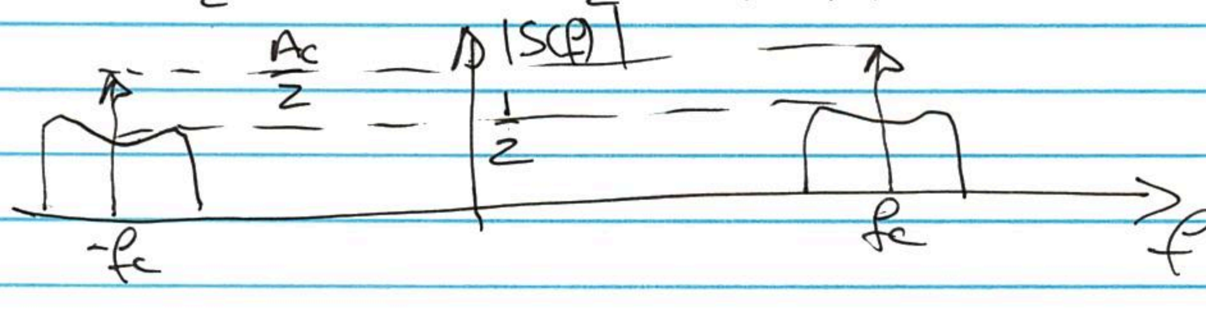
$$= \underbrace{m(t) \cdot \cos(2\pi f_c t)}_{\text{mod. signal}} + \underbrace{A_c \cdot \cos(2\pi f_c t)}_{\text{carrier}}$$

For non-coherent demodution, we need:

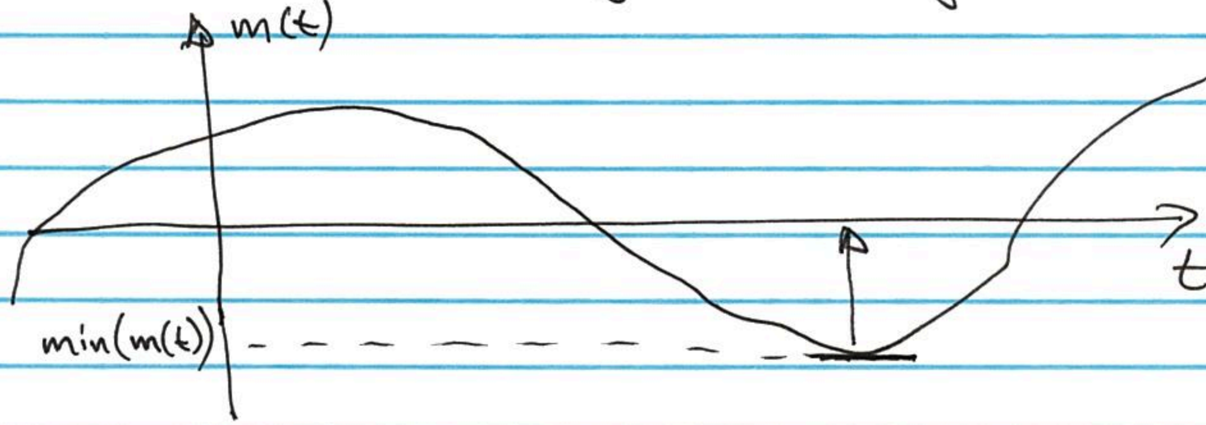
$$\boxed{A_c + m(t) \geq 0 \quad \text{for all } t}$$

Reminder: Spectrum $S(f)$

$$S(f) = \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c) + \frac{A_c}{2} \delta(f - f_c) + \frac{A_c}{2} \delta(f + f_c)$$



Time Domain: a typical signal



Condition $A_c + m(t) \geq 0$ for all t is satisfied if

$$A_c + \min(m(t)) \geq 0$$

$$\Rightarrow A_c \geq -\min(m(t))$$

b/c $\min(m(t)) < 0$ for zero-DC signals $\Rightarrow A_c \geq |\min(m(t))|$

To measure how far $A_c + m(t) > 0$, define modulation index:

$$a_{mod} = \frac{|\min(m(t))|}{A_c}$$

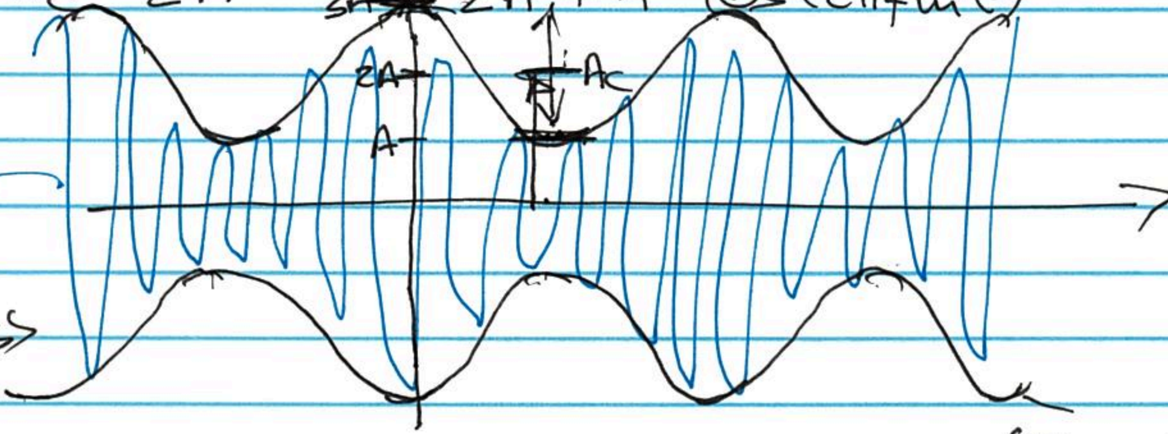
for $a_{mod} = 1 = 100\%$: $A_c + m(t) = 0$ for some

for $a_{mod} < 1$: $A_c + m(t) > 0$

for $a_{mod} > 1$: $A_c + m(t) < 0$ for some t

Illustrations: Let $m(t) = A \cdot \cos(2\pi f_m t)$

1.) $A_c = 2A$ $\quad 2A + A \cdot \cos(2\pi f_m t)$



$s(t)$

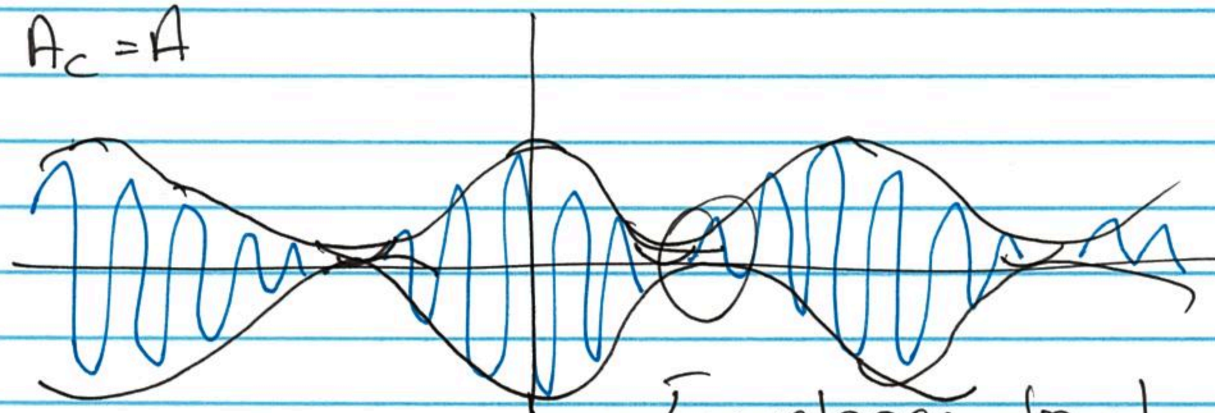
Envelopes don't cross

$-(2A + m(t))$

$A_c + m(t)$ — upper envelope
$-(A_c + m(t))$ — lower envelope
carrier oscillates b/w envelopes

mod. ind: $a_{mod} = \dfrac{|\min(m(t))|}{A_c} = \dfrac{+A}{2A} = 0.5$

2. $A_c = A$

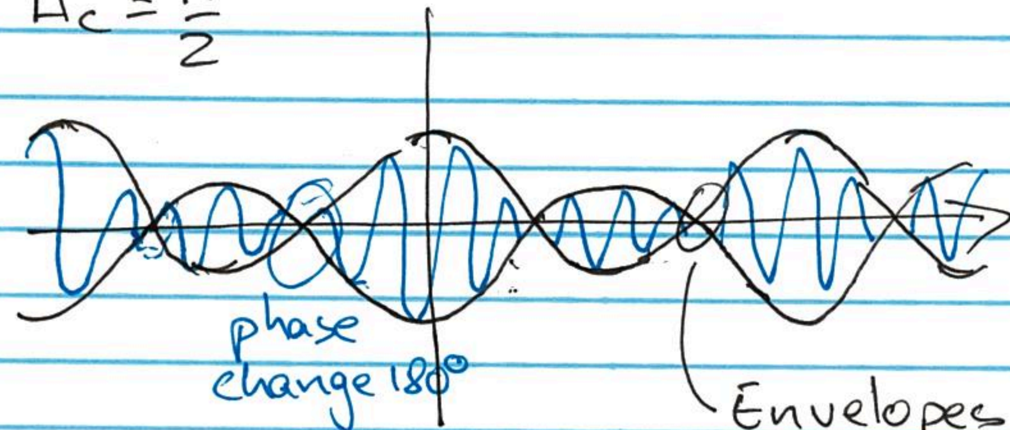


Envelopes touch

$$a_{mod} = \frac{|\min(m(t))|}{A_c} = \frac{|-A|}{A} = 1$$

100% mod. index

3.) $A_c = \frac{A}{2}$



$$a_{mod} = \frac{|\min(m(t))|}{A_c} = \frac{|-A|}{A/2} = 2$$

Non-coherent demodulator:

$$s(t) \longrightarrow \underset{\text{Rectifier}}{\boxed{|\cdot|}} \xrightarrow{g(t)} \underset{f_{cutoff} = B}{\boxed{\text{LPF}}} \longrightarrow \hat{m}(t)$$

B = signal BW

How can this possibly work?

Key: use of the non-linearity

After the rectifier:

$$y(t) = |s(t)|$$

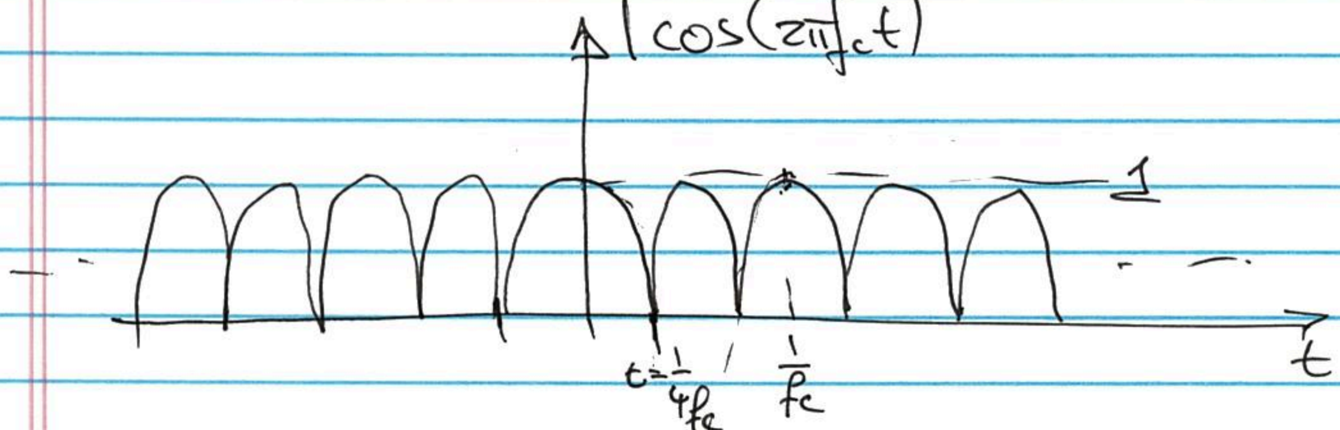
$$= |(A_c + m(t)) \cdot \cos(2\pi f_c t)|$$

$$= |\underbrace{A_c + m(t)}| \cdot |\cos(2\pi f_c t)|$$

> 0 if $a_{mod} < 1$

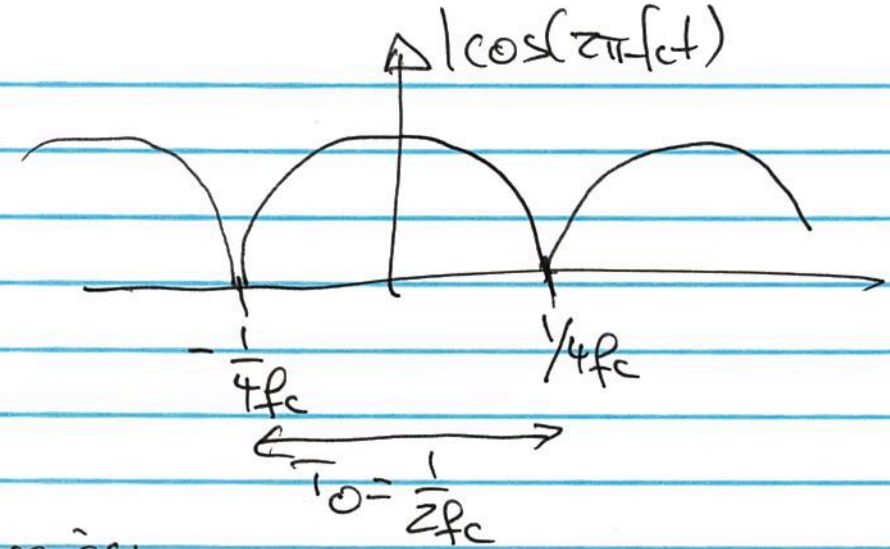
$$= (A_c + m(t)) \cdot |\cos(2\pi f_c t)|$$

if $a_{mod} \le 1$ then signal $A_c + m(t)$ is not affected by the non-linearity



To figure out what the LPF in receiver does, need $Y(f)$.

Plan:
- $A_c + m(t)$ (easy to Fourier X.f.)
- multiplication ⇒ conv. freq. domain
- $|\cos(2\pi f_c t)|$ ⇒ periodic ⇒ Fourier series ⇒ Fourier Xform



Fourier series:

$$|\cos(2\pi f_c t)| = \sum_{n=-\infty}^{\infty} X_n \cdot e^{j2\pi n f_0 t}$$

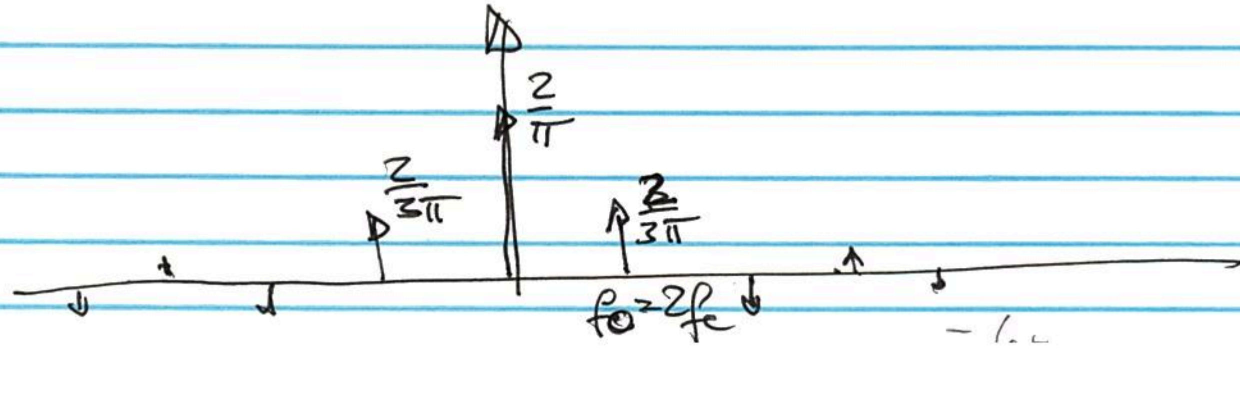
$$X_n = \frac{1}{T_0}\int_{-1/4f_c}^{1/4f_c} \cancel{|}\cos(2\pi f_c t)\cancel{|} \cdot e^{-j2\pi n f_0 t}\, dt$$

$\cos(2\pi f_c t) \geq 0$ over $-\frac{1}{4f_c} \leq t \leq \frac{1}{4f_c}$

$\vdots$

$$\boxed{X_n = \frac{2}{\pi} \cdot \frac{1}{4n^2-1} \cdot (-1)^{n+1}}$$

$$\Rightarrow |\cos(2\pi f_c t)| \Longleftrightarrow \sum_{n=-\infty}^{\infty} X_n \delta(f - nf_0)$$



- $A_c + m(t) \longleftrightarrow A_c\delta(f) + M(f)$
- multiplication $\longleftrightarrow$ convolution
- $|\cos(2\pi f_c t)| \longleftrightarrow$ see above

Therefore: