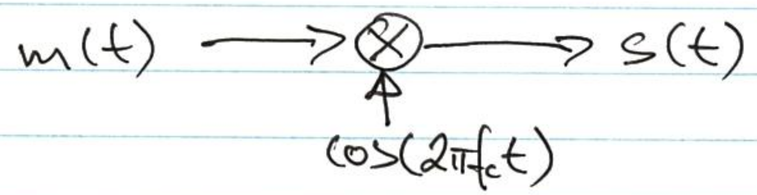
9/5/18

Demodulation of Modulated Signals

Last Time:

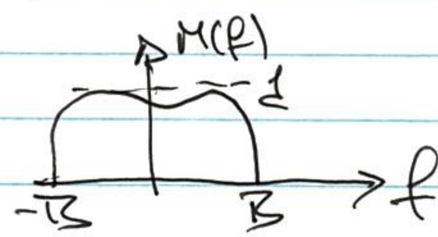


Time domain:

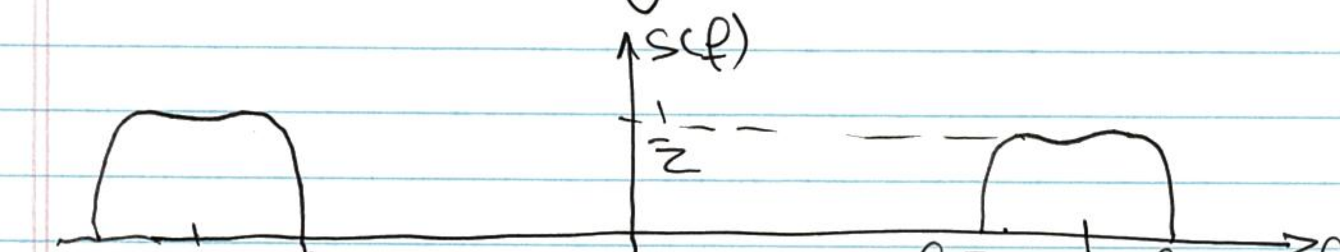
$$s(t) = m(t) \cdot \cos(2\pi f_c t)$$

Freq. domain:

$$S(f) = \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c)$$

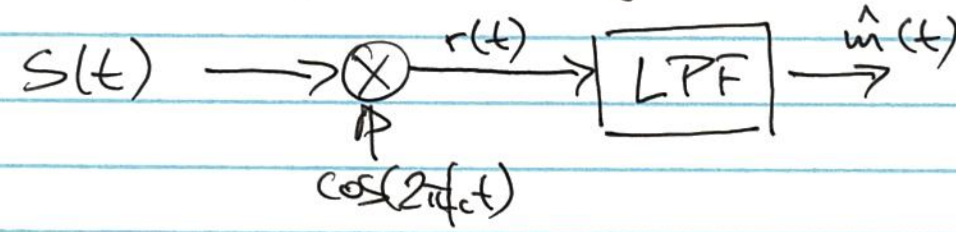


$\Downarrow$



Coherent demodulation:

The following system will recover the original signal:



This works, because:

$$r(t) = s(t) \cdot \cos(2\pi f_c t)$$

$$= \left(m(t) \cdot \cos(2\pi f_c t)\right) \cdot \cos(2\pi f_c t)$$

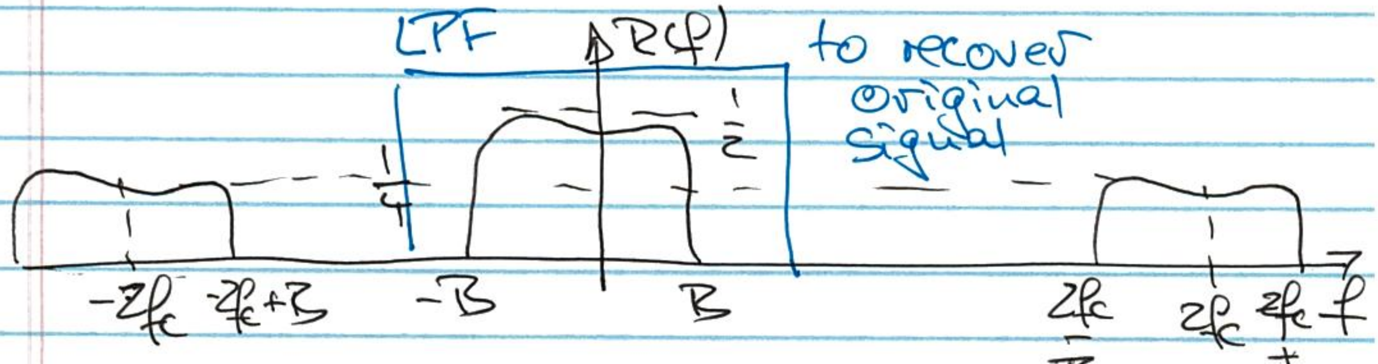
$$= m(t) \cdot \cos^2(2\pi f_c t) \qquad \Big| \cos^2 x = \tfrac{1}{2} + \tfrac{1}{2} \cos(2x)$$

$$= m(t) \cdot \tfrac{1}{2}\left(1 + \cos(4\pi f_c t)\right)$$

$$= \tfrac{1}{2} m(t) + \tfrac{1}{2} m(t) \cdot \cos(4\pi f_c t)$$

$$\Updownarrow$$

$$R(f) = \tfrac{1}{2} M(f) + \tfrac{1}{4} M(f - 2f_c) + \tfrac{1}{4} M(f + 2f_c)$$



A problem with coherent demodulation is that it requires carrier signals used by transmitter and receiver to be identical:

- same phase
- same frequency

This is <u>not</u> realistic:

- Carriers are generated from local oscillators
- different local oscillators will differ in phase and frequency
  + ~~they are~~ their frequency changes over time
- in addition propagation delay induces phase change

<u>Q:</u> What happens if carrier signals are not identical.

<u>Model for carrier at receiver:</u>

$$\cos(2\pi f_c t + \theta(t)) \Leftarrow$$

- eg $\theta(t) = \pi/2$
- or $\theta(t) = 2\pi f_d \cdot t$

demodulated signal with imperfect carrier:

$$\tilde{r}(t) = s(t) \cdot \cos(2\pi f_c t + \theta(t))$$

$$= (m(t) \cdot \cos(2\pi f_c t)) \cdot \cos(2\pi f_c t + \theta(t))$$

$\cos x \cdot \cos y = \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y)$

$$= \frac{1}{2} \cdot m(t) \cdot \cos(\theta(t)) + \frac{1}{2} m(t) \cdot \cos(4\pi f_c t + \theta(t))$$

After LPF: $\boxed{\hat{m}(t) = \frac{1}{2} m(t) \cdot \cos(\theta(t))}$

If $\theta(t) = 0 \Rightarrow$ no problem
perfect ~~syn~~ coherence

phase error: If $\theta(t) = \frac{\pi}{2} \Rightarrow \hat{m}(t) = 0$ signal vanishes

Frequency error (e.g. $f_d = 1\text{Hz}$): If $\theta(t) = 2\pi f_d t \Rightarrow \hat{m}(t) = \frac{1}{2} m(t) \cdot \cos(2\pi f_d \cdot t)$

signal fades in and out.

<u>Observation:</u> For coherent demodulation the receiver's oscillator must be synchronized with the carrier signal:
- in phase <u>and</u>
- in frequency

To address this problem, we have two options:

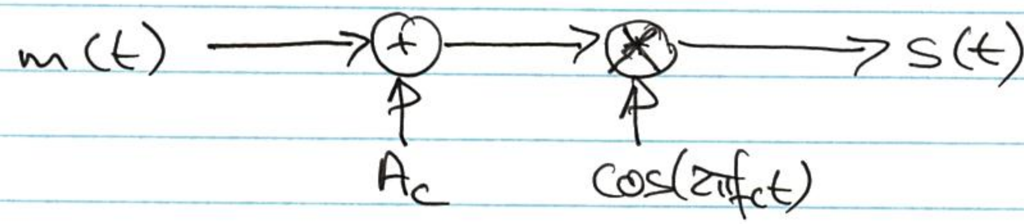
- non-coherent demodulation
- synchronization to a carrier transmitted with signal

For both of these, the carrier itself is transmitted along with the modulated signal.

$\Rightarrow$ conventional AM

Conventional AM:

Transmitter:

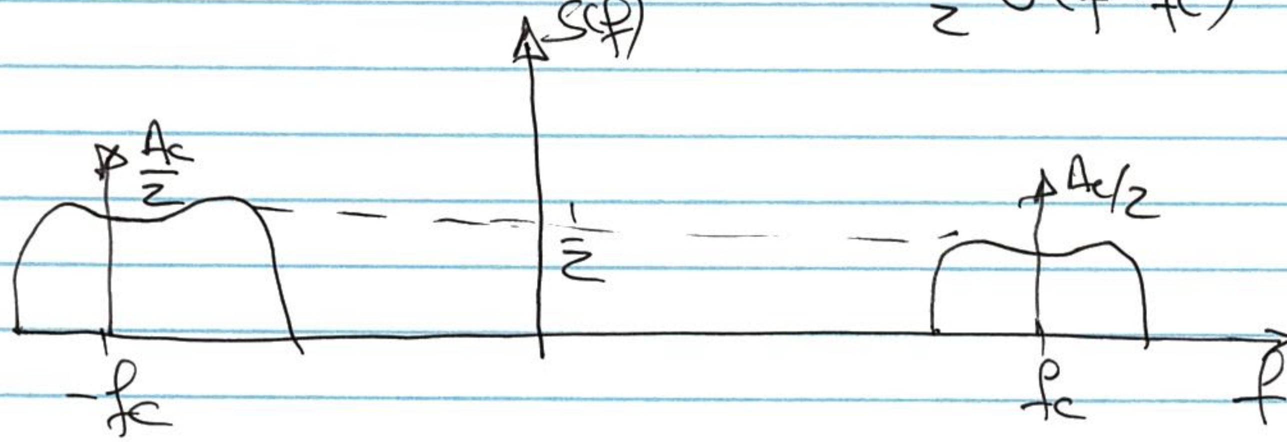


$$\Rightarrow s(t) = (m(t) + A_c) \cdot \cos(2\pi f_c t)$$

$$= m(t) \cdot \cos(2\pi f_c t) + A_c \cdot \cos(2\pi f_c t)$$

Fourier Transform:

$$S(f) = \frac{1}{2} M(f-f_c) + \frac{1}{2} M(f+f_c) + \frac{A_c}{2}\delta(f-f_c) + \frac{A_c}{2}\delta(f+f_c)$$



This is called

- double-sideband AM (DSB-AM
- or conventional AM
- when $A_c = 0$ ⇒ no carrier transmitted

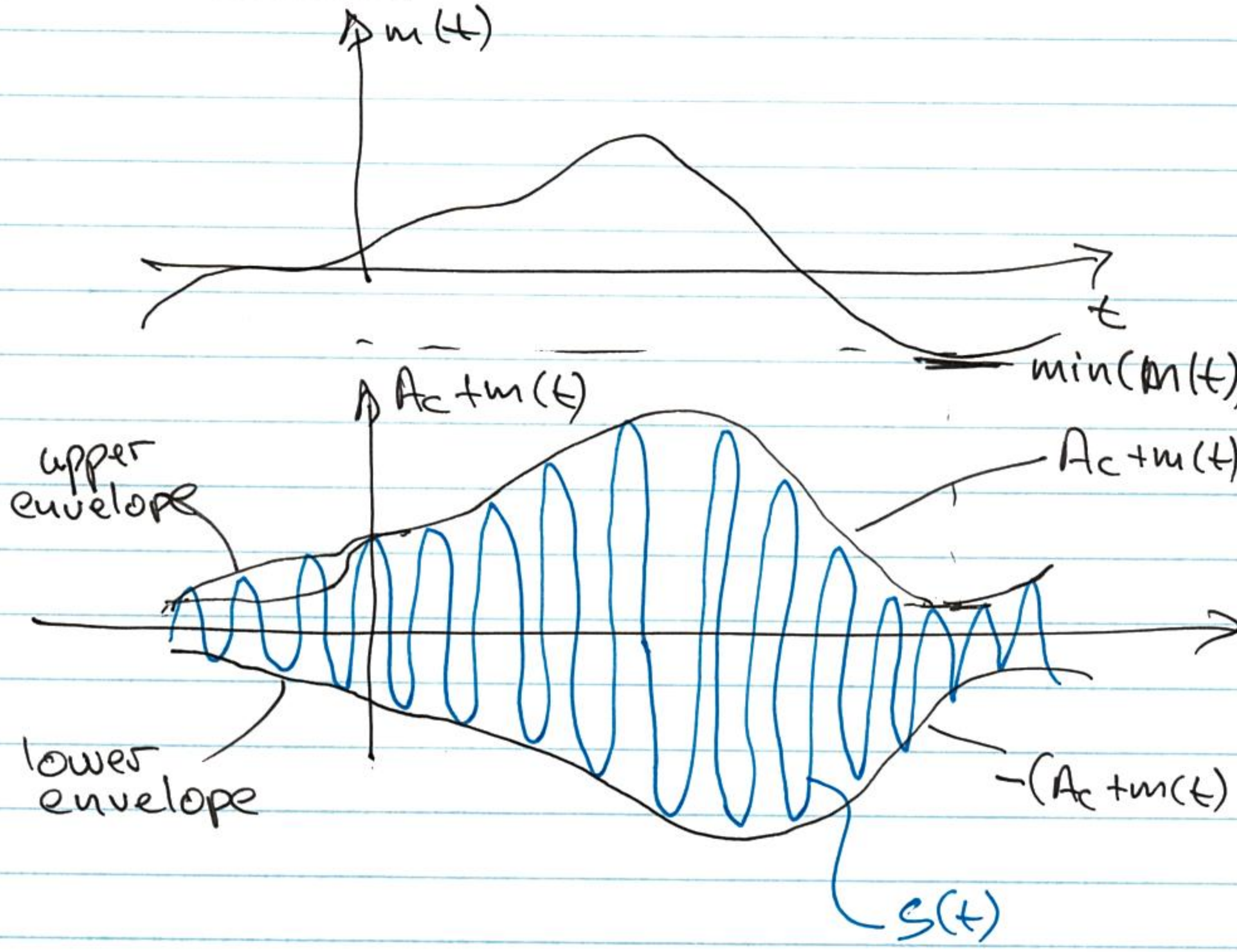
DSB-AM-SC (suppressed carrier)

Important case:

A_c is chosen such that

$$\boxed{A_c + m(t) \geq 0 \quad \text{for all } t}$$

Illustration:



Definition: modulation index

$$a_{mod} = \frac{|\min(m(t)|}{A_c}$$