8/29/18

Amplitude Modulation (AM)
(with suppressed carrier)

Plan: • a little bit of Fourier Transform rev.
• Frequency shifting ⇒ AM

Objective: - A fundamental problem in communications is moving a signal to a different location in spectrum

- The underlying tool to describe this process is the Fourier transform.

Reminder: Fourier transform $X(f)$ of signal $x(t)$:

$$\boxed{X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt}$$

Inverse transform:

$$\boxed{x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df}$$

Notation: $x(t) \longleftrightarrow X(f)$

Notes:

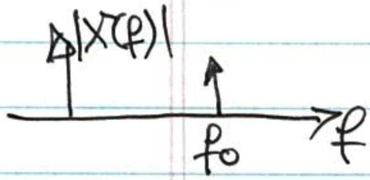
- The transform expessions are valid/defined if signals are of finite energy:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$$

- Infinite energy signals have Fourier transforms containing impulses

Examples: (of infinite energy signals)

a) complex exponential signals:



$$x(t) = e^{j2\pi f_0 t} \longleftrightarrow X(f) = \delta(f - f_0)$$

special case: $f_0 = 0$

$$\Rightarrow \quad x(t) = 1 \longleftrightarrow X(f) = \delta(f)$$

b) impulse function/signal

$$x(t) = \delta(t - \tau) \longleftrightarrow X(f) = e^{-j2\pi f \tau}$$

special case: $\tau = 0$

$$x(t) = \delta(t) \longleftrightarrow X(f) = 1$$

c) <u>periodic signals</u>

can be written as a Fourier series:

$$s(t) = \sum_{n=-\infty}^{\infty} s_n \cdot e^{j2\pi n \cdot f_0 t}$$

where $f_0 = \frac{1}{T_0}$

T_0 = period of signal

$$s_n = \frac{1}{T_0} \left\langle s(t), e^{j2\pi n f_0 t} \right\rangle$$

$$= \frac{1}{T_0} \int_{-\infty}^{\infty} s(t) \cdot e^{-j2\pi n f_0 t} \, dt$$

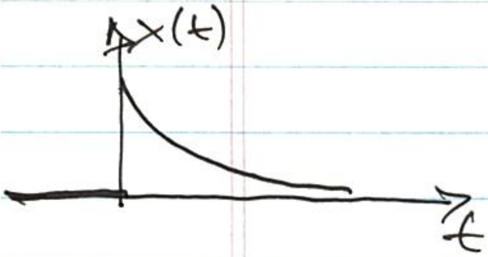
$$s(t) = \sum_{n=-\infty}^{\infty} s_n \cdot e^{j2\pi n f_0 t}$$

$$\updownarrow$$

$$S(f) = \sum_{n=-\infty}^{\infty} s_n \cdot \delta(f - n f_0)$$

Exercise:

Find the Fourier transform of



$$x(t) = \begin{cases} e^{-\alpha t} & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad \alpha > 0$$

$$= e^{-\alpha t} \cdot I_{(0,\infty)}(t)$$

where $I_{(a,b)}(t) = \begin{cases} 1 & a \leq t \leq b \\ 0 & \text{else} \end{cases}$

Solution:

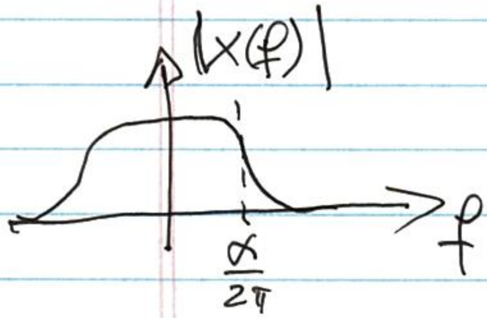
$$X(f) = \int_{-\infty}^{\infty} \underbrace{x(t)}_{\parallel} e^{-j2\pi f t} \, dt$$

$$= \int_{-\infty}^{\infty} \overbrace{e^{-\alpha t} I_{(0,\infty)}(t)} \, e^{-j2\pi f t} \, dt$$

$$= \int_{0}^{\infty} e^{-\alpha t} e^{-j2\pi f t} \, dt$$

$$= \int_{0}^{\infty} e^{-(\alpha + j2\pi f)t} \, dt$$

$$= \left. \frac{e^{-(\alpha + j2\pi f)t}}{-(\alpha + j2\pi f)} \right|_{0}^{\infty}$$



$$= 0 - \frac{1}{-(\alpha + j2\pi f)} = \boxed{\frac{1}{\alpha + j2\pi f}}$$

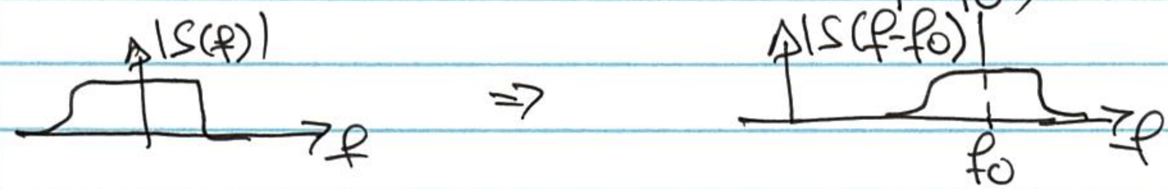
Frequency Translation Property:

Let $s(t) \longleftrightarrow S(f)$

Question: $s(t) \cdot e^{j2\pi f_0 t} \longleftrightarrow \,?$

$$\int_{-\infty}^{\infty} \left(s(t) \cdot e^{j2\pi f_0 t}\right) \cdot e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} s(t) \cdot e^{-j2\pi (f-f_0) \cdot t}\, dt$$

$$= S(f-f_0)$$



Modulation Property:

Let $s(t) \longleftrightarrow S(f)$

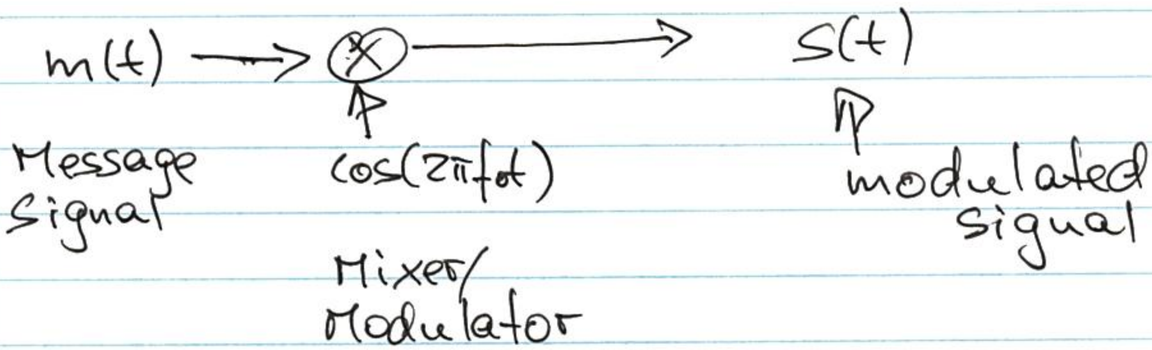
Q: $s(t) \cdot \cos(2\pi f_0 t) \longleftrightarrow \,?$

b/c $\cos(2\pi f_0 t) = \frac{1}{2} e^{j2\pi f_0 t} + \frac{1}{2} e^{-j2\pi f_0 t}$

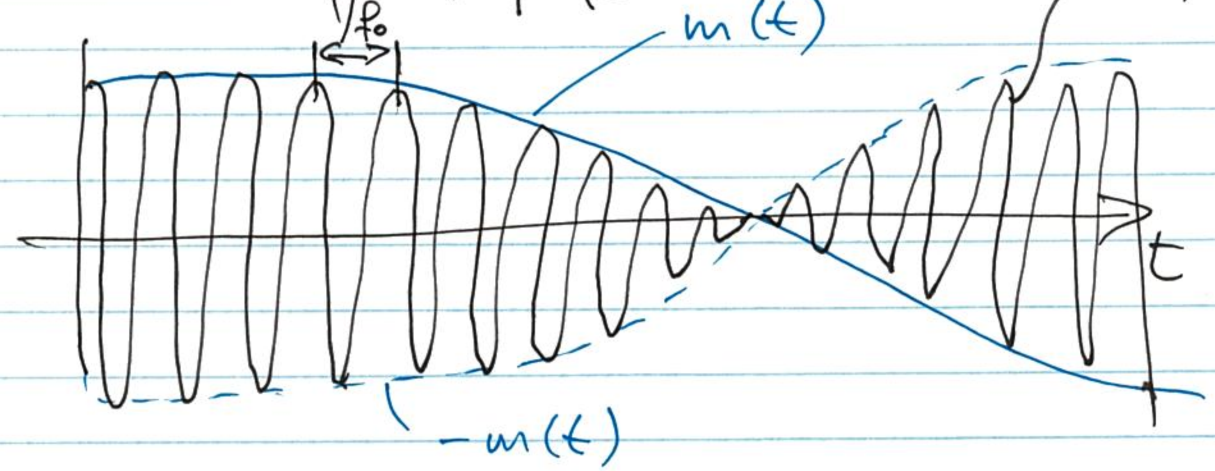
and Frequency Translation
and Linearity of Fourier Transform

$$s(t) \cdot \cos(2\pi f_0 t) \longleftrightarrow \frac{1}{2} S(f-f_0) + \frac{1}{2} S(f+f_0)$$

Block diagram of amplitude modulator.



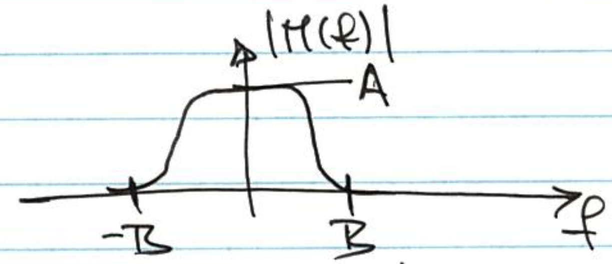
carrier freq: f_0



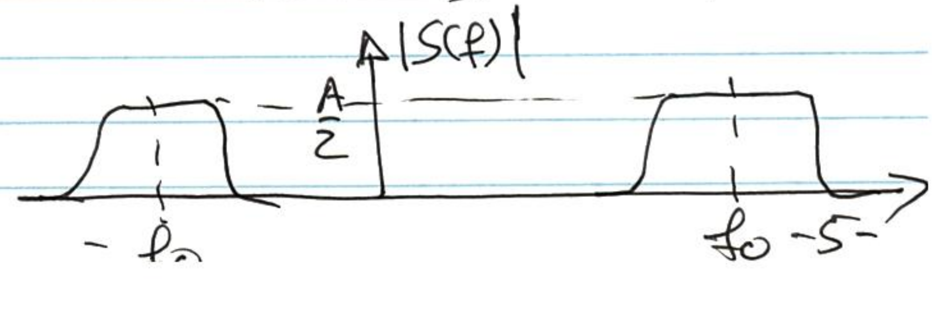
Notes: • f_0 is called the carrier frequency

• In AM, $f_0 \gg B$ (B = highest freq. in $M(f)$)

Baseband ~~Sig~~ spectrum



Modulated (Passband) Signal



Example: 1. Sinusoidal Message

$$m(t) = \cos(2\pi f_M t) \longleftrightarrow M(f) = \frac{1}{2}\delta(f-f_M) + \frac{1}{2}\delta(f+f_M)$$

Amplitude modulation:

$$s(t) = m(t)\cdot\cos(2\pi f_0 t)$$

Q: $S(f) = ?$

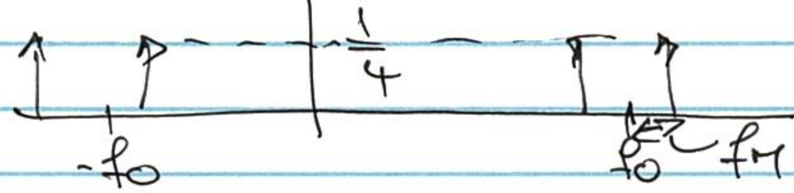
a) via trig identities:

$$s(t) = \cos(2\pi f_M t)\cdot\cos(2\pi f_0 t)$$

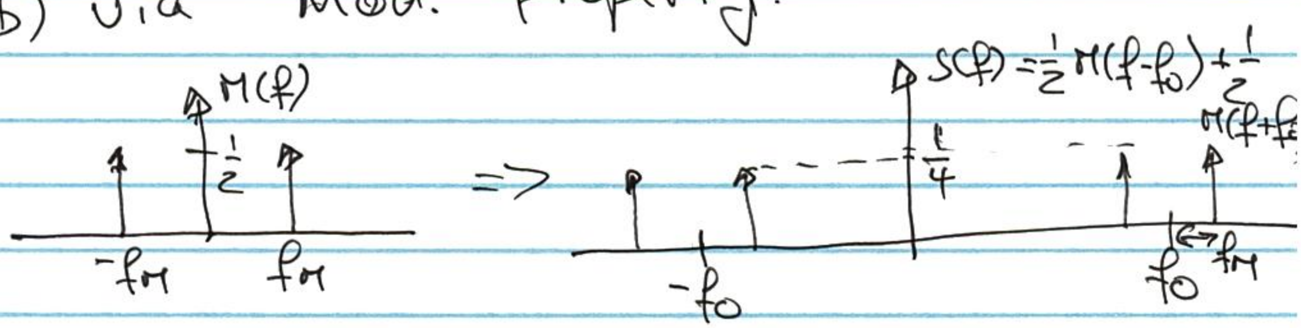
$\cos a \cdot \cos b = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b)$

$$= \frac{1}{2}\cos(2\pi(f_0-f_M)t) + \frac{1}{2}\cos(2\pi(f_0+f_M)t)$$

$$\Rightarrow S(f) = \frac{1}{4}\cdot\Big[\delta(f-(f_0-f_M)) + \delta(f+(f_0-f_M)) + \delta(f-(f_0+f_M)) + \delta(f+(f_0+f_M))\Big]$$



b) via mod. property:



Example 2: One-sided exponential

$$m(t) = e^{-\alpha t} \cdot \mathbb{I}_{(0,\infty)}(t)$$

$$\updownarrow$$

$$M(f) = \frac{1}{\alpha + j2\pi f}$$

$$s(t) = m(t) \cdot \cos(2\pi f_0 t)$$

$$\updownarrow$$

$$S(f) = ?$$

From modulation property:

$$S(f) = \frac{1}{2} M(f-f_c) + \frac{1}{2} M(f+f_c)$$

$$= \frac{1}{2} \left(\frac{1}{\alpha + j2\pi(f-f_c)} + \frac{1}{\alpha + j2\pi(f+f_c)} \right)$$