Lecture: From Time-Domain to Frequency-Domain and back

Time-domain and Frequency-domain

- Signals are *naturally* observed in the time-domain.
- A signal can be illustrated in the time-domain by plotting it as a function of time.
- The frequency-domain provides an alternative perspective of the signal based on sinusoids:
  - Starting point: arbitrary signals can be expressed as sums of sinusoids (or equivalently complex exponentials).
  - The frequency-domain representation of a signal indicates which complex exponentials must be combined to produce the signal.
  - Since complex exponentials are fully described by amplitude, phase, and frequency it is sufficient to just specify a list of theses parameters.
    - Actually, we list pairs of complex amplitudes ($Ae^{j\phi}$) and frequencies f and refer to this list as $X(f)$.

Time-domain and Frequency-domain

- It is possible (but not necessarily easy) to find $X(f)$ from $x(t)$: this is called Fourier or spectrum analysis.
- Similarly, one can construct $x(t)$ from the spectrum $X(f)$: this is called Fourier synthesis.
- Notation: $x(t) \leftrightarrow X(f)$.
- Example (from last time):
  - **Time-domain:** signal

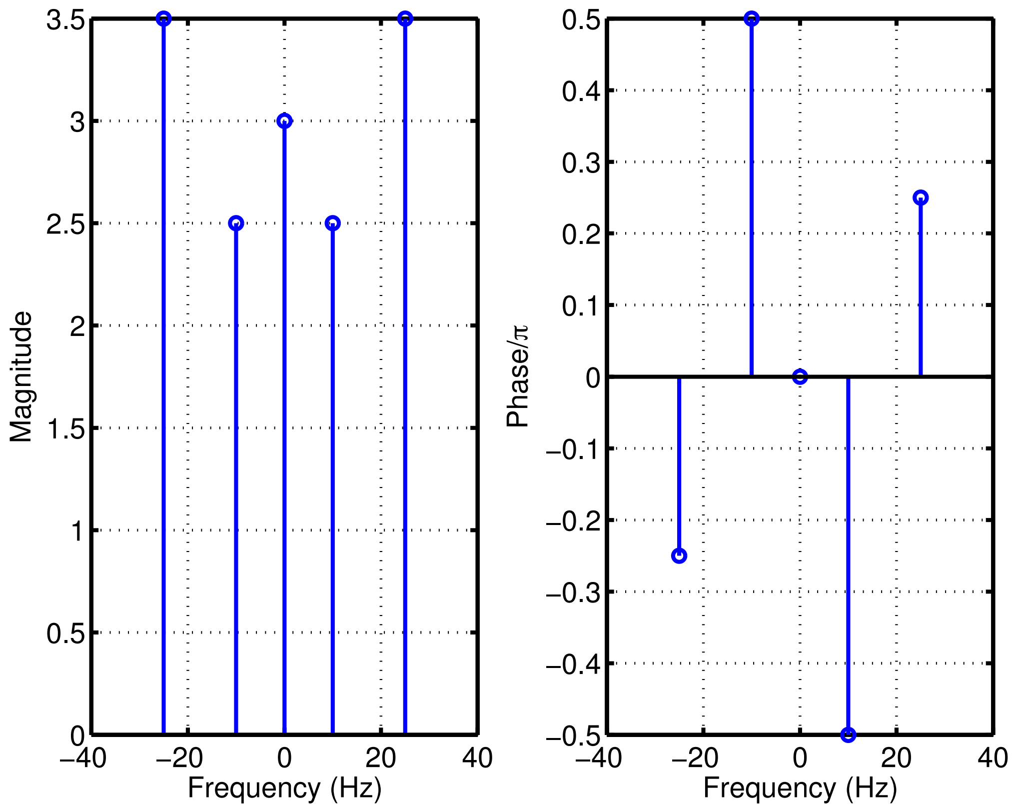
$$x(t) = 3 + 5\cos(20\pi t - \pi/2) + 7\cos(50\pi t + \pi/4).$$

  - **Frequency Domain:** spectrum

$$X(f) = \{(3,0), \quad (\tfrac{5}{2}e^{-j\pi/2}, 10), (\tfrac{5}{2}e^{j\pi/2}, -10), (\tfrac{7}{2}e^{j\pi/4}, 25), (\tfrac{7}{2}e^{-j\pi/4}, -25)\}$$

Plotting a Spectrum

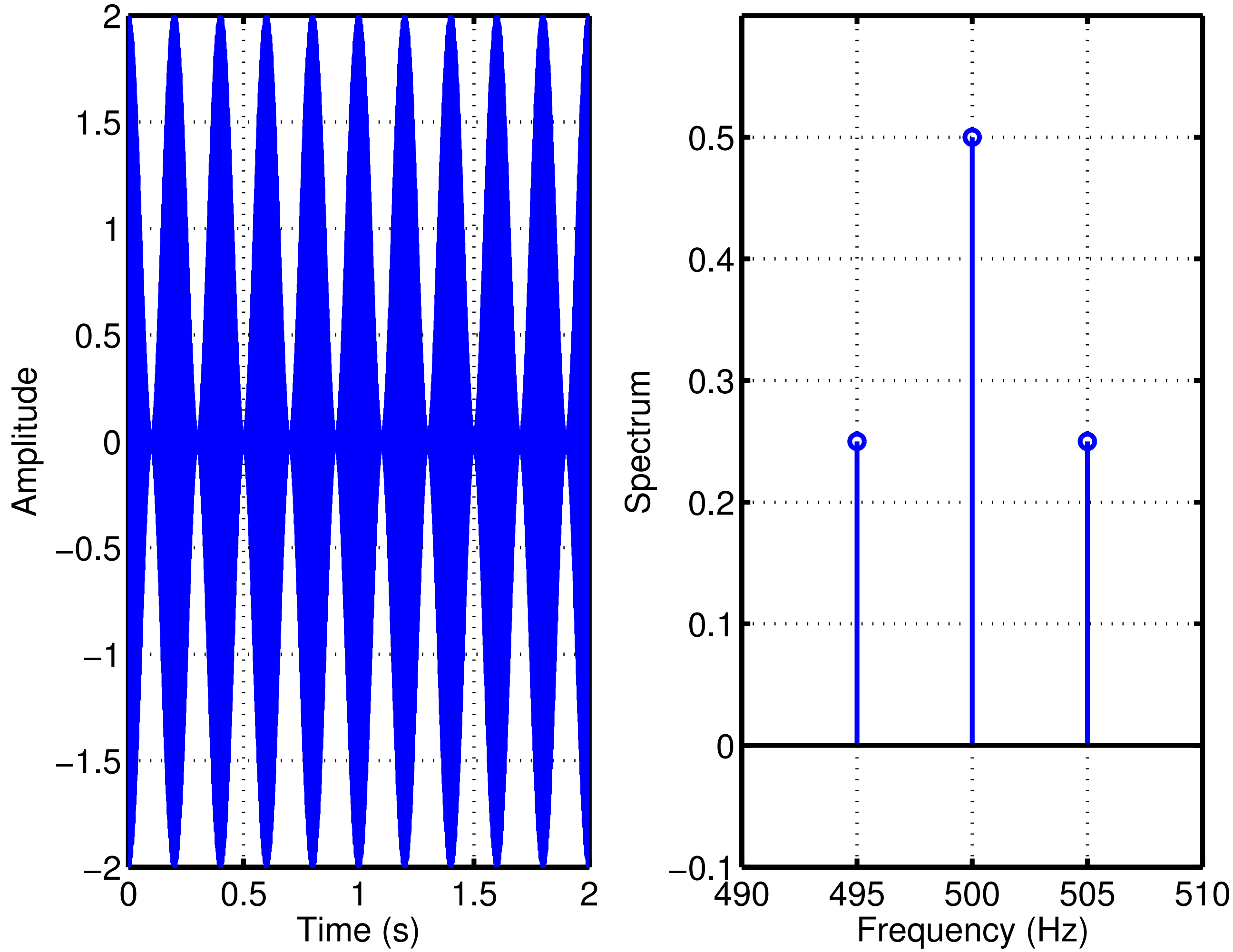
- To illustrate the spectrum of a signal, one typically plots the magnitude versus frequency.
- Sometimes the phase is plotted versus frequency as well.



Why Bother with the Frequency-Domain?

- In many applications, the frequency contents of a signal is very important.
  - For example, in radio communications signals must be limited to occupy only a set of frequencies allocated by the FCC.
  - Hence, understanding and analyzing the spectrum of a signal is crucial from a regulatory perspective.
- Often, features of a signal are much easier to understand in the frequency domain. (Example on next slides).
- We will see later in this class, that the frequency-domain interpretation of signals is very useful in connection with linear, time-invariant systems.
  - Example: A low-pass filter retains low frequency components of the spectrum and removes high-frequency components.

Example: Original signal



Example: Corrupted signal