Lecture: Introduction to Sinusoids
The general formula for a sinusoidal signal is

\[ x(t) = A \cdot \cos(2\pi ft + \phi). \]

- **A**, **f**, and **ϕ** are parameters that characterize the sinusoidal signal.
  - **A** - Amplitude: determines the height of the sinusoid.
  - **f** - Frequency: determines the number of cycles per second.
  - **ϕ** - Phase: determines the location of the sinusoid.
The formula for this sinusoid is:

\[ x(t) = 3 \cos(2\pi \cdot 50 \cdot t + \pi/4). \]
The Significance of Sinusoidal Signals

- Fundamental building blocks for describing arbitrary signals.
  - General signals can be expresssed as sums of sinusoids (Fourier Theory)
  - Provides bridge to frequency domain.
- Sinusoids are *special signals* for linear filters (eigenfunctions).
- Sinusoids occur naturally in many situations.
  - They are solutions of differential equations of the form
    \[
    \frac{d^2 x(t)}{dt^2} + ax(t) = 0.
    \]
- Much more on these points as we proceed.
Background: The cosine function

The properties of sinusoidal signals stem from the properties of the cosine function:

- **Periodicity**: \( \cos(x + 2\pi) = \cos(x) \)
- **Eveness**: \( \cos(-x) = \cos(x) \)
- **Ones of cosine**: \( \cos(2\pi k) = 1 \), for all integers \( k \).
- **Minus ones of cosine**: \( \cos(\pi(2k + 1)) = -1 \), for all integers \( k \).
- **Zeros of cosine**: \( \cos(\frac{\pi}{2}(2k + 1)) = 0 \), for all integers \( k \).
- **Relationship to sine function**: \( \sin(x) = \cos(x - \frac{\pi}{2}) \) and \( \cos(x) = \sin(x + \frac{\pi}{2}) \).
Amplitude

- The amplitude $A$ is a scaling factor.
- It determines how large the signal is.
- Specifically, the sinusoid oscillates between $+A$ and $-A$. 
Sinusoidal Signals

Frequency and Period

- Sinusoids are \textit{periodic} signals.
- The frequency $f$ indicates how many times the sinusoid repeats per second.
- The duration of each cycle is called the \textit{period} of the sinusoid. It is denoted by $T$.
- The relationship between frequency and period is
  \[ f = \frac{1}{T} \text{ and } T = \frac{1}{f}. \]
Phase and Delay

- The phase $\phi$ causes a sinusoid to be shifted sideways.
- A sinusoid with phase $\phi = 0$ has a maximum at $t = 0$.
- A sinusoid that has a maximum at $t = t_1$ can be written as

$$x(t) = A \cdot \cos(2\pi f(t - t_1)).$$

- Expanding the argument of the cosine leads to

$$x(t) = A \cdot \cos(2\pi ft - 2\pi ft_1).$$

- Comparing to the general formula for a sinusoid reveals

$$\phi = -2\pi ft_1 \text{ and } t_1 = \frac{-\phi}{2\pi f}.$$
Sinusoidal Signals

\[ T = \frac{1}{f} \]

Time (s)
Exercise

1. Plot the sinusoid

\[ x(t) = 2 \cos(2\pi \cdot 10 \cdot t + \pi/2) \]

between \( t = -0.1 \) and \( t = 0.2 \).

2. Find the equation for the sinusoid in the following plot