	Introduction to Sampling
00	00
	000000
	000000000000000000000000000000000000000
	•
	00000

The Sampling Theorem

We have analyzed the relationship between the frequency f of a sinusoid and the sampling rate f_s.

- We saw that the ratio f/f_s must be less than 1/2, i.e., $f_s > 2 \cdot f$. Otherwise aliasing or folding occurs.
- This insight provides the first half of the famous sampling theorem

A continuous-time signal x(t) with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2 \cdot f_{max}$.

This very import result is attributed to Claude Shannon and Harry Nyquist.



Reconstructing a Signal from Samples

- The sampling theorem suggests that the original continuous-time signal x(t) can be recreated from its samples x[n].
 - Assuming that samples were taken at a high enough rate.
 - This process is referred to as reconstruction or D-to-C conversion (discrete-time to continuous-time conversion).
- In principle, the continous-time signal is reconstructed by placing a suitable pulse at each sample location and adding all pulses.
 - The amplitude of each pulse is given by the sample value.



Suitable Pulses

Suitable pulses include

Rectangular pulse (zero-order hold):

$$p(t) = \begin{cases} 1 & \text{for } -T_s/2 \le t < T_s/2 \\ 0 & \text{else.} \end{cases}$$

Triangular pulse (linear interpolation)

$$p(t) = \begin{cases} 1 + t/T_s & \text{for } -T_s \le t \le 0\\ 1 - t/T_s & \text{for } 0 \le t \le T_s\\ 0 & \text{else.} \end{cases}$$



Reconstruction

The reconstructed signal $\hat{x}(t)$ is computed from the samples and the pulse p(t):

$$\hat{x}(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot p(t - nT_s).$$

- The reconstruction formula says:
 - ▶ place a pulse at each sampling instant $(p(t nT_s))$,
 - scale each pulse to amplitude x[n],
 - add all pulses to obtain the reconstructed signal.



	Introduction to Sampling	
00	00	
	000000	
	000000000000000000000000000000000000000	
	0	
	00000	

Ideal Reconstruction

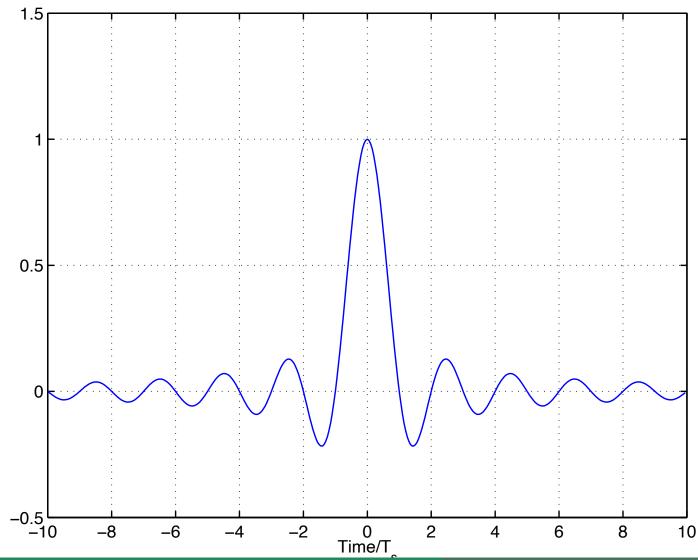
- Reconstruction with the above pulses will be pretty good.
 - Particularly, when the sampling rate is much greater than twice the signal frequency (significant oversampling).
- However, reconstruction is not perfect as suggested by the sampling theorem.
- To obtain perfect reconstruction the following pulse must be used:

$$p(t) = \frac{\sin(\pi t/T_s)}{\pi t/T_s}$$

- This pulse is called the sinc pulse.
- Note, that it is of infinite duration and, therefore, is not practical.
 - In practice a truncated version may be used for excellent reconstruction.



The sinc pulse



MASON UNIVERSITY

©2009-2019, B.-P. Paris

ECE 201: Intro to Signal Analysis