Amplitude Modulation

- Amplitude Modulation is used in communication systems.
- The objective of amplitude modulation is to move the spectrum of a signal $m(t)$ from low frequencies to high frequencies.
  - The message signal $m(t)$ may be a piece of music; its spectrum occupies frequencies below 20 KHz.
  - For transmission by an AM radio station this spectrum must be moved to approximately 1 MHz.

Amplitude Modulation

- Conventional amplitude modulation proceeds in two steps:
  1. A constant A is added to $m(t)$ such that $A + m(t) > 0$ for all t.
  2. The sum signal $A + m(t)$ is multiplied by a sinusoid $\cos(2\pi f_c t)$, where f_c is the radio frequency assigned to the station.
- Consequently, the transmitted signal has the form:

$$x(t) = (A + m(t)) \cdot \cos(2\pi f_c t).$$

Amplitude Modulation

- We are interested in the spectrum of the AM signal.
- However, we cannot compute $X(f)$ for arbitrary message signals $m(t)$.
- For the special case $m(t) = \cos(2\pi f_m t)$ we can find the spectrum.
  - To mimic the radio case, f_m would be a frequency in the audible range.
- As before, we will first need to express the AM signal $x(t)$ as a sum of sinusoids.

Amplitude Modulated Signal

- For $m(t) = \cos(2\pi f_m t)$, the AM signal equals

$$x(t) = (A + \cos(2\pi f_m t)) \cdot \cos(2\pi f_c t).$$

- This simplifies to

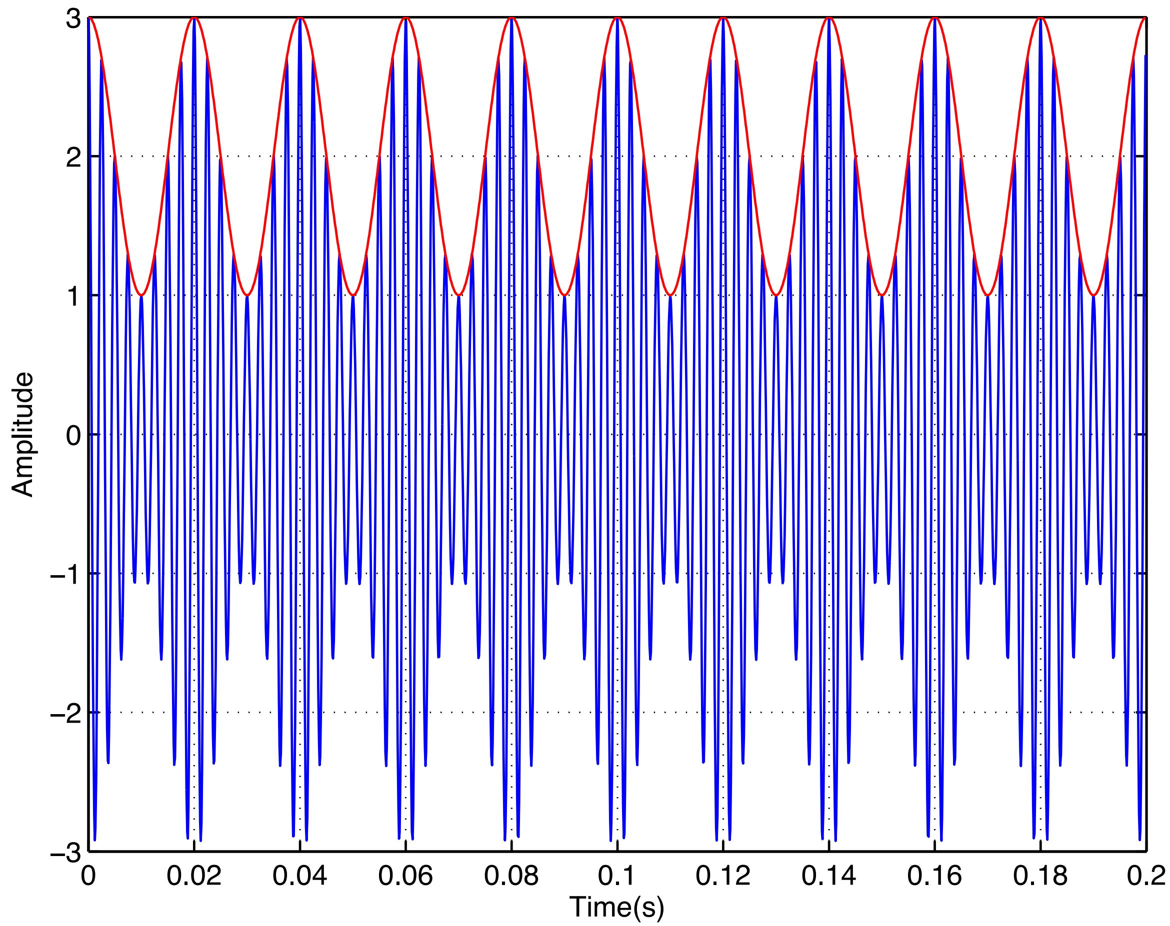
$$x(t) = A \cdot \cos(2\pi f_c t) + \cos(2\pi f_m t) \cdot \cos(2\pi f_c t).$$

- Note that the second term of the sum is a beat notes signal with frequencies f_m and f_c.
- We know that beat notes can be written as a sum of sinusoids with frequencies equal to the sum and difference of f_m and f_c:

$$x(t) = A \cdot \cos(2\pi f_c t) + \frac{1}{2} \cos(2\pi (f_c + f_m) t) + \frac{1}{2} \cos(2\pi (f_c - f_m) t).$$

Plot of Amplitude Modulated Signal

For $A = 2$, $fm = 50$, and $fc = 400$, the AM signal is plotted below.



Spectrum of Amplitude Modulated Signal

- The AM signal is given by

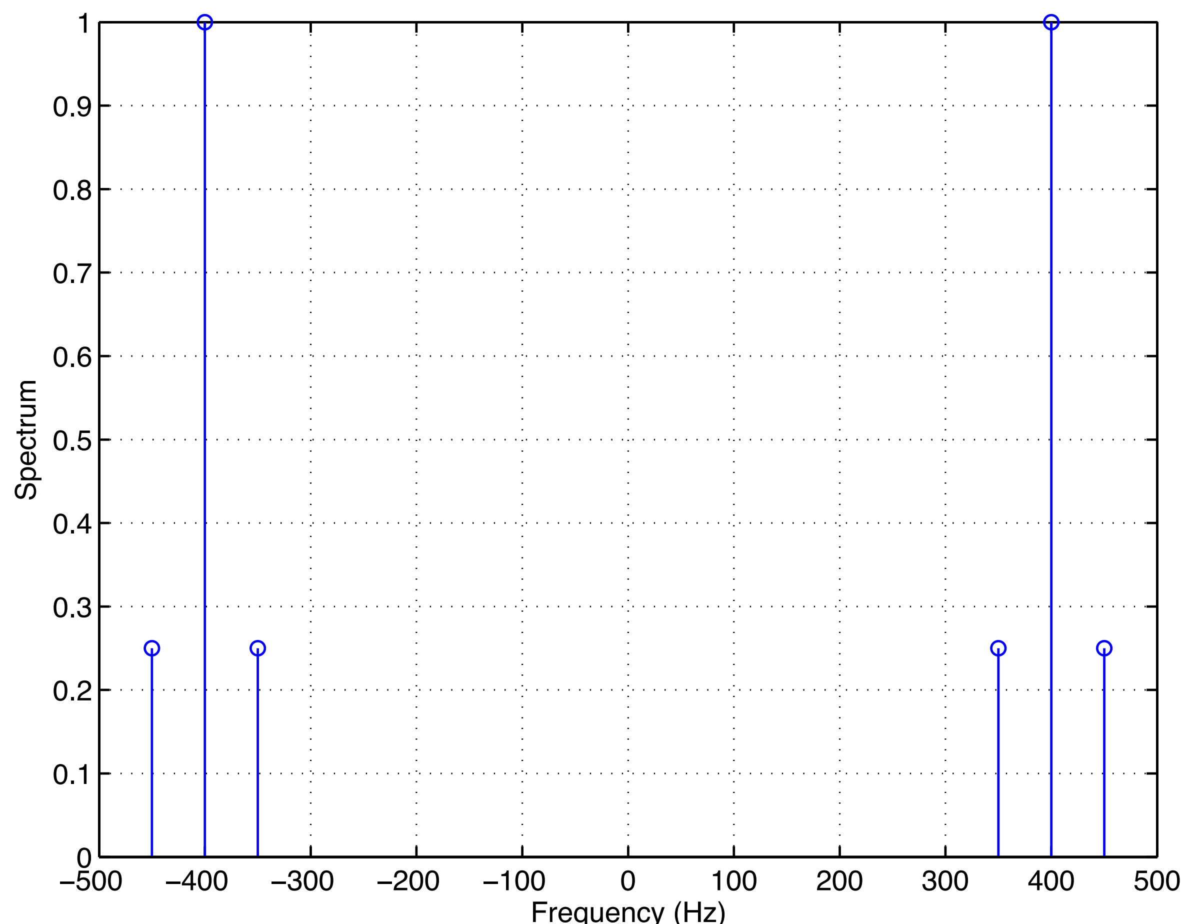
$$x(t) = A \cdot \cos(2\pi f_c t) + \frac{1}{2}\cos(2\pi(f_c + f_m)t) + \frac{1}{2}\cos(2\pi(f_c - f_m)t).$$

- Thus, its spectrum is

$$X(f) = \{ \quad (\tfrac{A}{2}, f_c), (\tfrac{A}{2}, -f_c), \\ (\tfrac{1}{4}, f_c + f_m), (\tfrac{1}{4}, -f_c - f_m), (\tfrac{1}{4}, f_c - f_m), (\tfrac{1}{4}, -f_c + f_m)\}$$

Spectrum of Amplitude Modulated Signal

For $A = 2$, $fm = 50$, and $fc = 400$, the spectrum of the AM signal is plotted below.



Spectrum of Amplitude Modulated Signal

- It is interesting to compare the spectrum of the signal before modulation and after multiplication with $\cos(2\pi f_c t)$.
- The signal $s(t) = A + m(t)$ has spectrum

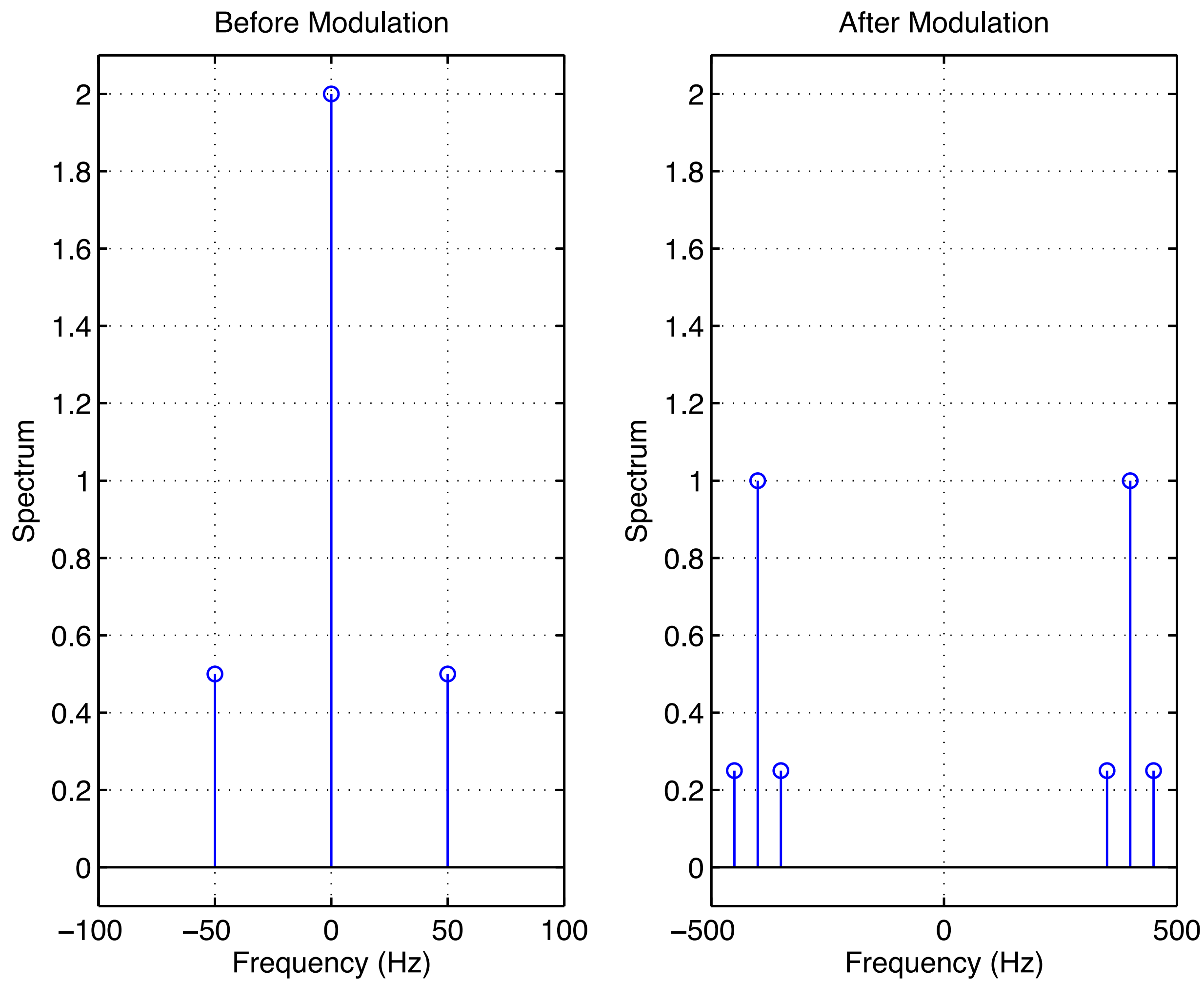
$$S(f) = \{(A, 0), (\frac{1}{2}, 50), (\frac{1}{2}, -50)\}.$$

- The modulated signal $x(t)$ has spectrum

$$X(f) = \{ \quad (\tfrac{A}{2}, 400), (\tfrac{A}{2}, -400), \\ (\tfrac{1}{4}, 450), (\tfrac{1}{4}, -450), (\tfrac{1}{4}, 350), (\tfrac{1}{4}, -350)\}$$

- Both are plotted on the next page.

Spectrum before and after AM



Spectrum before and after AM

- Comparison of the two spectra shows that amplitude modulation indeed moves a spectrum from low frequencies to high frequencies.
- Note that the shape of the spectrum is precisely preserved.
- Amplitude modulation can be described concisely by stating:
  - Half of the original spectrum is shifted by f_c to the right, and the other half is shifted by f_c to the left.
- **Question:** How can you get the original signal back so that you can listen to it.
  - This is called demodulation.