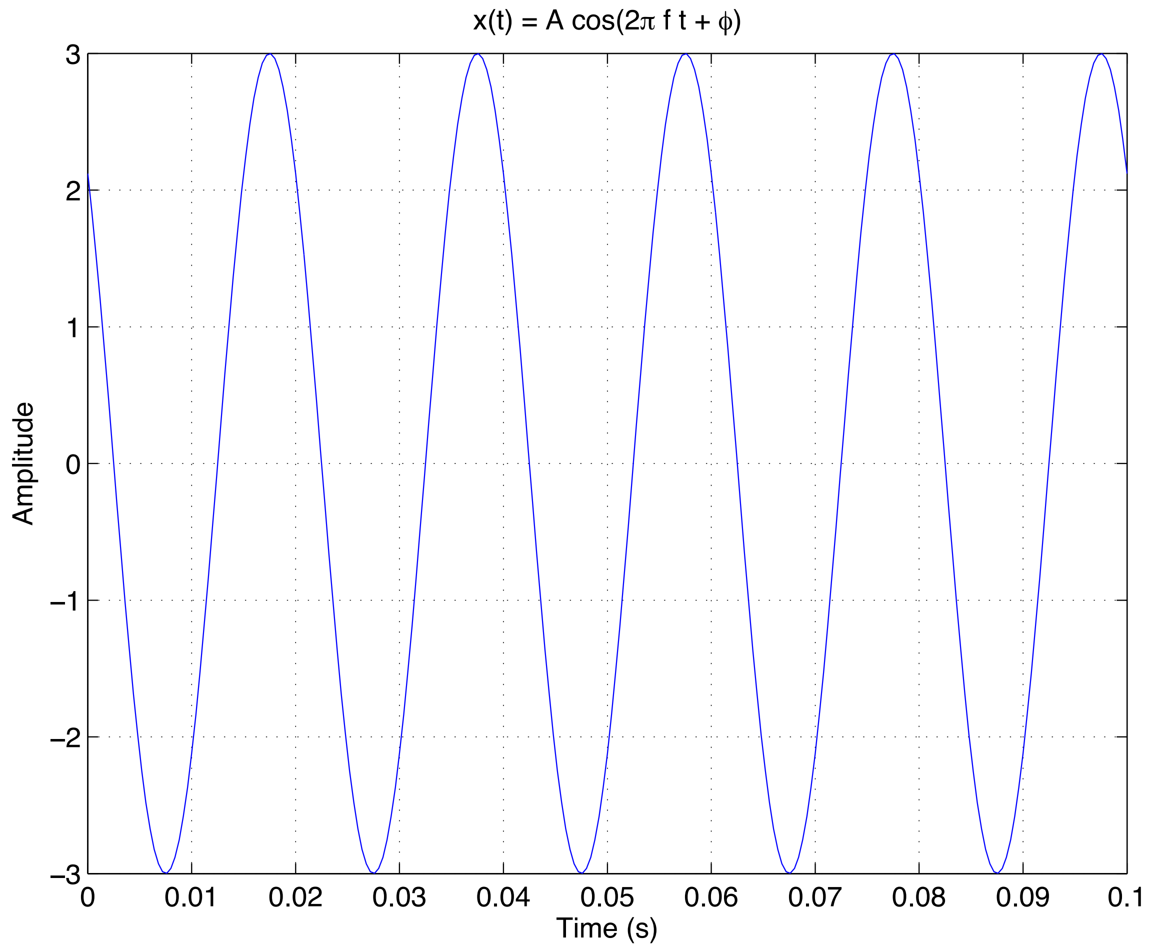
Lecture: Introduction to Sinusoids

The Formula for Sinusoidal Signals

- The general formula for a sinusoidal signal is

$$x(t) = A \cdot \cos(2\pi ft + \phi).$$

- A, f, and ϕ are parameters that characterize the sinusoidal sinal.
  - A - Amplitude: determines the height of the sinusoid.
  - f - Frequency: determines the number of cycles per second.
  - ϕ - Phase: determines the location of the sinusoid.



- The formula for this sinusoid is:

$$x(t) = 3 \cdot \cos(2\pi \cdot 50 \cdot t + \pi/4).$$

The Significance of Sinusoidal Signals

- Fundamental building blocks for describing arbitrary signals.
  - General signals can be expresssed as sums of sinusoids (Fourier Theory)
  - Provides bridge to frequency domain.
- Sinusoids are *special signals* for linear filters (eigenfunctions).
- Sinusoids occur naturally in many situations.
  - They are solutions of differential equations of the form

$$\frac{d^2x(t)}{dt^2} + ax(t) = 0.$$

- Much more on these points as we proceed.

Background: The cosine function

- The properties of sinusoidal signals stem from the properties of the cosine function:
  - **Periodicity:** $\cos(x + 2\pi) = \cos(x)$
  - **Eveness:** $\cos(-x) = \cos(x)$
  - **Ones** of cosine: $\cos(2\pi k) = 1$, for all integers k.
  - **Minus ones** of cosine: $\cos(\pi(2k + 1)) = -1$, for all integers k.
  - **Zeros** of cosine: $\cos(\frac{\pi}{2}(2k + 1)) = 0$, for all integers k.
  - Relationship to **sine function:** $\sin(x) = \cos(x - \pi/2)$ and $\cos(x) = \sin(x + \pi/2)$.

Amplitude

- The amplitude A is a *scaling factor*.
- It determines how large the signal is.
- Specifically, the sinusoid oscillates between $+A$ and $-A$.

Frequency and Period

- Sinusoids are periodic signals.
- The frequency f indicates how many times the sinusoid repeats per second.
- The duration of each cycle is called the period of the sinusoid.
  It is denoted by T.
- The relationship between frequency and period is

$$f = \frac{1}{T} \text{ and } T = \frac{1}{f}.$$

Phase and Delay

- The phase ϕ causes a sinusoid to be shifted sideways.
- A sinusoid with phase $\phi = 0$ has a maximum at $t = 0$.
- A sinusoid that has a maximum at $t = t_1$ can be written as

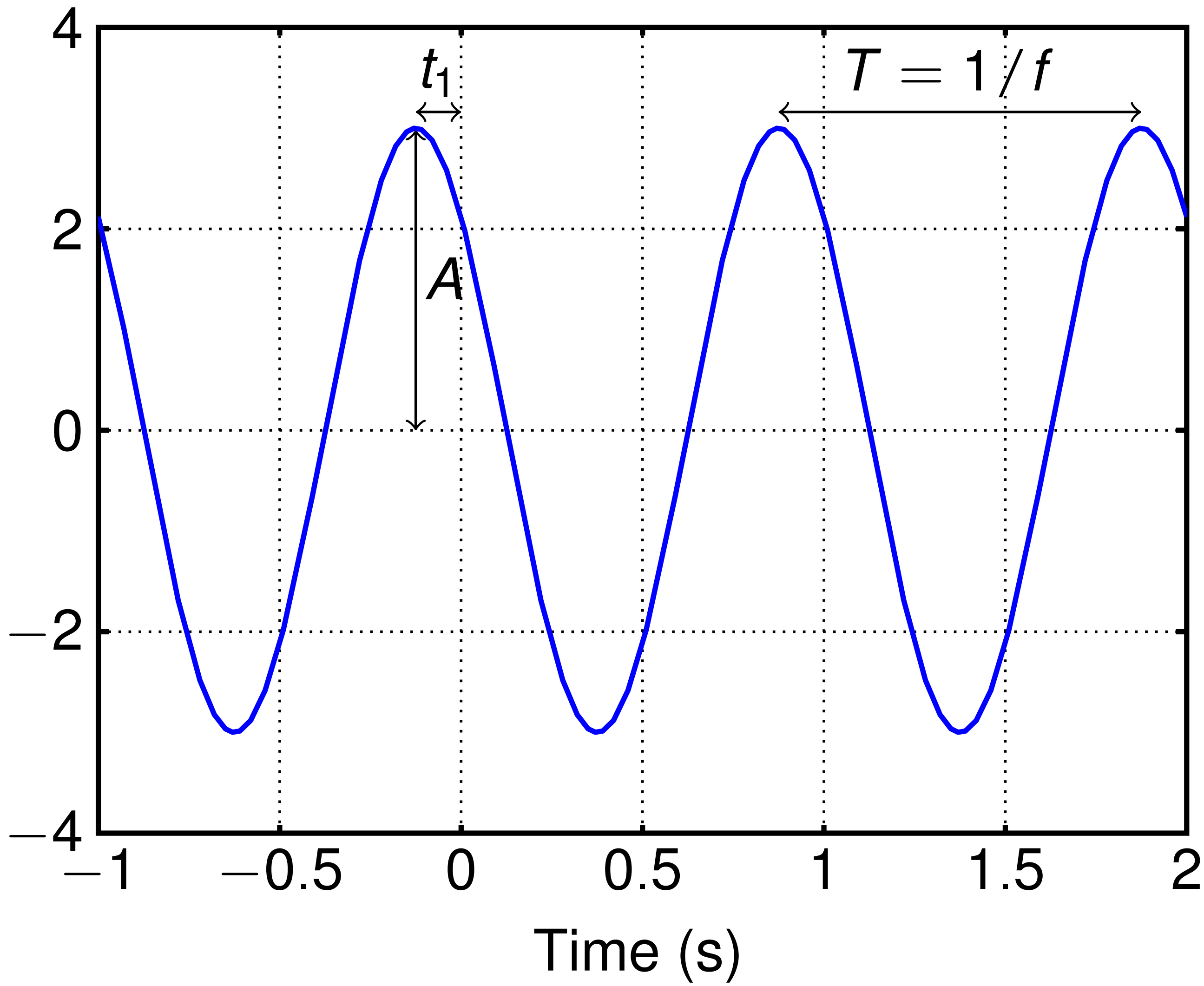
$$x(t) = A \cdot \cos(2\pi f (t - t_1)).$$

- Expanding the argument of the cosine leads to

$$x(t) = A \cdot \cos(2\pi f t - 2\pi f t_1).$$

- Comparing to the general formula for a sinusoid reveals

$$\phi = -2\pi f t_1 \text{ and } t_1 = \frac{-\phi}{2\pi f}.$$


4
2
0
−2
−4
−1
−0.5
0
0.5
1
1.5
2
Time (s)
t_1
$T = 1/f$
A

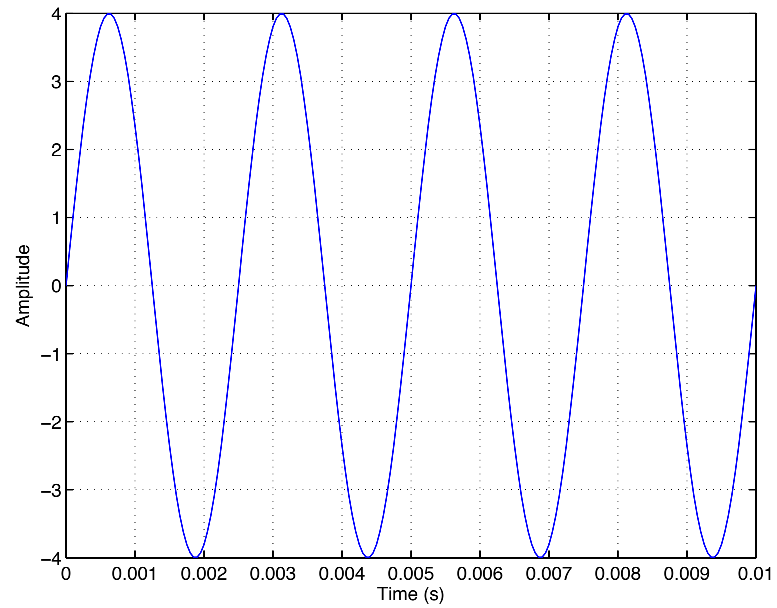
Exercise

1. Plot the sinusoid

$$x(t) = 2\cos(2\pi \cdot 10 \cdot t + \pi/2)$$

   between $t = -0.1$ and $t = 0.2$.

2. Find the equation for the sinusoid in the following plot



Vectors and Matrices

- MATLAB is specialized to work with vectors and matrices.
- Most MATLAB commands take vectors or matrices as arguments and perform looping operations automatically.
- Creating vectors in MATLAB:

  directly:

  ```
  x = [ 1, 2, 3 ];
  ```

  using the increment (:) operator:

  ```
  x = 1:2:10;
  ```

  produces a vector with elements

  `[1, 3, 5, 7, 9]`.

  using MATLAB commands For example, to read a .wav file

  ```
  [ x, fs] = wavread('music.wav');
  ```

Plot a Sinusoid

```
%% parameters
A   = 3;
f   = 50;
phi = pi/4;

fs  = 50*f;

%% generate signal
% 5 cycles with 50 samples per cycle
tt = 0 : 1/fs : 5/f;
xx = A*cos(2*pi*f*tt + phi);

%% plot
plot(tt,xx)
xlabel( 'Time (s)' )    % labels for x and y axis
ylabel( 'Amplitude' )
title( 'x(t) = A cos(2\pi f t + \phi)')
```