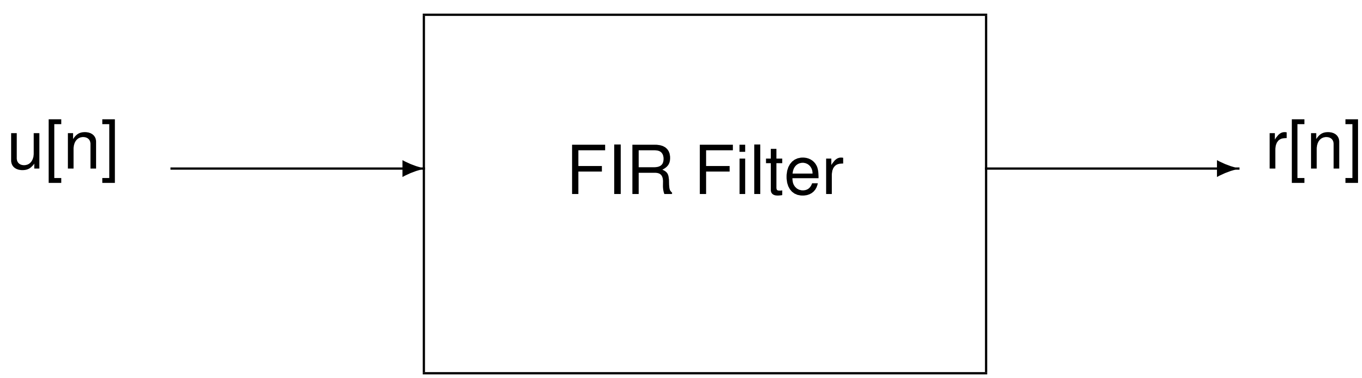
Unit Step Sequence and Unit Step Response

- The signal with samples

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0, \\ 0 & \text{for } n < 0 \end{cases}$$

  is called the unit-step sequence or unit-step signal.
- The output of an FIR filter when the input is the unit-step signal ($x[n] = u[n]$) is called the unit-step response $r[n]$.



Unit-Step Response of the 3-Point Averager

- Input signal: $x[n] = u[n]$.
- Output signal: $r[n] = \frac{1}{3} \sum_{k=0}^{2} u[n-k]$.

n	-1	0	1	2	3	...
$u[n]$	0	1	1	1	1	...
$\frac{1}{3}u[n]$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	...
$+\frac{1}{3}u[n-1]$	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	...
$+\frac{1}{3}u[n-2]$	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	...
$r[n]$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	1	...

Unit-Impulse Sequence and Unit-Impulse Response

- The signal with samples

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0, \\ 0 & \text{for } n \neq 0 \end{cases}$$

  is called the unit-impulse sequence or unit-impulse signal.
- The output of an FIR filter when the input is the unit-impulse signal ($x[n] = \delta[n]$) is called the unit-impulse response, denoted $h[n]$.
- Typically, we will simply call the above signals simply impulse signal and impulse response.
- We will see that the impulse-response captures all characteristics of a FIR filter.
  - This implies that impulse response is a very important concept!

Unit-Impulse Response of a FIR Filter

- Input signal: $x[n] = \delta[n]$.
- Output signal: $h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$.

n	-1	0	1	2	3	...	M
$\delta[n]$	0	1	0	0	0	...	0
$b_0 \cdot \delta[n]$	0	b_0	0	0	0	...	0
$+b_1 \cdot \delta[n-1]$	0	0	b_1	0	0	...	0
$+b_2 \cdot \delta[n-2]$	0	0	0	b_2	0	...	0
$\vdots$				$\vdots$			
$+b_M \cdot \delta[n-M]$	0	0	0	0	0	...	b_M
$h[n]$	0	b_0	b_1	b_2	b_3	...	b_M

Important Insights

- For an FIR filter, the impulse response equals the sequence of filter coefficients:

$$h[n] = \begin{cases} b_n & \text{for } n = 0, 1, \ldots, M \\ 0 & \text{else.} \end{cases}$$

- Because of this relationship, the system relationship for an FIR filter can also be written as

$$\begin{aligned} y[n] &= \sum_{k=0}^{M} b_k x[n-k] \\ &= \sum_{k=0}^{M} h[k] x[n-k] \\ &= \sum_{-\infty}^{\infty} h[k] x[n-k]. \end{aligned}$$

- The operation $y[n] = h[n] * x[n] = \sum_{-\infty}^{\infty} h[k]x[n-k]$ is called convolution; it is a **very, very** important operation.

Exercise

1. Find the impulse response $h[n]$ for the FIR filter with difference equation
$$y[n] = 2 \cdot x[n] + x[n-1] - 3 \cdot x[n-3].$$
2. Compute the output signal, when the input signal is $x[n] = u[n]$.
3. Compute the output signal, when the input signal is $x[n] = \exp(-\alpha n) \cdot u[n]$.