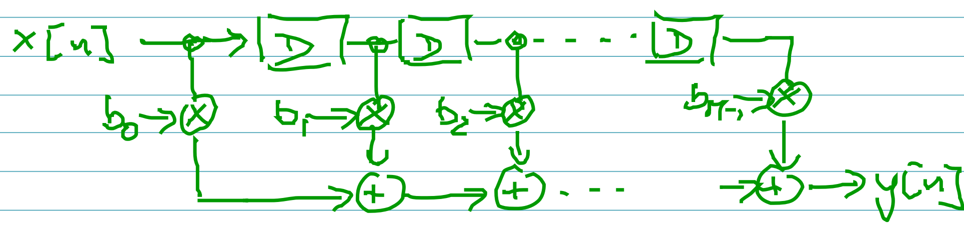
11/22/16 **Block Diagrams**

Reminders:

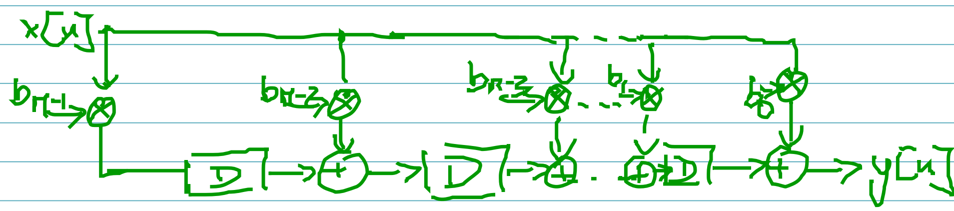
Want to implement systems that realize:

$$y[n] = \sum_{k=0}^{M-1} b_k \, x[n-k] \quad \text{(FIR Filter)}$$

Direct Form:



Transposed Form:



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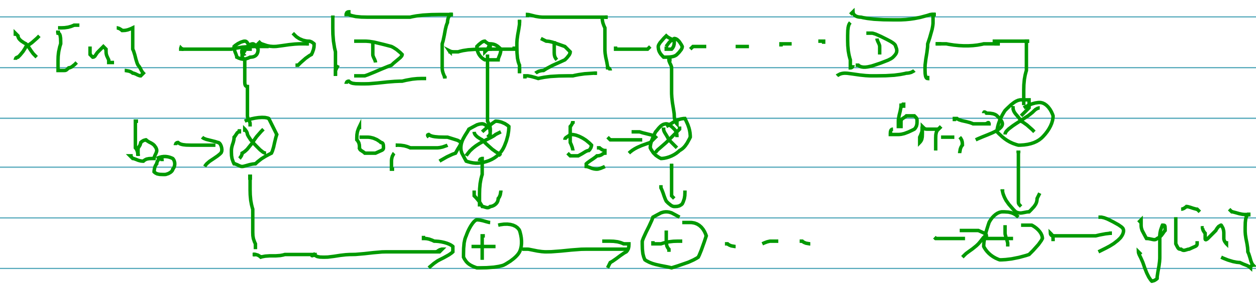

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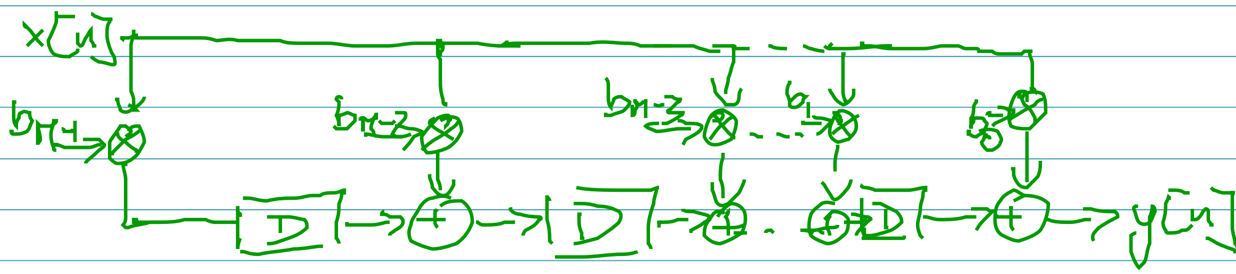
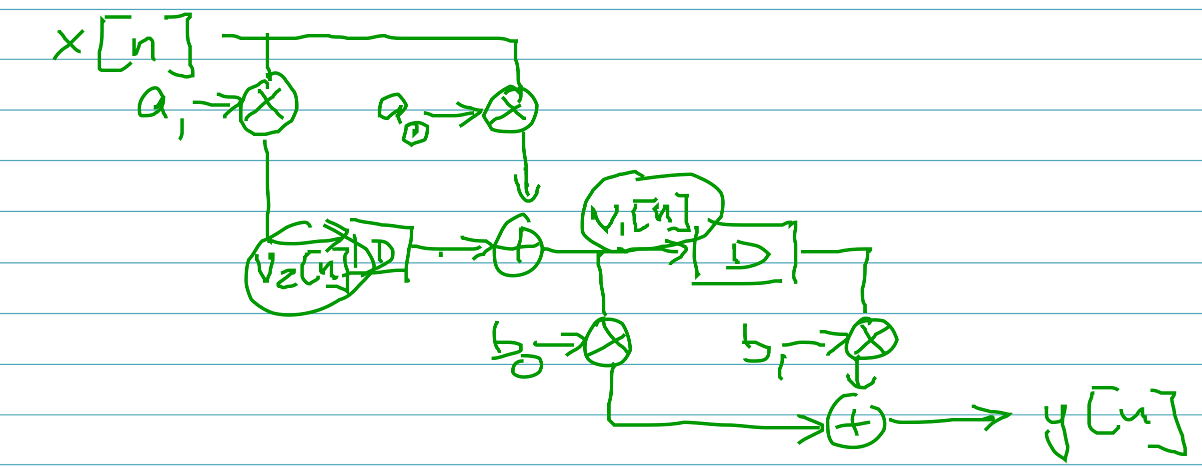


Example:



1.) Label temporary signals at inputs to all delays

2.) write signal equations

1) $y[n] = b_0 v_1[n] + b_1 \cdot v_1[n-1]$

2) $v_1[n] = a_0 x[n] + v_2[n-1]$

3) $v_2[n] = a_1 x[n]$

3.) Solve signal equations for $y[n]$ in terms of $x[n]$

3 & 2) : 4) $v_1[n] = \underbrace{a_0 x[n] + \underbrace{a_1 x[n-1]}_{= v_2[n-1]}}_{= v_1[n]}$

4 & 1) : 5) $y[n] = b_0 \cdot (a_0 x[n] + a_1 x[n-1]) + b_1 \cdot \underbrace{(a_0 x[n-1] + a_1 x[n-2])}_{= v_1[n-1]}$

$= b_0 a_0 x[n] + (b_0 a_1 + b_1 a_0)\, x[n-1] + b_1 a_1 x[n-2]$

Double check:

Paths from $x[n]$ to $y[n]$:

- no delay: $a_0 \cdot b_0$
- one delays: $a_0 \cdot b_1 + a_1 b_0$
- two delays: $a_1 \cdot b_1$

coefficients of difference equation

Impulse response:

$$h[n] = \{a_0 b_0,\ a_0 b_1 + a_1 b_0,\ a_1 b_1\}$$

Connecting Systems

Two LTI systems:

$$S_1 \rightarrow h_1[n]$$
$$S_2 \rightarrow h_2[n]$$

Cascade / Series connection:

$$x[n] \rightarrow \underbrace{\boxed{h_1[n]} \rightarrow \boxed{h_2[n]}}_{h[n]} \rightarrow y[n]$$

Q: What is the impulse response of the cascade?

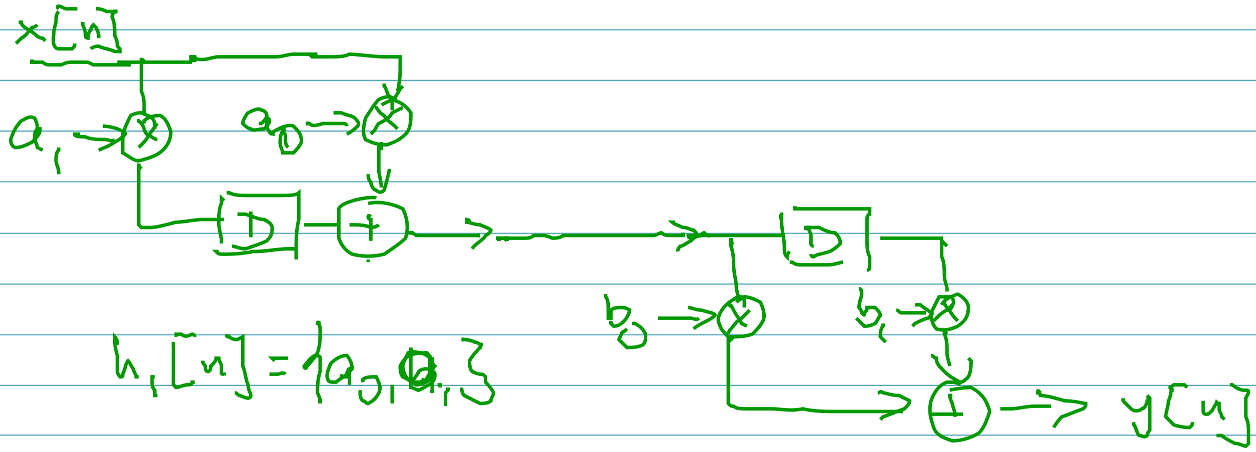
Approach: Feed impulse into system
i.e. $x[n] = \delta[n]$

Then, $y[n] = h[n]$

$$\delta[n] \rightarrow \boxed{h_1[n]} \xrightarrow{h_1[n]} \boxed{h_2[n]} \rightarrow h_1[n] * h_2[n]$$

$$\Rightarrow \boxed{h[n] = h_1[n] * h_2[n]}$$

Example



$$h_1[n] = \{a_0, a_1\}$$

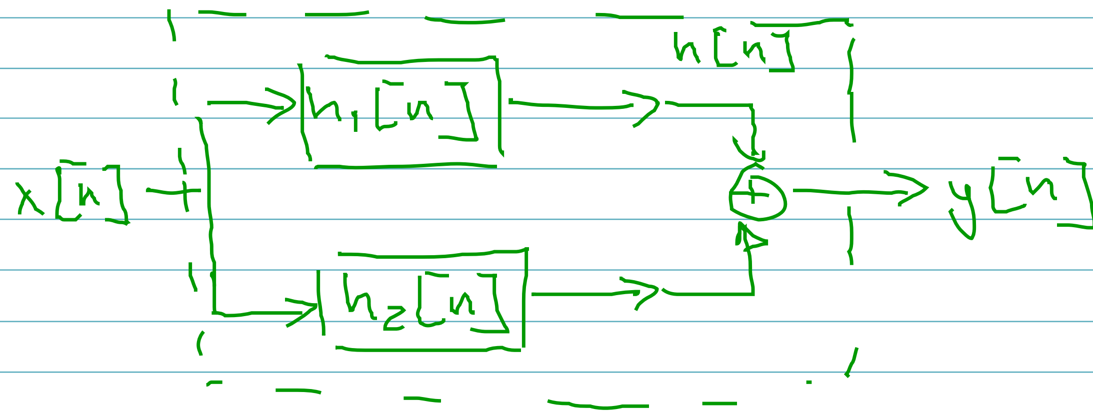
$$h_2[n] = \{b_0, b_1\}$$

$$h[n] = \{a_0, a_1\} * \{b_0, b_1\}$$

$h_1[n]$	a_0	a_1	
$b_0 \cdot h_1[n]$	$b_0 a_0$	$b_0 a_1$	
$b_1 h_1[n-1]$		$b_1 a_0$	$b_1 \cdot a_1$
$h[n]$	$b_0 a_0$	$b_0 a_1 + b_1 a_0$	$b_1 a_1$

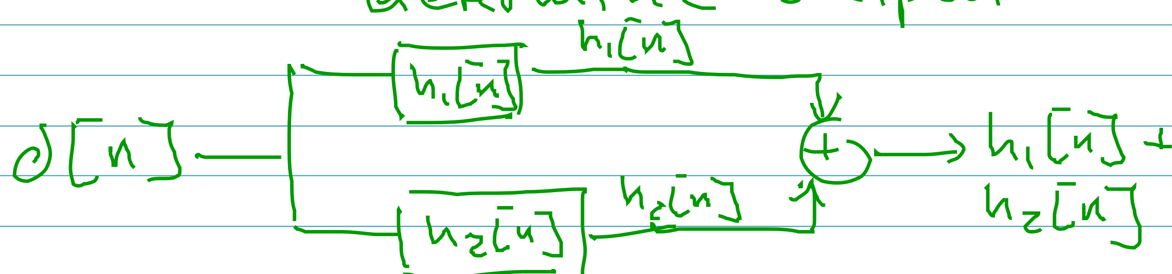
$$\Rightarrow h[n] = \{b_0 a_0,\ b_0 a_1 + b_1 a_0,\ b_1 a_1\}$$

Parallel connection:



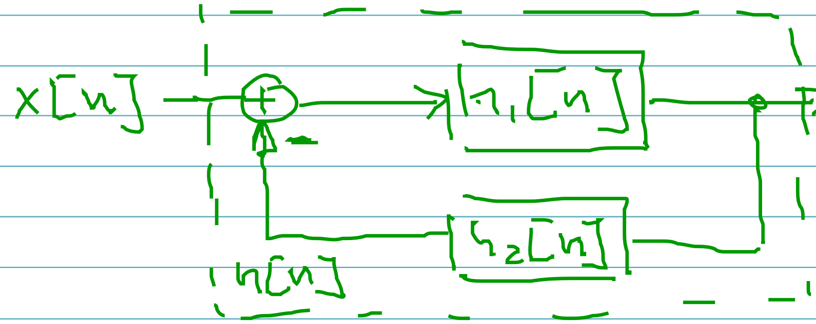
Q: What is the impulse response of this system?

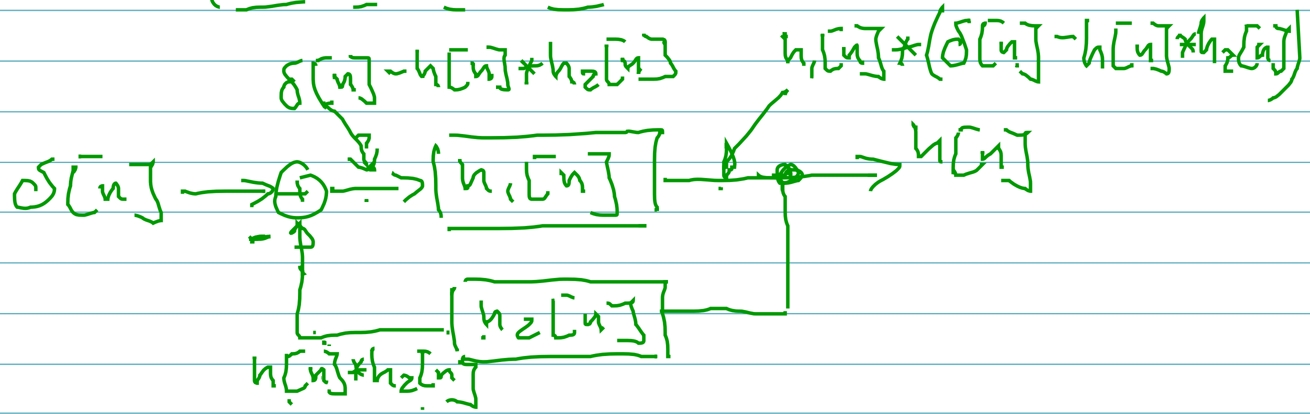
Approach: Let $x[n] = \delta[n]$ and determine output



$$\boxed{h[n] = h_1[n] + h_2[n]}$$

Feedback Connection





h[n] is the solution of:

$$h[n] = h_1[n] * (\delta[n] - h[n] * h_2[n])$$

$$\Rightarrow \boxed{h[n] = h_1[n] - h[n] * h_1[n] * h_2[n]}$$

Question:

FIR Filter: $\to$ 3pt averager

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

What if: $x[n] = e^{j2\pi\hat{f}n}$?

$$\Rightarrow y[n] = \frac{1}{3}\cdot\left(\underset{x[n]}{e^{j2\pi\hat{f}n}} + \underset{x[n-1]}{e^{j2\pi\hat{f}(n-1)}} + \underset{x[n-2]}{e^{j2\pi\hat{f}(n-2)}}\right)$$

$$= \frac{1}{3}\cdot e^{j2\pi\hat{f}n}\cdot\left(1 + e^{-j2\pi\hat{f}} + e^{-j4\pi\hat{f}}\right)$$

$$= \underbrace{\frac{1}{3}\cdot\left(1+e^{-j2\pi\hat{f}}+e^{-j4\pi\hat{f}}\right)}_{\text{Magnitude and Phase of output signal + depends of } \hat{f}}\cdot \underbrace{e^{j2\pi\hat{f}n}}_{\substack{\text{complex exponential} \\ \text{frequency same as input}}}$$

- Magnitude and Phase of output signal
+ depends of $\hat{f}$

complex exponentia

frequency same as input

Magnitude and Phase:

$$H(\hat{f}) = \frac{1}{3}\left(1 + e^{-j2\pi\hat{f}} + e^{-j4\pi\hat{f}}\right)$$

$$= \frac{1}{3}e^{-j2\pi\hat{f}}\cdot\left(e^{j2\pi\hat{f}} + 1 + e^{-j2\pi\hat{f}}\right)$$

$$e^{j2\pi\hat{f}} + e^{-j2\pi\hat{f}} = 2\cdot\cos(2\pi\hat{f})$$

$$= e^{-j2\pi\hat{f}}\cdot\frac{1}{3}\left(1 + 2\cos(2\pi\hat{f})\right)$$