12/8/16 **Properties of Frequency Response**

Frequency Responses are periodic with period 1 in $\hat{f}$ (or 2π in $\hat{\omega}$)

Impulse response $h[n]$

$$\Rightarrow H(\hat{f}) = \sum_k h[k] \cdot e^{-j2\pi\hat{f}\cdot k}$$

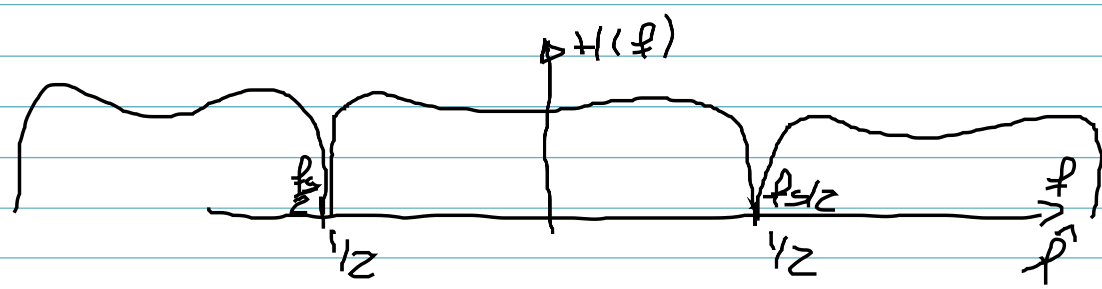
To show periodicity with period 1,

$$H(\hat{f}+1) = H(\hat{f})$$

$$H(\hat{f}+1) = \sum_k h[k] \cdot e^{-j2\pi(\hat{f}+1)\cdot k}$$

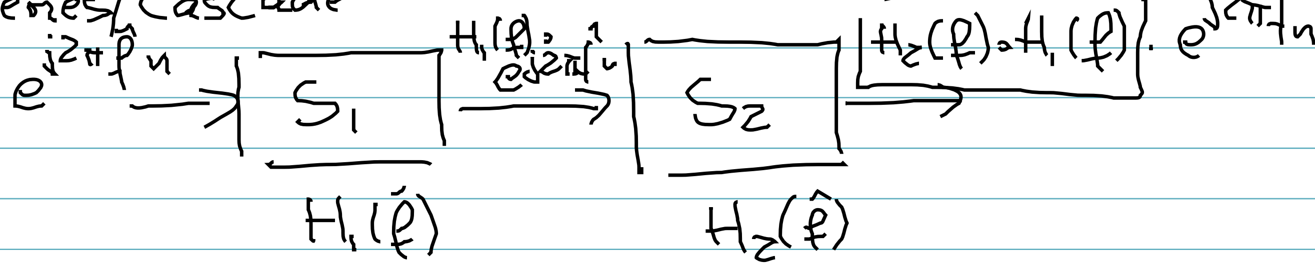
$$= \underbrace{\sum_k h[k] \cdot e^{-j2\pi\hat{f}\cdot k}}_{=H(\hat{f})} \cdot \underbrace{e^{-j2\pi k}}_{=1} = H(\hat{f})$$

$$x(t) \longrightarrow \boxed{\text{C-D converter}} \longrightarrow \boxed{H(\hat{f})} \longrightarrow \boxed{\text{D-C converter}} \longrightarrow y(t)$$



Connecting Systems

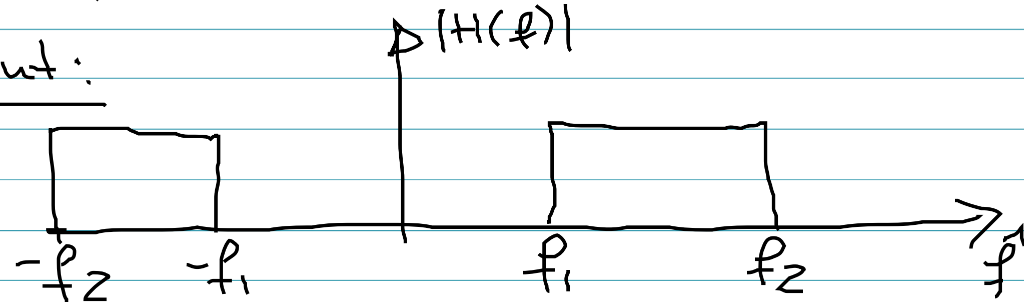
Series/Cascade:



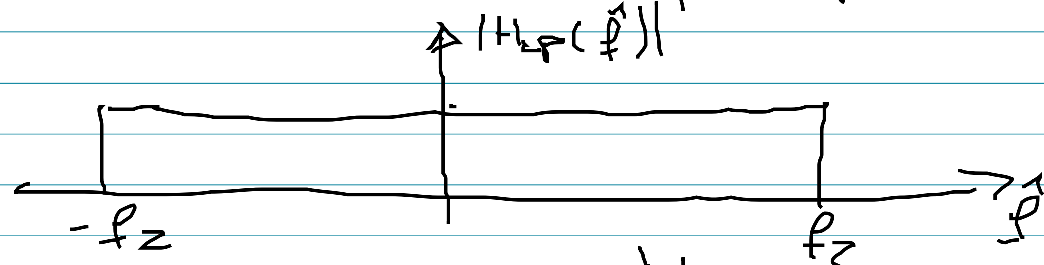
Q: What is Frequency response of cascade?

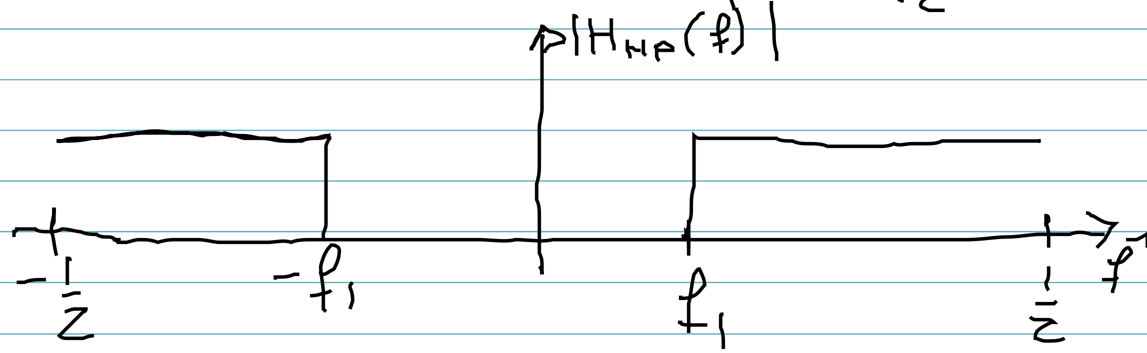
$$\boxed{H(\hat{f}) = H_1(\hat{f})\cdot H_2(\hat{f})}$$

Want:

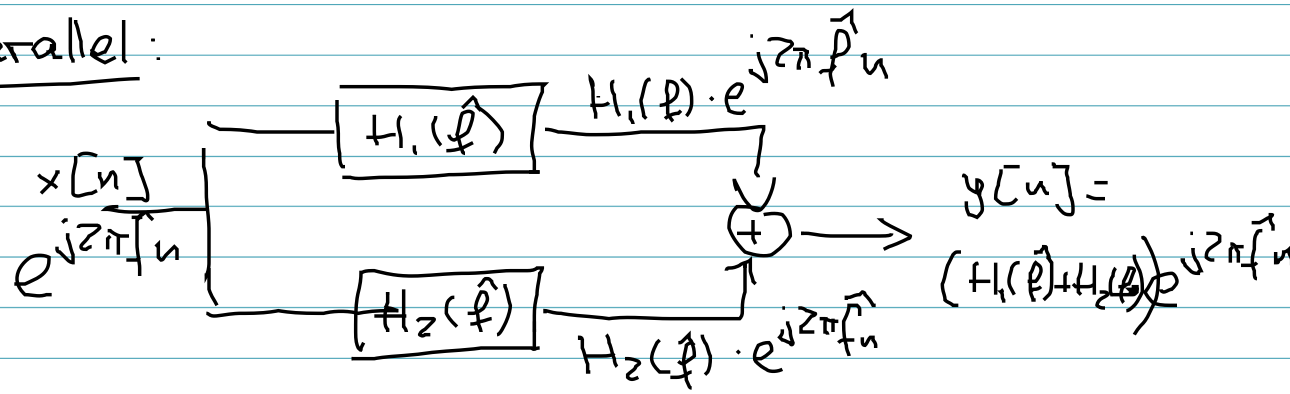


Realize this via cascade of lowpass & Highpass





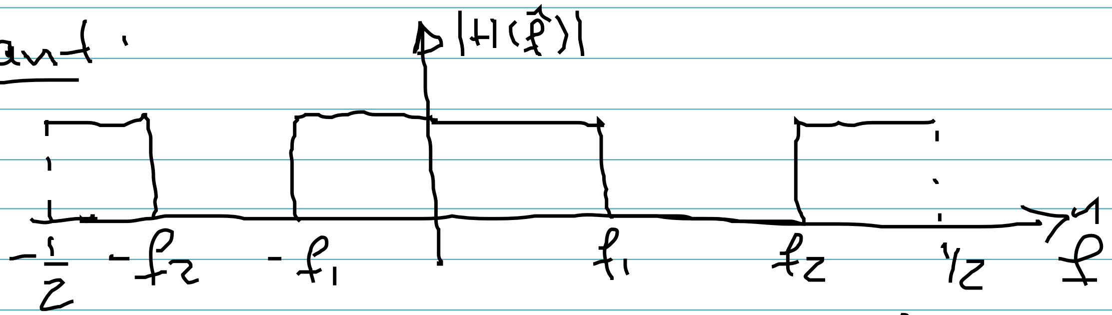
Parallel:



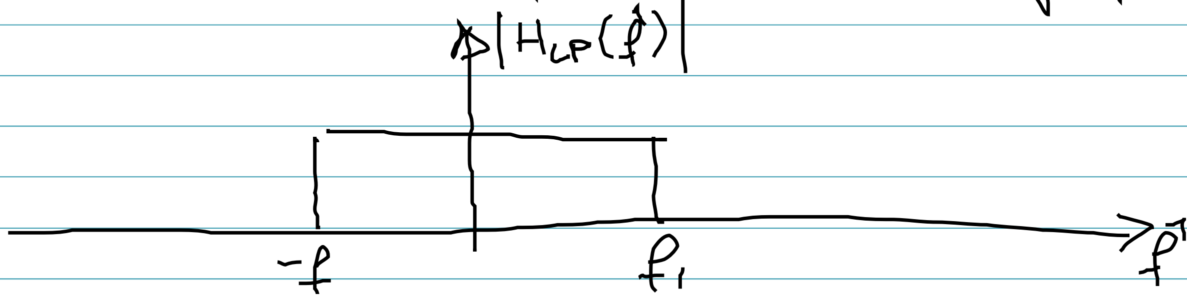
Parallel:

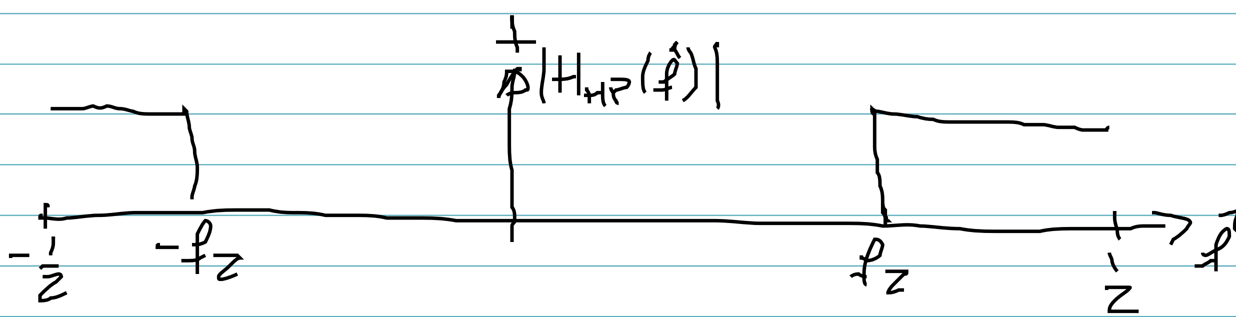
$$H(\hat{f}) = H_1(\hat{f}) + H_2(\hat{f})$$

Want:

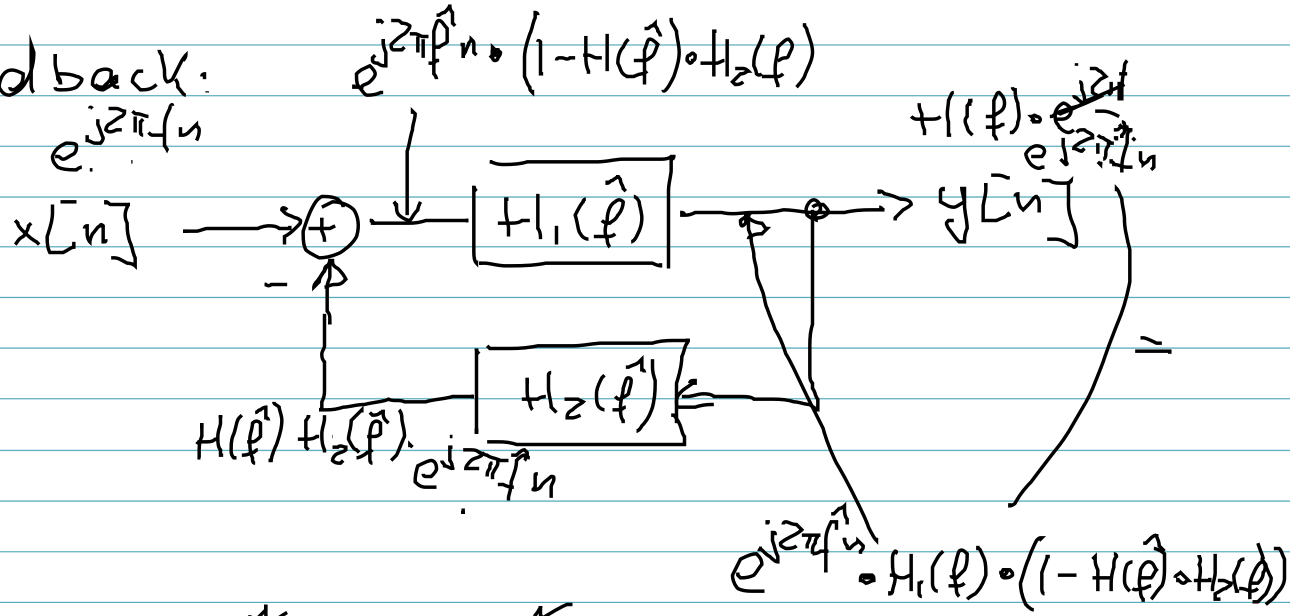


Realize this with parallel low & high pass





Feedback:



$$\Rightarrow \underline{H(\hat{f})} \cdot \cancel{e^{j2\pi \hat{f} n}} = \cancel{e^{j2\pi \hat{f} n}} \cdot H_1(\hat{f}) \cdot (1 - \underline{H(\hat{f})} \cdot H_2(\hat{f}))$$

$$\Rightarrow \underbrace{H(\hat{f}) + H(\hat{f}) \cdot H_1(\hat{f}) \cdot H_2(\hat{f})} = H_1(\hat{f}) \cdot H_2(\hat{f})$$

$$\Rightarrow \boxed{H(\hat{f}) = \frac{H_1(\hat{f}) \cdot H_2(\hat{f})}{1 + H_1(\hat{f}) \cdot H_2(\hat{f})}}$$

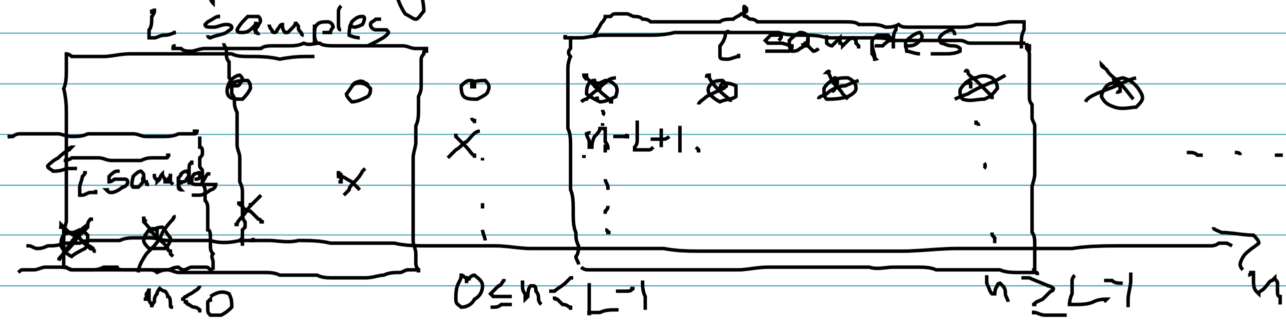
Question: L-point averager

- impulse response
- convolution
- frequency respone

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k]$$

$$\Rightarrow h[n] = \{\underbrace{\tfrac{1}{L}, \tfrac{1}{L}, \ldots \tfrac{1}{L}}_{L \text{ terms}}\}$$

Input signal $x[n] = u[n]$



$n<0$: all samples in window are equal to zero $\rightarrow y[n] = 0$

$n \geq L-1$: $\Rightarrow n-L+1 \geq 0 \Rightarrow$ all samples in window equal to 1 $\Rightarrow y[n] = 1$

$0 \leq n \leq L-1$: samples at ~~n~~ $0, 1, \ldots, n$ are equal to 1

$\Rightarrow$ $n+1$ samples are equal to 1

$$\Rightarrow y[n] = \frac{n+1}{L}$$

$$H(\hat{f}) = \sum_{k=0}^{L-1} \frac{1}{L} \cdot e^{-j2\pi\hat{f}\cdot k}$$

$$= \frac{1}{L} \cdot \sum_{k=0}^{L-1} \left(e^{-j2\pi\hat{f}}\right)^k \quad \leftarrow \text{geometric series}$$

$$S = \sum_{k=0}^{L-1} a^k = \frac{a^L - 1}{a - 1} \qquad a = e^{-j2\pi\hat{f}}$$

$$\Rightarrow H(\hat{f}) = \frac{1}{L} \cdot \frac{e^{-j2\pi\hat{f}L} - 1}{e^{-j2\pi\hat{f}} - 1}$$

$$= \frac{1}{L} \frac{e^{-j2\pi\hat{f}L/2}}{e^{-j2\pi\hat{f}1/2}} \cdot \frac{e^{-j2\pi\hat{f}L/2} - e^{j2\pi\hat{f}L/2}}{e^{-j2\pi\hat{f}1/2} - e^{j2\pi\hat{f}1/2}}$$

$$= \boxed{\frac{1}{L} e^{-j2\pi\hat{f}(L-1)/2} \cdot \frac{\sin(2\pi\hat{f}L/2)}{\sin(2\pi\hat{f}1/2)}}$$

Dirichlet Form

Discrete-time equivalent of sinc(·)?

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$