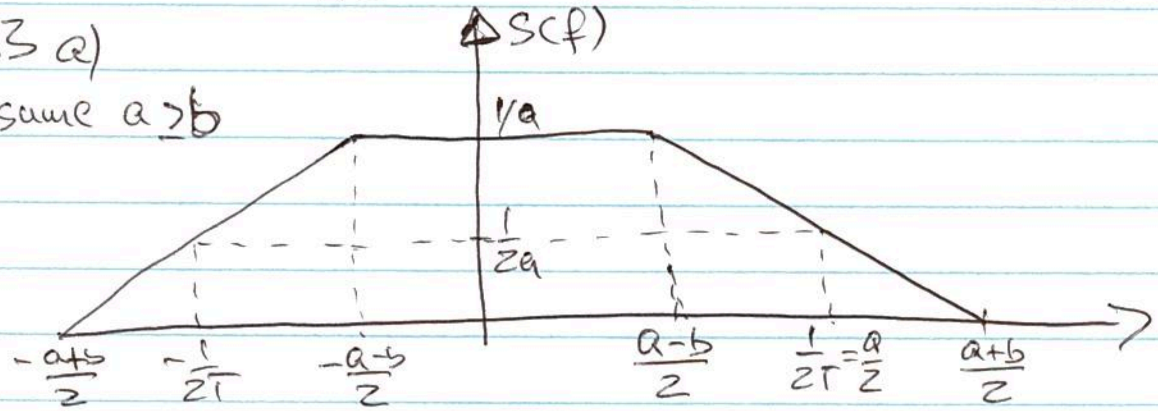
Dr. Paris

ECE 460

HW 8 - Solution

P.4.3 a)

assume $a \geq b$



b) Since this is a baseband channel, bandwidth is the one-sided BW:

Given: • channel BW = 400 Hz $\overset{!}{=} \frac{a+b}{2}$ (1)

• Symbol rate $= \frac{1}{T} = \frac{1200 \text{ bits/s}}{\log_2 4 \ \frac{\text{bits}}{\text{symbol}}} = 600 \frac{\text{symb}}{\text{sec}}$

$\Rightarrow \boxed{a = \frac{1}{T} = 600 \text{ Hz}}$ (2) (see Fig. a)

Also note that by the Nyquist criterion sinc$(a t_n)$ must have zeros at $t_n = n \cdot T, n \neq 0$

$\Rightarrow \pi a \cdot t_n \overset{!}{=} n \cdot \pi \quad (\sin(n \cdot \pi) = 0)$

$\Rightarrow \pi \cdot a \cdot n \cdot T \overset{!}{=} n \cdot \pi \Rightarrow a = \frac{1}{T}.$

From (1) and (2): $b = 200$ Hz

c) This is a passband channel ⇒ two-sided BW

Given: • channel BW = 20 MHz $\overset{!}{=} a + b$ (1)

• Symbol rate $\frac{1}{T} = \frac{60 \text{M bit/s}}{\log_2 64 \ \frac{\text{bits}}{\text{symb.}}} = 10\text{M} \frac{\text{symb}}{\text{s}}$

$\Rightarrow a = \frac{1}{T} = 10$ MHz (2)

$\Rightarrow b = 10$ MHz (from (1)&(2))

d) $s(t) = \text{sinc}(at) \cdot \text{sinc}(bt) = \dfrac{\sin(\pi a t) \cdot \sin(\pi b t)}{\pi^2 ab\, t^2}$

decays like $\frac{1}{t^2}$

$\Rightarrow$ worst case ISI (with timing error) $\sim \sum_{n=1}^{\infty} \frac{1}{(nT)^2} = \frac{1}{T} \cdot \frac{\pi^2}{6}$ is finite!

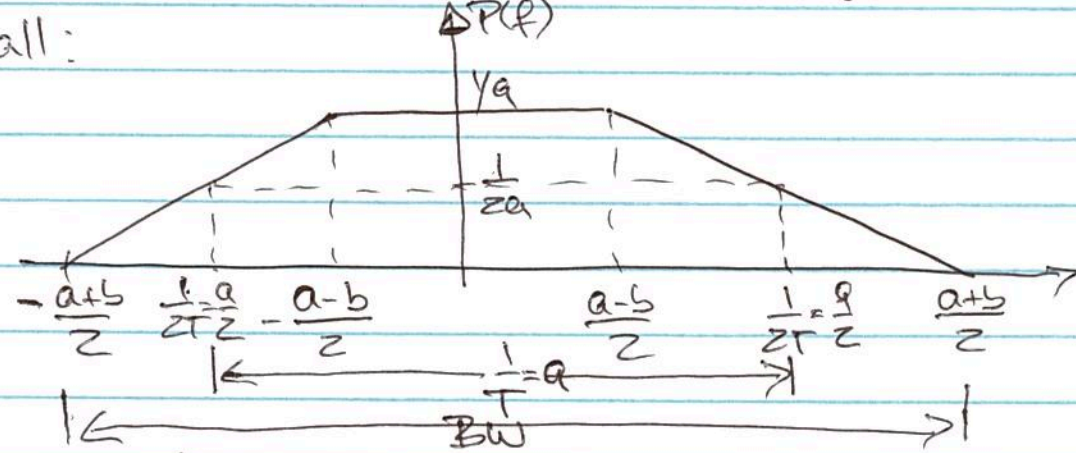
In comparison, for $p(t) = \text{sinc}(at) = \dfrac{\sin(\pi a t)}{\pi a t}$

decays like $\frac{1}{t}$

$\Rightarrow$ worst case ISI (with timing error) $\sim \sum_{n=1}^{\infty} \frac{1}{nT} = \infty$!!

P.4.7 a) Given:
- Excess BW = 50 % (two-sided)
- symbol rate: $\frac{1}{T} = \dfrac{40\,\text{Mb/s}}{\log_2 16 \,\frac{b}{\text{symb}}} = 10\,\text{MHz}$

Recall:



$\Rightarrow a = \frac{1}{T} = 10\,\text{MHz}$

$BW = a + b \overset{!}{=} \frac{1}{T} \cdot (1 + 50\%) = 15\,\text{MHz}$ (50%: excess BW)

$\Rightarrow b = 5\,\text{MHz}$

b) The two signaling formats have symbol rates:

$$\bullet \quad \frac{1}{T_1} = \frac{40\text{ Mb/s}}{\log_2 16 \ \frac{b}{\text{symb}}} = 10\text{ MHz}$$

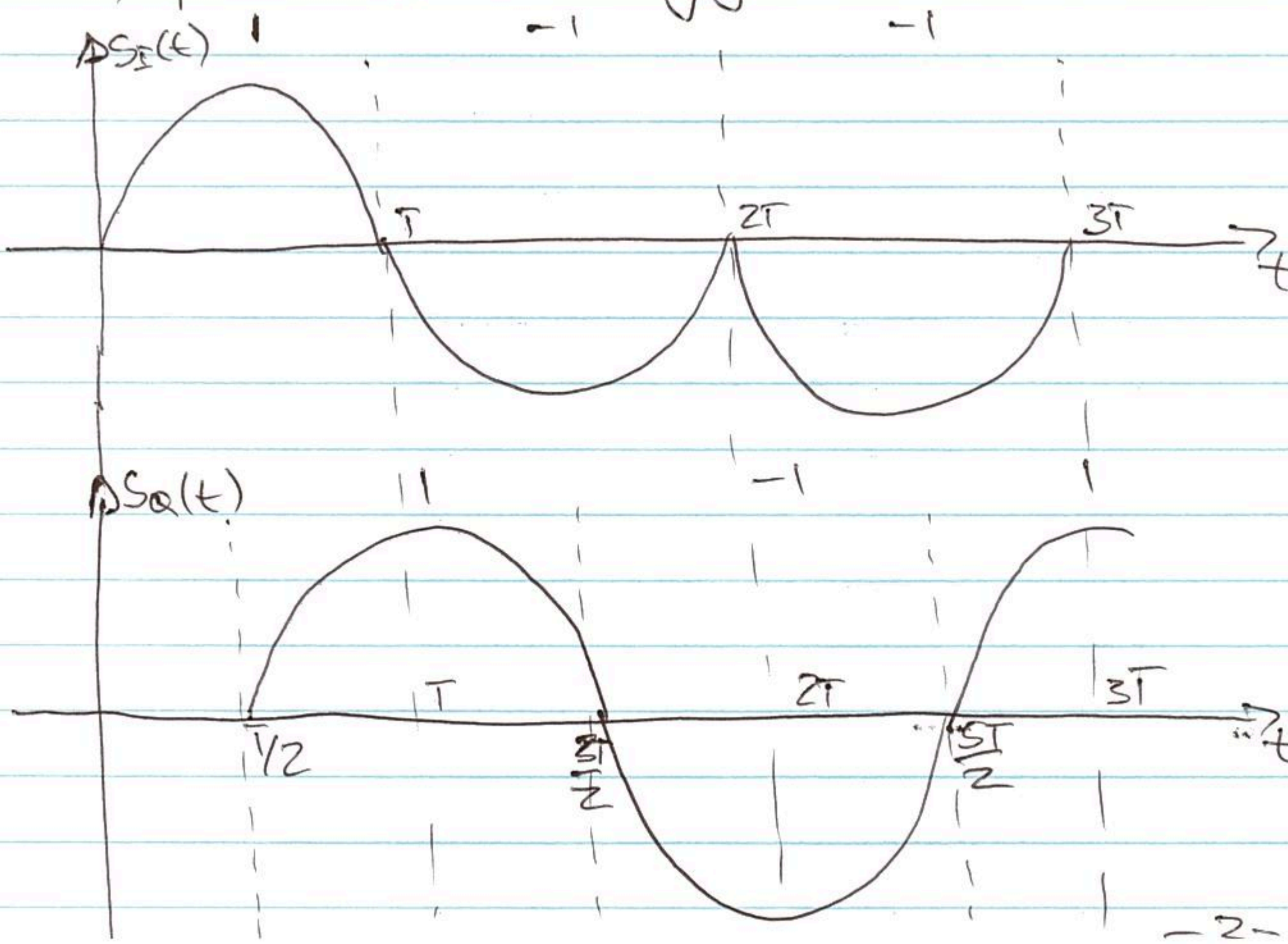
$$\bullet \quad \frac{1}{T_2} = \frac{18\text{ Mb/s}}{\log_2 8 \ \frac{b}{\text{symb}}} = 6\text{ MHz}$$

The Nyquist condition can be met for both signals by choosing

$$a = 10\text{ MHz}$$

$$b = 6\text{ MHz}$$

4.17 a) possible I & Q siggals:



b) MSK has a constant envelope (magnitude) because:

i) in-phase signal ~~are~~ is composed of segments of $\pm \sin(\cdot)$

ii) quadrature signal is composed of segments of $\pm \cos(\cdot)$, due to $\frac{T}{2}$ delay.

$$\Rightarrow \text{Magnitude} = \sqrt{(\pm \sin(\cdot))^2 + (\pm \cos(\cdot))^2} = 1$$

c) This is still a linearly modulated signal. Thus,

$$S(f) = \frac{\sigma_b^2}{T} \cdot |P(f)|^2$$

with $\sigma_b^2 = 2$ and $|P(f)| = \frac{2\cos(\pi f)}{1-4f^2}$ (see P.4.1)

d)

MSK 99% containment BW	:	$2.36 \cdot \frac{1}{T}$
Minimum Nyquist BW	:	$1/T$
OQPSK w/ rect. pulses	:	$20.4 \cdot \frac{1}{T}$

(Note; the 99% containment BWs in the book appear to be off by factor 2, see pp. 164 & 166)

e) Plot should look exactly like Fig. 4.6.

MATLAB Solution in P4_7.m in
http://www.spec.gmu.edu/~pparis/classes/resources-460/

P.4.21

QPSK: symbol rate $\frac{1}{T} = \frac{10\,\text{Mbit/s}}{\log_2 4 \ \frac{\text{bits}}{\text{symb.}}} = 5\,\text{MHz}$

$\Rightarrow BW = \frac{1}{T} \cdot (1 + \underset{\text{excess BW}}{50\%}) = 7.5\,\text{MHz}$

64 QAM: symbol rate $\frac{1}{T} = \frac{10\,\text{Mbit/s}}{\log_2 64 \ \frac{\text{bits}}{\text{symb.}}} = 1.66\,\text{MH}$

$\Rightarrow BW = \frac{1}{T} \cdot (1 + 0.5) = 2.5\,\text{MHz}$

P.5

Pulse	ISI (no time error)	ISI (time error)	99% BW ($\cdot \frac{1}{T}$)
Rect.	0	0	20.4 (*)
Half-Sine	0	0.02	2.4
Sq. Half-sine	0	0.04	2.8
Gaussian	1.51	1.51	0.58
Sinc	0	0.31	0.98
Trapezoid (50%)	0	0.10	1.20
Raised Cos (50%)	0	0.13	1.10

(*) Because of the very slow sidelobe decay, the numerical estimate is not very accurate

Note: The values for 99% BW on p. 164 and 166 of the book appear to be off by factor 2