ECE 460
Dr. Paris

HW#5 - Solution

Problem 3.21

a) The time constant RC of the lowpass filter must satisfy

$$\frac{1}{f_c} \ll RC \ll \frac{1}{B}$$

$f_c = 200\,\text{KHz}$ is given. The signal is periodic with period 2ms; thus, the fundamental frequency $f_0 = \frac{1}{2\text{ms}} = 0.5\,\text{KHz}$. We choose to set the bandwidth B to the frequency of the 3^{rd} harmonic, $B = 3f_0 = 1.5\,\text{KHz}$. This yields:

$$\frac{1}{200}\,\text{ms} \ll RC \ll \frac{2}{3}\,\text{ms}$$

and a reasonable choice is $RC = 0.1\,\text{ms}$.
With $R = 500\,\Omega$, $C = \frac{0.1\,\text{ms}}{500\,\Omega} = \frac{1}{2}\,\text{mF} = 500\,\mu\text{F}$

b)

Received passband signal $s_p(t) = (A + m(t)) \cdot \cos(2\pi\, 200\text{KHz}\, t)$. The outputs of a quadrature demodulator with frequency 201 KHz are:

$$u_c(t) = LPF\{(A+m(t)) \cdot \cos(2\pi\, 200\text{KHz}\, t) \cdot \cos(2\pi\, 201\text{KHz}$$

$$= LPF\{\tfrac{1}{2}(A+m(t)) \cdot (\cos(2\pi t) + \cos(2\pi\, 401 t))\}$$

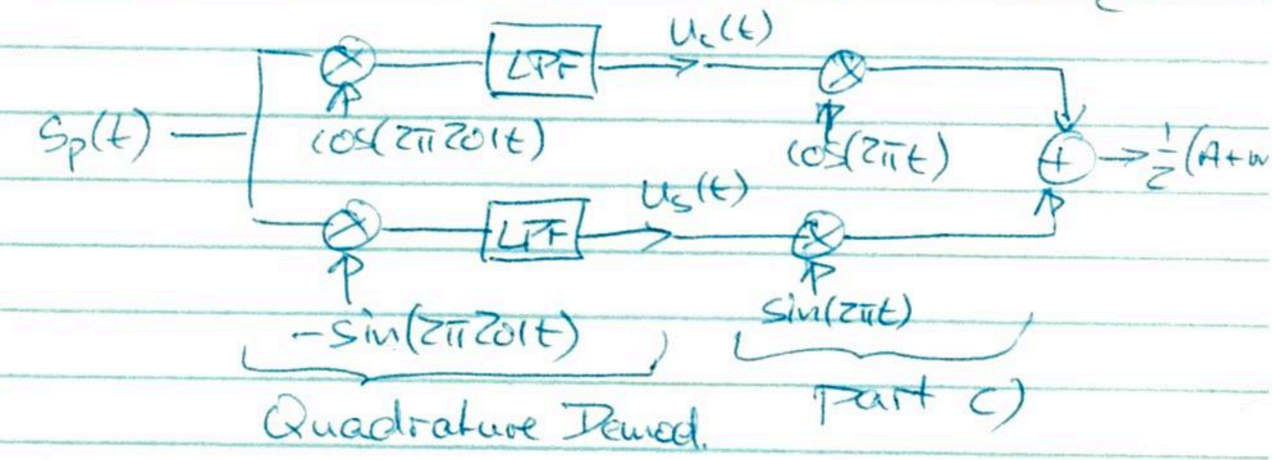
$$= \cancel{LPF}\ \tfrac{1}{2}(A + m(t)) \cdot \cos(2\pi t)$$

Similarly:

$$u_s(t) = -\tfrac{1}{2}(A + m(t)) \cdot \sin(2\pi t)$$

c) Objective is to recover $\frac{1}{2}(A+m(t))$. By inspection:

$$u_c(t)\cdot\cos(2\pi t) - u_s(t)\cdot\sin(2\pi t) = \frac{1}{2}(A+m(t))$$



2.) It is easy to verify that $x[n]$ is given by:

$$x[n] = (x[0]-2)\cdot(1-2\mu)^n + 2$$

with $x[0]=0$:

$$x[n] = 2 - 2\cdot(1-2\mu)^n$$

a) From the above n s.t. $|x[n]-2| < 10^{-5}$ is given by

$$n \geq \frac{\ln(10^{-5}\cdot\frac{1}{2})}{\ln(|1-2\mu|)}$$

μ	0.01	0.05	0.1	0.25	0.5	0.75	0.9	0.9[illegible]	0.99
$n \geq$	605	116	55	18	1	18	55	116	605

b) With $\mu=0.5$, it takes exactly one step to reach $x[n]=2$. The update rule becomes $x[n+1]=2$, independent of $x[n]$.

c) When $\mu = 1$, the update rule becomes

$$x[n+1] = 4 - x[n]$$

and $x[n]$ jumps back-and-forth betwee $x[0]$ and $4 - x[0]$ (e.g. between 0 and 4 if $x[0] = 0$). It never converges

d) When $\mu < 0$ or $\mu > 1$, then $x[n]$ diverge. It grows exponentially to ∞. When $\mu > 1$, $x[n]$ is alternating between positive and negative values (this is also apparen from $x[n] = (x[0] - 2) \cdot (1 - 2\mu)^n + 2$; the base $(1 - 2\mu)$ must have an absolute valu less than 1 for convergence.)

3.

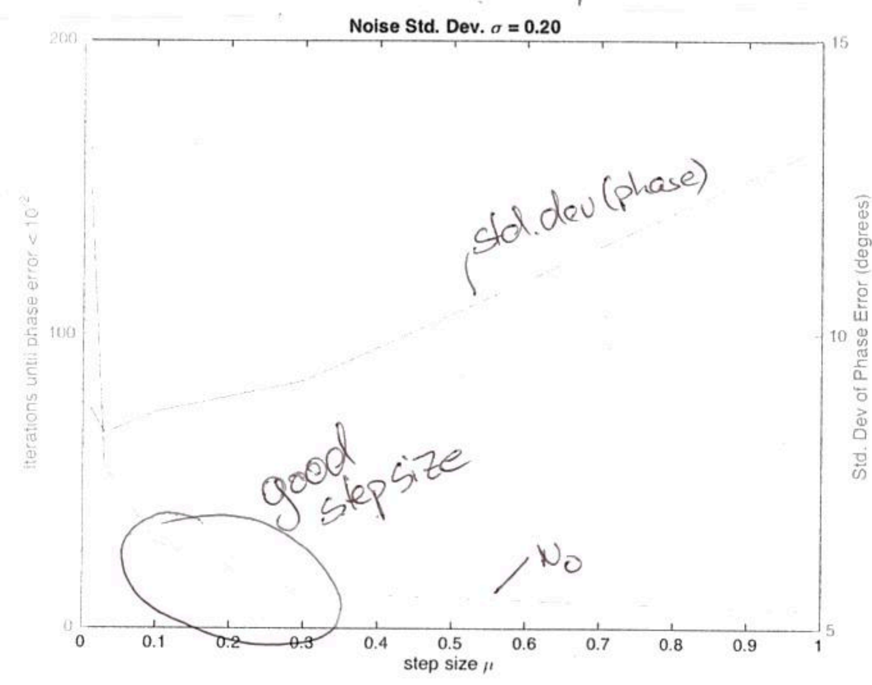
a) to measure the iteration when phase(r) is first within 10^{-2} of 0 degrees, I added the line

$$N0 = find(abs(angle(r)) < 0.01, 1, 'first');$$

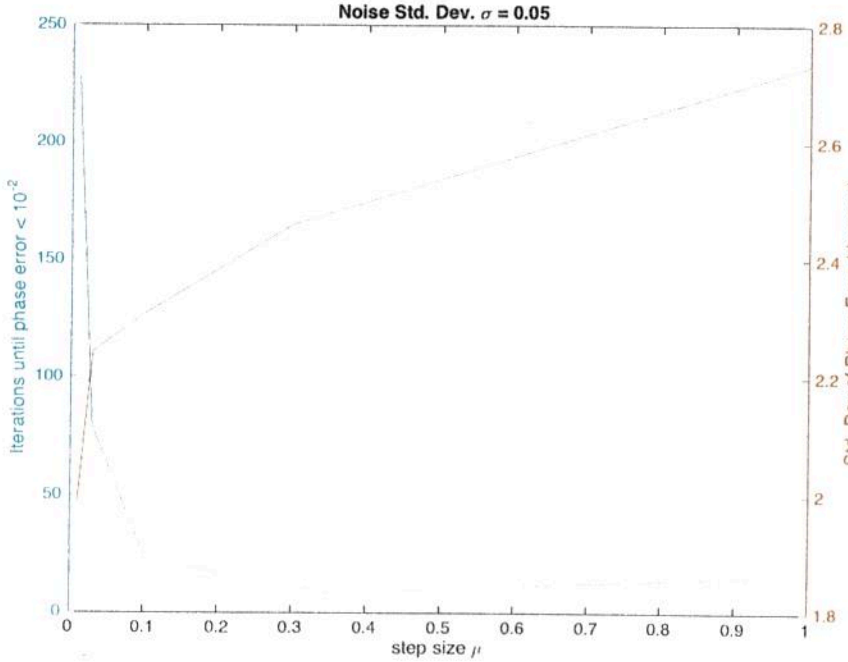
Similarly, to compute the standard deviation after initial convergence,

$$s\_theta = std(angle(r(N0 : N0+100)));$$

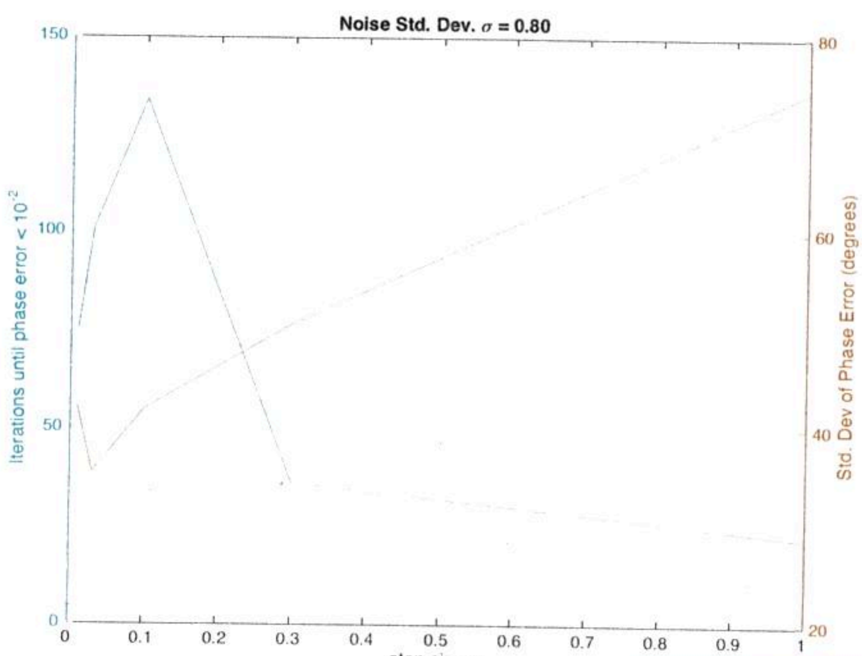
b) The plot below shows the initial convergence time N_0 and standard deviation as a function of step size μ.
For small step sizes, convergence is slow.
For large stepsizes, ~~noise~~ phase error is large.



Low noise: $\sigma = 0.05$ - significantly smaller phase error good step size $\mu = 0.1$. Marginal improvement in converges for $\mu > 0.1$ at the expense of greater noise.

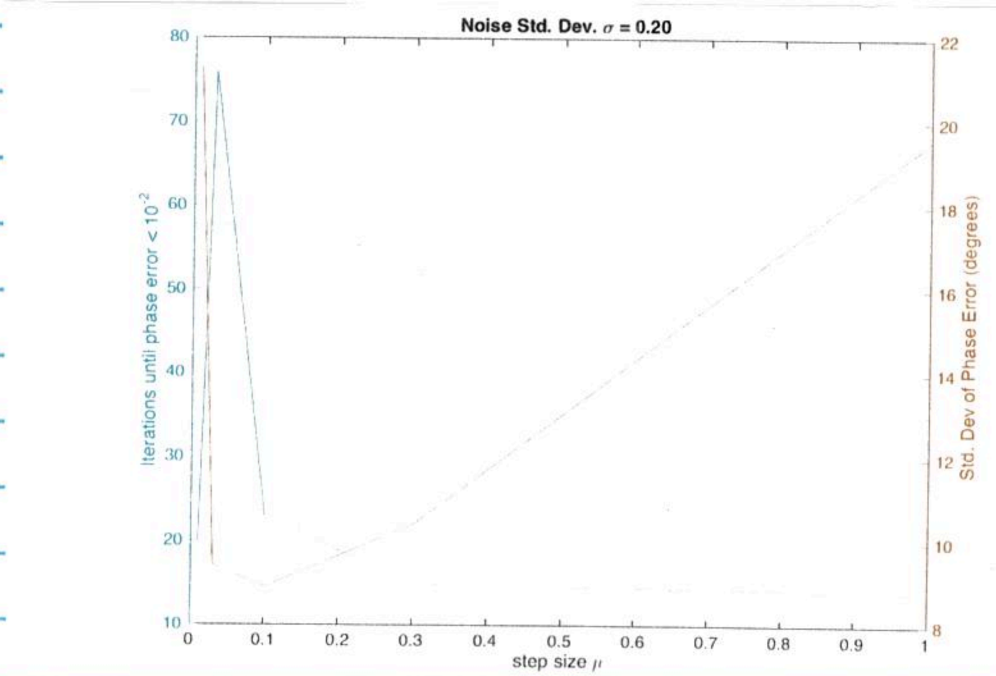


High noise: $\sigma = 0.8$: very large phase error (>20°)
Need very small step size $\mu \leq 0.1$ to control phase error.

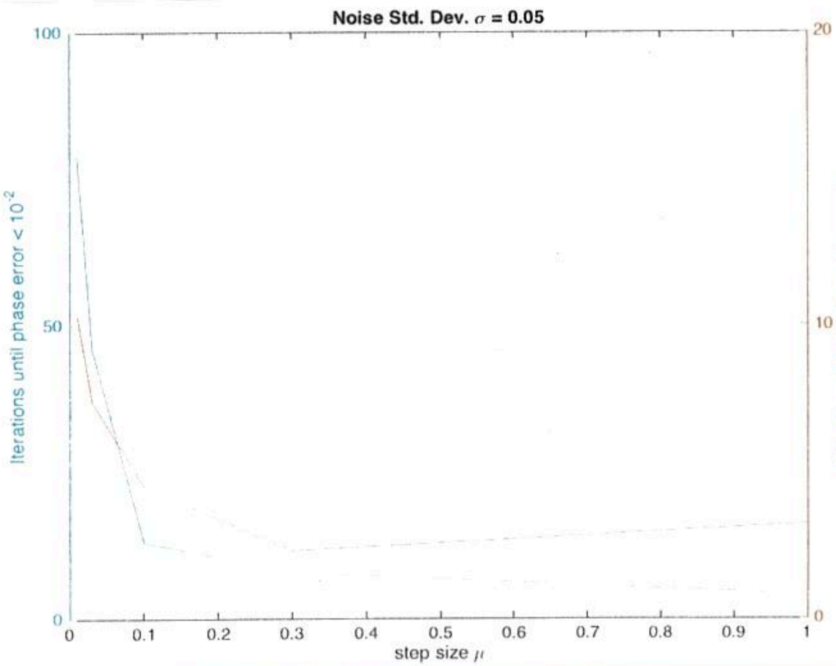


4. In general, tracking frequency error in addition to phase creates larger phase error.

Moderate noise ($\sigma = 0.2$): Again, fundamental trade-off between convergence and phase error (noise) is apparent. Best phase error at $\mu = 0.1$ with acceptable speed of convergence



Low noise ($\sigma = 0.05$): Best phase noise for $\mu = 0.3$ with good speed of convergence



High noise ($\sigma = 0.8$) ; very high phase error ($> 40^0$). Must keep μ small ($\mu = 0.1$) However, too small step size prevents proper tracking. For $\mu > 0.3$ no convergence.