ECE 460
Dr. Paris

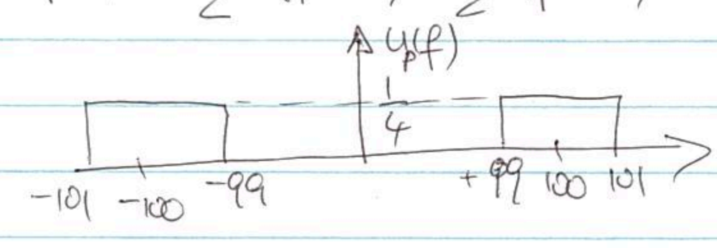
HW#3 - Solution

2.14

a) $u_p(t) = a(t) \cdot \cos(2\pi\, 100 t)$ $\qquad a(t) = \text{sinc}(2t$

$\updownarrow \qquad\qquad\qquad\qquad \updownarrow$

$U_p(f) = \frac{1}{2} A(f-100) + \frac{1}{2} A(f+100) \qquad A(f) = \frac{I_{[-1,1]}(f}{2}$



$\Rightarrow$ occupied Band: 99 - 101 MHz

b) $u_p(t) = a(t) \cdot \cos(2\pi 100 t)$

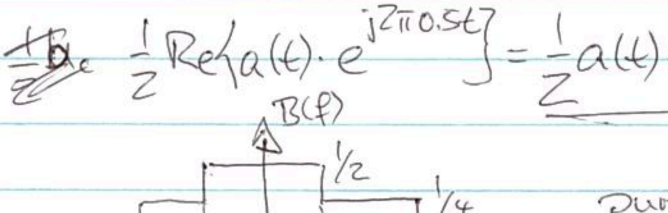
$= \text{Re}\{a(t) \cdot e^{j2\pi 100 t}\}$

switch reference to 99.5MHz

$= \text{Re}\{\underbrace{a(t) \cdot e^{j2\pi 0.5 t}}_{\text{baseband equivalent}} \cdot e^{j2\pi 99.5 t}\}$

Output of Lowpass filter (after $\cos(2\pi 99.5t)$):

$b(t) = \frac{1}{2} \text{Re}\{a(t) \cdot e^{j2\pi 0.5 t}\} = \underline{\frac{1}{2} a(t) \cdot \cos(2\pi\, 0.5 t)}$

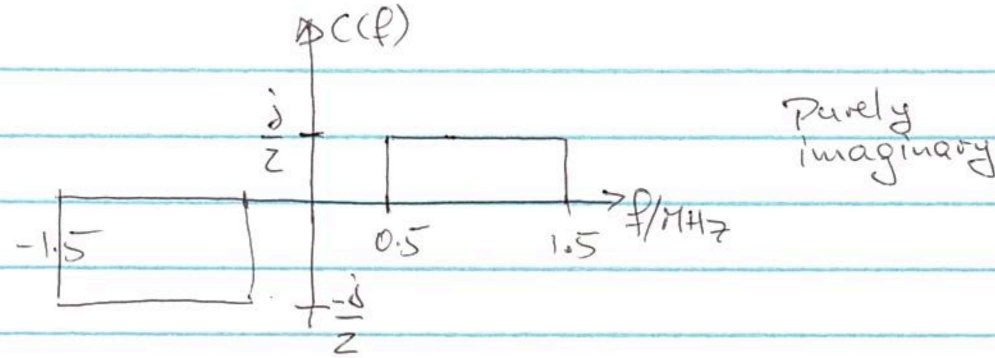


purely real

c) Similarly:

$c(t) = -\frac{1}{2} \cdot \text{Im}\{a(t) \cdot e^{j2\pi 0.5 t}\}$

$= -\frac{1}{2} a(t) \cdot \sin(2\pi 0.5 t)$



d) $b(t) = \frac{1}{2} a(t) \cdot \cos(2\pi\, 0.5t)$

$c(t) = \frac{1}{2} a(t) \cdot \sin(2\pi\, 0.5t)$

$\Rightarrow b(t) \cdot \cos(2\pi\, 0.5t) = \frac{1}{2} a(t) \cdot \cos^2(2\pi\, 0.5t)$

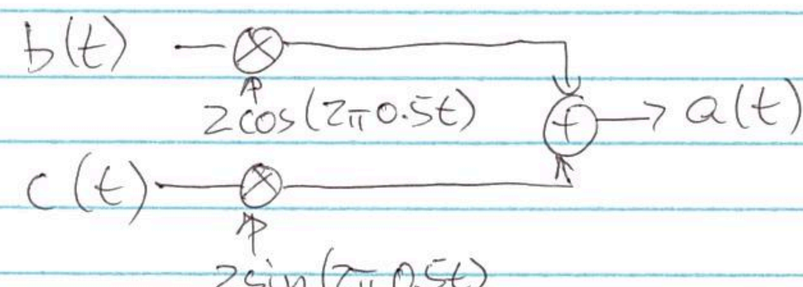
and $c(t) \cdot \sin(2\pi\, 0.5t) = \frac{1}{2} a(t) \cdot \sin^2(2\pi\, 0.5t)$

$$\Rightarrow b(t) \cdot \cos(2\pi\, 0.5t) + c(t) \cdot \sin(2\pi\, 0.5t) =$$

$$= \frac{1}{2} a(t) \cdot \left(\cos^2(2\pi\, 0.5t) + \sin^2(2\pi\, 0.5t)\right)$$

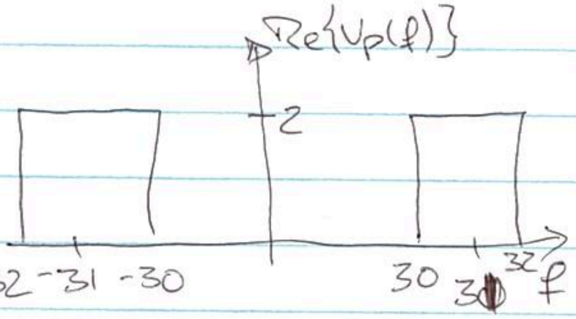
$$= \frac{1}{2} a(t)$$

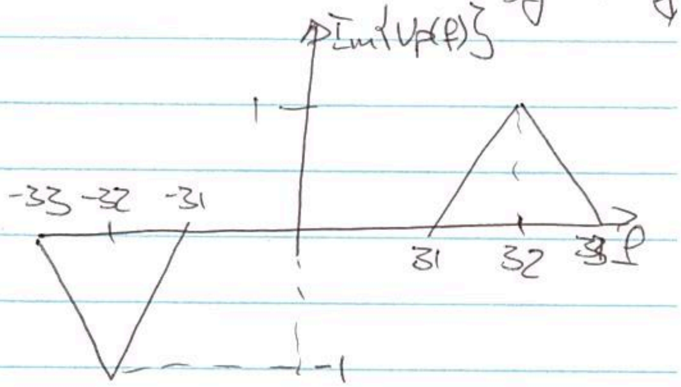
To recover $a(t)$:



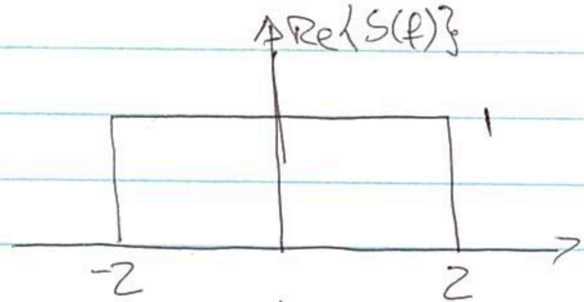
2.17 a) • $V_p(f)$ is given for positive f.

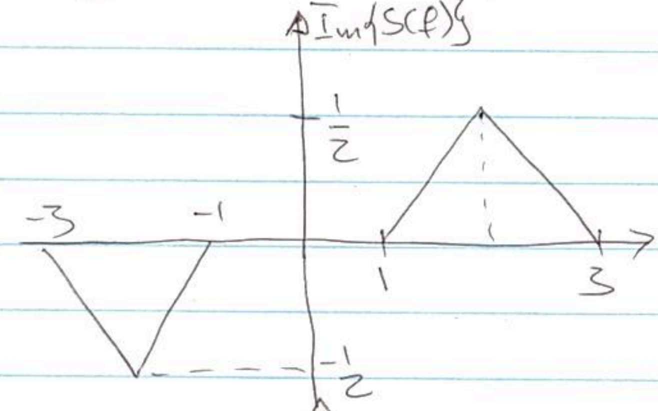
• $v_p(t)$ is real $\Rightarrow V_p(f) = V_p^*(-f)$ (conjugate symmetry)





b) $s(t) = v_p(t) \cdot \cos(2\pi 30 t) \longleftrightarrow \frac{1}{2} V_p(f-30) + \frac{1}{2} V_p(f+30) = S(f)$





$4 \cdot \mathrm{sinc}(4t)$

$-\mathrm{sinc}^2(t) \cdot \sin(2\pi 2t)$

$\Rightarrow s(t) = 4 \cdot \mathrm{sinc}(4t) - \mathrm{sinc}^2(t) \cdot \sin(2\pi 2t)$

2.18 $u_p(t) = I_{[-1,1]}(t) \cdot \cos(2\pi 50 t)$

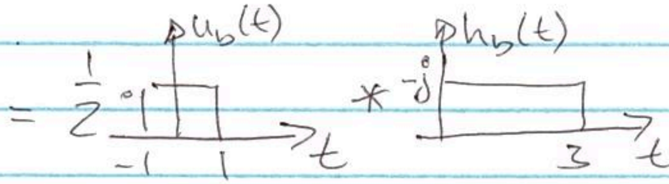
$h_p(t) = I_{[0,3]}(t) \cdot \sin(2\pi 50 t)$

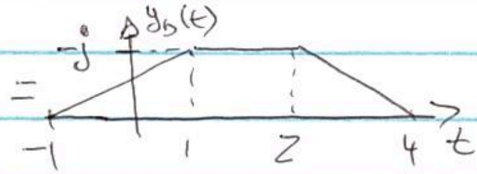
(a) Baseband equivalent signals

$u_b(t) = I_{[-1,1]}(t) \qquad h_b(t) = -j \cdot I_{[0,3]}(t)$

2.) Convolve baseband signals

$$y_b(t) = \frac{1}{2} \cdot u_b(t) * h_b(t)$$





$y_b(t) = -j \cdot p(t)$

Let $p(t)$ denote the shape of tropezoid above.

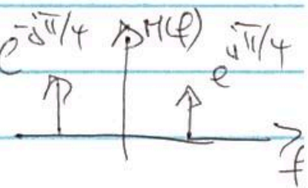
3.) Then, passband signal:

$$y_p(t) = \mathrm{Re}\{y_b(t) \cdot e^{j 2\pi 50 t}\}$$

$$= p(t) \cdot \sin(2\pi 50 t)$$

3.2.) This problem is poorly worded:

a) $m(t) = 2\cos(2\pi t + \pi/4)$

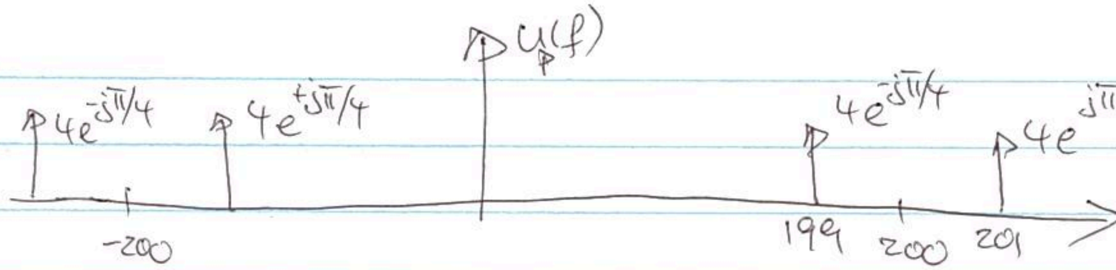


$$= e^{j\pi/4} e^{j2\pi t} + e^{-j\pi/4} \cdot e^{-j2\pi t}$$

$$M(f) = e^{j\pi/4}\delta(f-1) + e^{-j\pi/4}\delta(f+1)$$

$$u_p(t) = 8 \cdot m(t) \cdot \cos(2\pi 200 t)$$

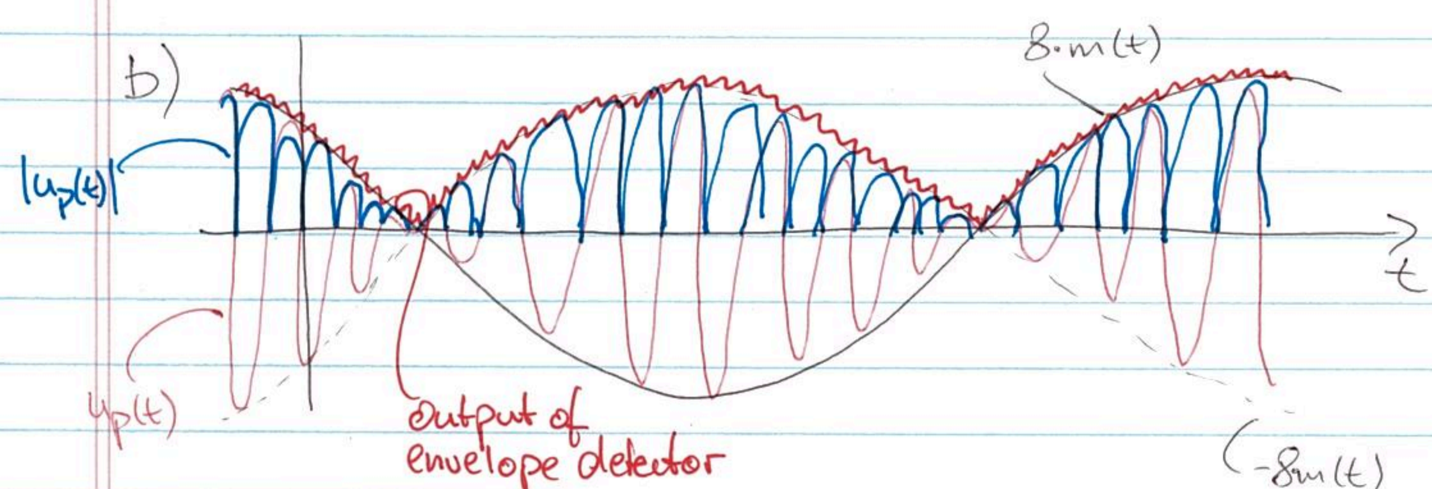
$$U_p(f) = 4 \cdot M(f-200) + 4 \cdot M(f+200)$$



Power of each of the four tones is $4^2 = 16$

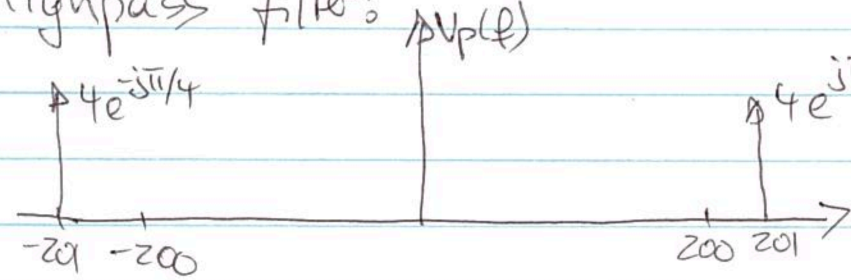
$\Rightarrow$ total power: $4 \cdot 16 = 64$

b)



Output from ideal envelope detector: $8 \cdot |m(t)|$

c) After highpass filter:



$$\Rightarrow v_p(t) = 8 \cdot \cos(2\pi\, 201t + \pi/4)$$

$$= 8 \cdot \cos(2\pi\, 200t + 2\pi t + \pi/4)$$

$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$

$$= 8 \cdot \cos(2\pi t + \pi/4) \cdot \cos(2\pi\, 200t) - 8 \cdot \sin(2\pi t + \pi/4) \cdot \sin(2\pi\, 200t)$$

$\Rightarrow v_c(t) = 8\cos(2\pi t + \pi/4)$ $\quad v_s(t) = 8 \cdot \sin(2\pi t + \pi/4)$