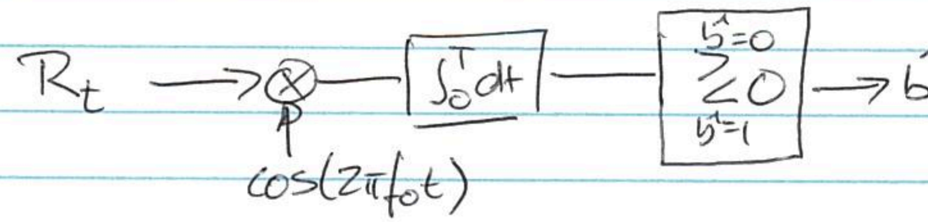
Dr. Paris

HW 11 Solution

ECE 460

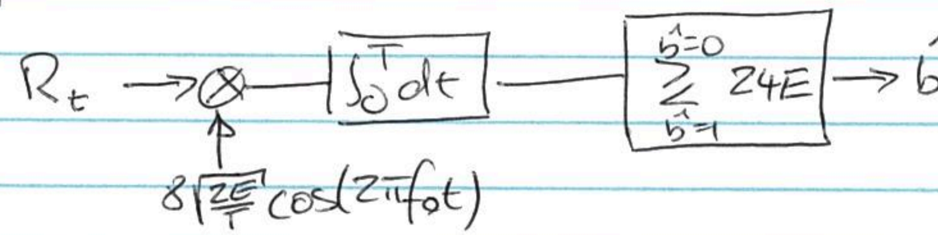
1. a)



b) $E_0 = E_1 = -r_{01} = 25E$

$$\Rightarrow P_e = Q\left(\sqrt{\frac{100E}{2N_0}}\right) = Q\left(\sqrt{\frac{50E}{N_0}}\right)$$

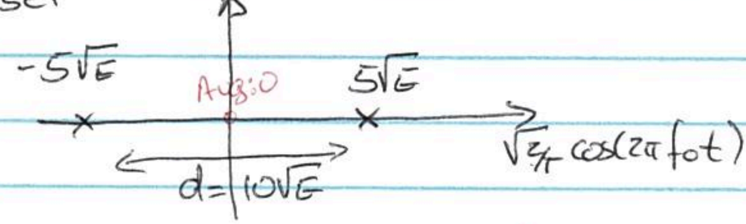
c) opt. receiver



$E_0 = 49E \quad E_1 = 1 \quad r_{01} = -7E$

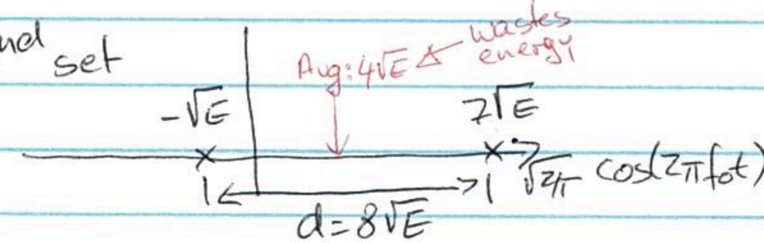
$$P_e = Q\left(\sqrt{\frac{64E}{2N_0}}\right) = Q\left(\sqrt{\frac{32}{N_0}}\right)$$

d) 1st set



e) $E_b = \frac{1}{2}(E_0 + E_1)$
$= 25E$

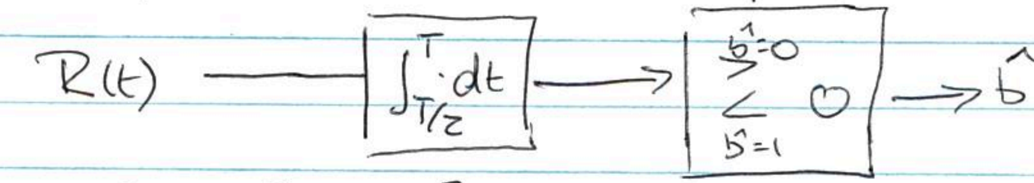
2nd set



$E_b = \frac{1}{2}(E_0 + E_1)$
$= 25E$

1st set is better; for same transmitted energy (25E), 1st set has better distance $\Rightarrow$ better P_e

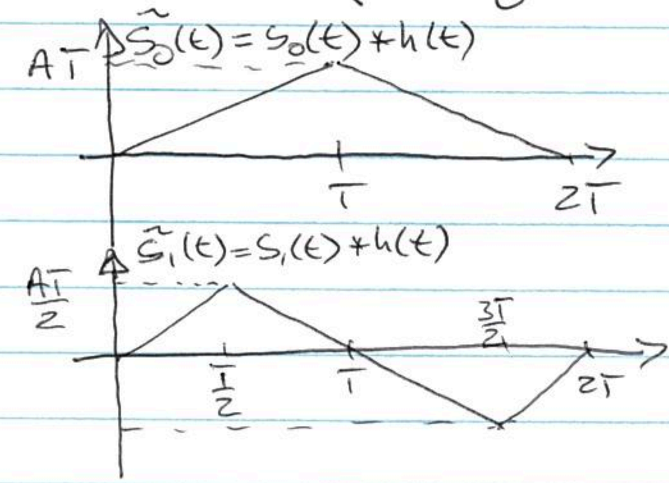
2\. a) Opt. receiver can be simplified to: (why?)



$$E_0 = E_1 = A^2 T \qquad r_{01} = 0$$

$$\Rightarrow P_e = Q\left(\sqrt{\frac{2A^2T}{2N_0}}\right) = Q\left(A\sqrt{T/N_0}\right)$$

b)



c) The receiver in (a) is not optimal for the signals $\tilde{s}_0(t)$ and $\tilde{s}_1(t)$.

$$P_e = Q\left(\frac{\langle \tilde{s}_0(t) - \tilde{s}_1(t), g(t)\rangle}{\sqrt{2N_0 \, \|g(t)\|^2}}\right) \quad \text{with: } g(t) = \begin{cases} 1, & \frac{T}{2} \le t \le T \\ 0, & \text{else} \end{cases}$$

$$\langle \tilde{s}_0(t) - \tilde{s}_1(t), g(t)\rangle = \int_{T/2}^{T} \tilde{s}_0(t) - \tilde{s}_1(t) \, dt$$

$$= \int_{T/2}^{T} A\cdot t - (AT - At) \, dt$$

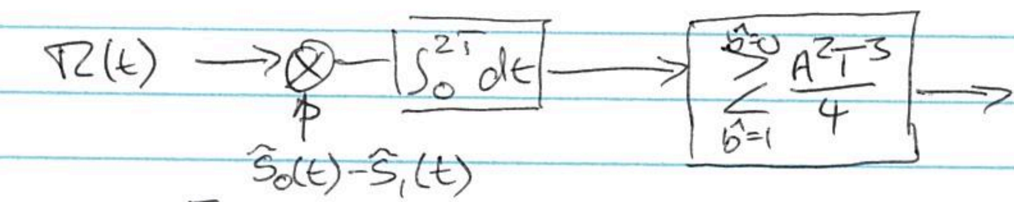
$$= \int_{T/2}^{T} 2At - AT \, dt$$

$$= \frac{2At^2}{2} - ATt \Big|_{T/2}^{T} = \frac{AT^2}{4}$$

$$\|g(t)\|^2 = \int_{T/2}^{T} 1^2 dt = \frac{T}{2}$$

$$\Rightarrow P_e = Q\left(\frac{AT^2/4}{\sqrt{2N_0 \cdot T/2}}\right) = Q\left(\frac{AT}{4}\cdot\sqrt{\frac{T}{N_0}}\right) = Q\left(AT\cdot\sqrt{\frac{T}{16\cdot N_0}}\right)$$

d) Opt. receiver correlates with $\tilde{s}_0(t) - \tilde{s}_1(t)$! Note, this requires that $h(t)$ is known.



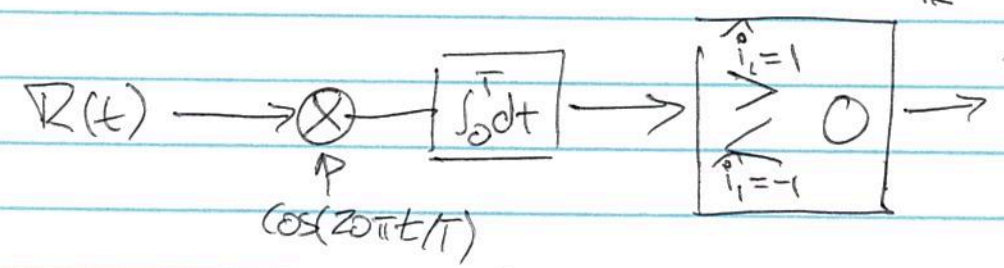
$$\tilde{E}_0 = 2\cdot\int_0^T (At)^2 dt = \frac{2}{3}A^2T^3 \qquad \tilde{r}_{01} = 0$$

$$\tilde{E}_1 = 4\cdot\int_0^{T/2} (At)^2 = \frac{1}{6}A^2T^3 \qquad \tilde{K} = \frac{1}{2}(\tilde{E}_0 - \tilde{E}_1) = \frac{A^2T^3}{4}$$

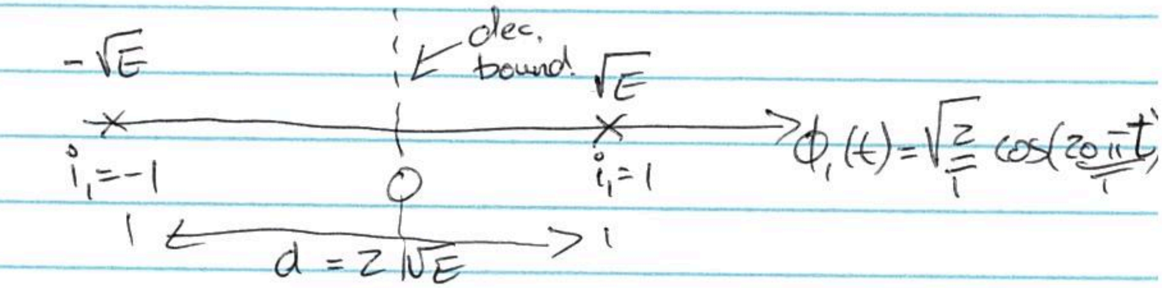
$$P_e = Q\left(\sqrt{\frac{\tilde{E}_0 + \tilde{E}_1}{2N_0}}\right) = Q\left(AT\cdot\sqrt{\frac{5\cdot T}{12\cdot N_0}}\right)$$

This is much (namely $\frac{20}{3}$ times better SNR) than (c

3. a)

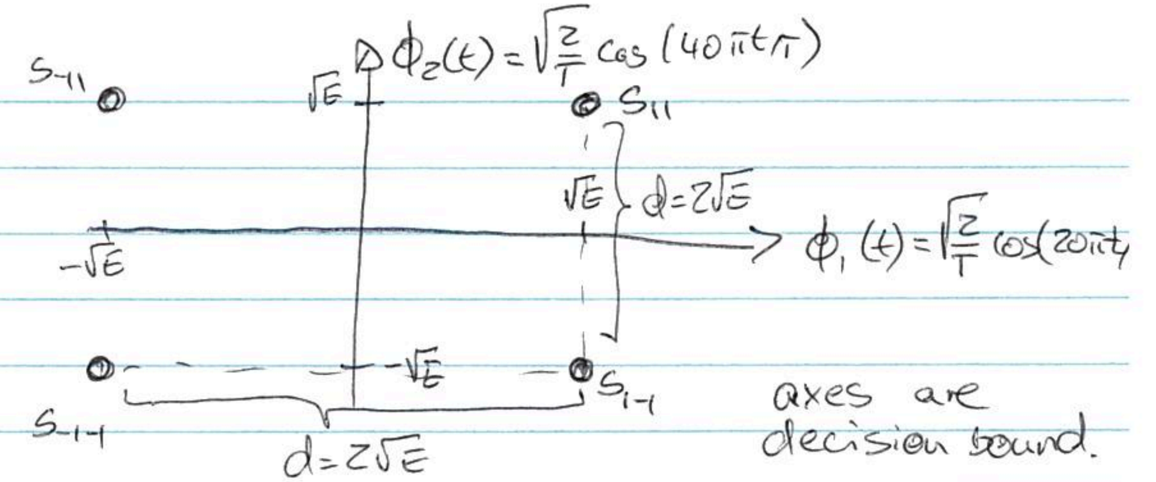


b)



$$P_e = Q\left(\frac{d}{\sqrt{2N_0}}\right) = Q\left(\frac{2\sqrt{E}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

c)

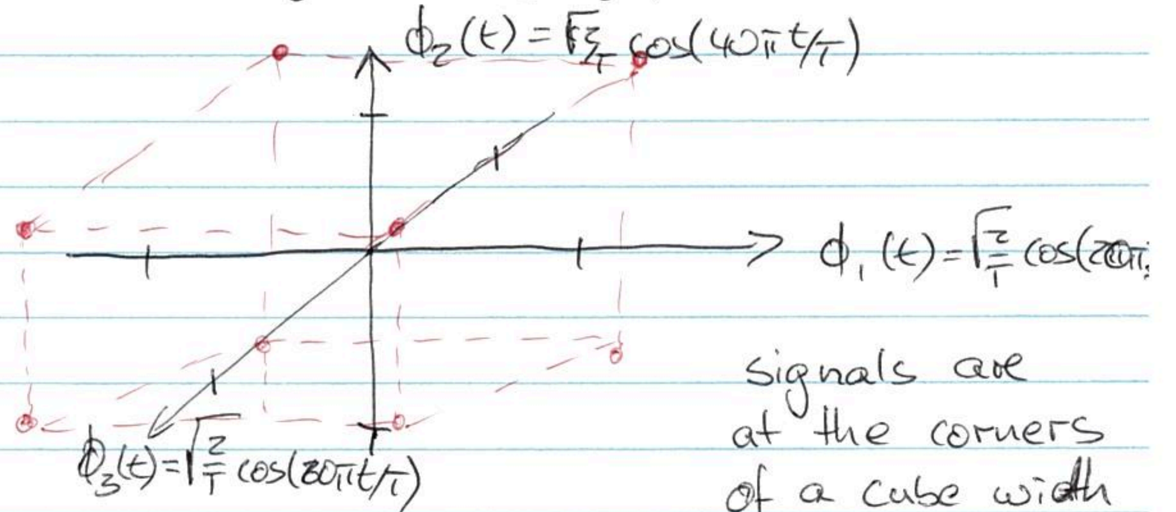


axes are decision bound.

$$P_e = 1 - \left(1 - Q\left(\sqrt{\frac{2E}{N_0}}\right)\right)^2$$

$$= 2Q\left(\sqrt{\frac{2E}{N_0}}\right) - Q^2\left(\sqrt{\frac{2E}{N_0}}\right)$$

d)



signals are at the corners of a cube with sides equal to $d = 2\sqrt{E}$

$$P_e = 1 - \left(1 - Q\left(\sqrt{\frac{2E}{N_0}}\right)\right)^3$$

$$= 3Q\left(\sqrt{\frac{2E}{N_0}}\right) - 3Q^2\left(\sqrt{\frac{2E}{N_0}}\right) + Q^3\left(\sqrt{\frac{2E}{N_0}}\right)$$

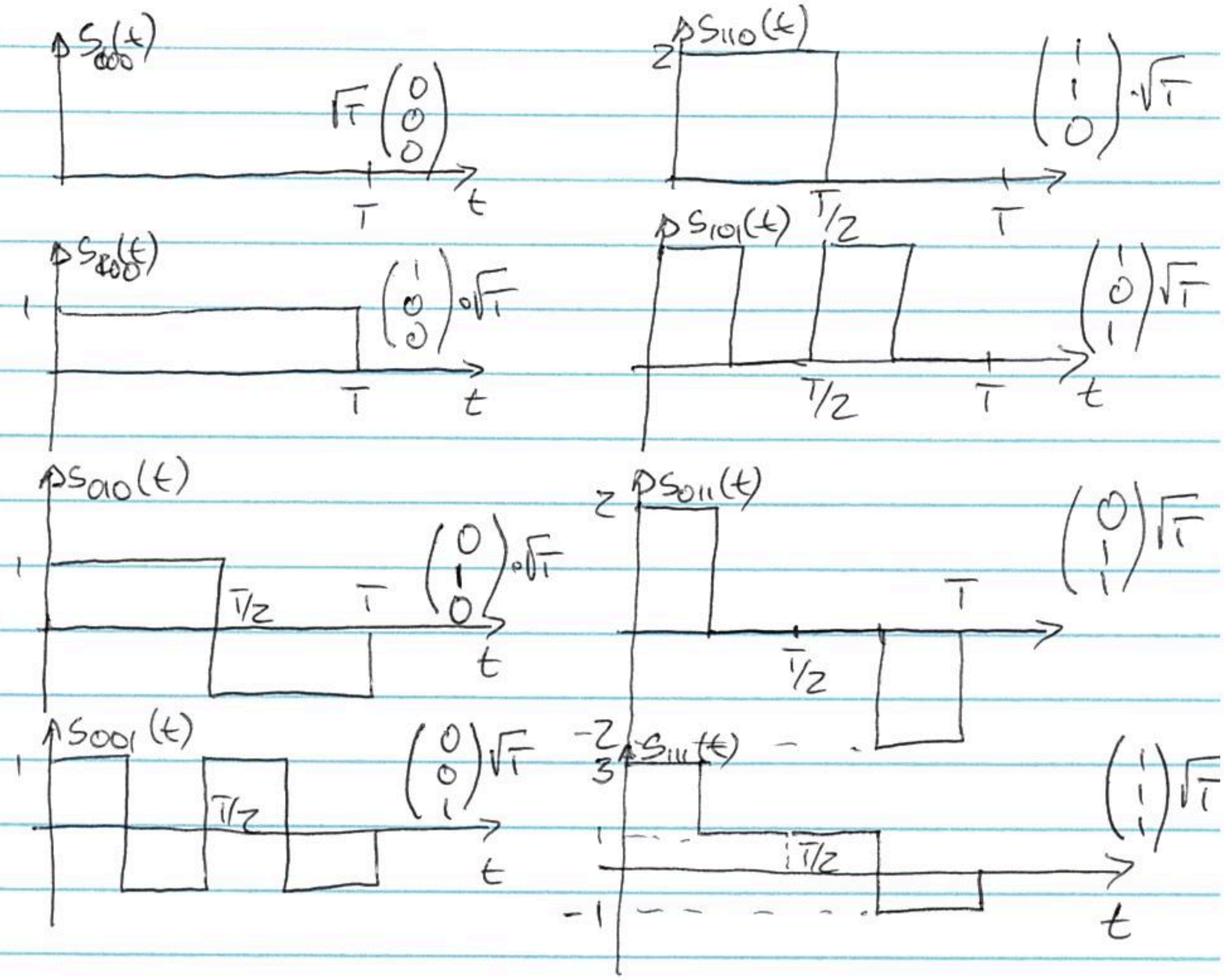
e) General expression for error probability

signals are the corners of N-dimensional hypercube.

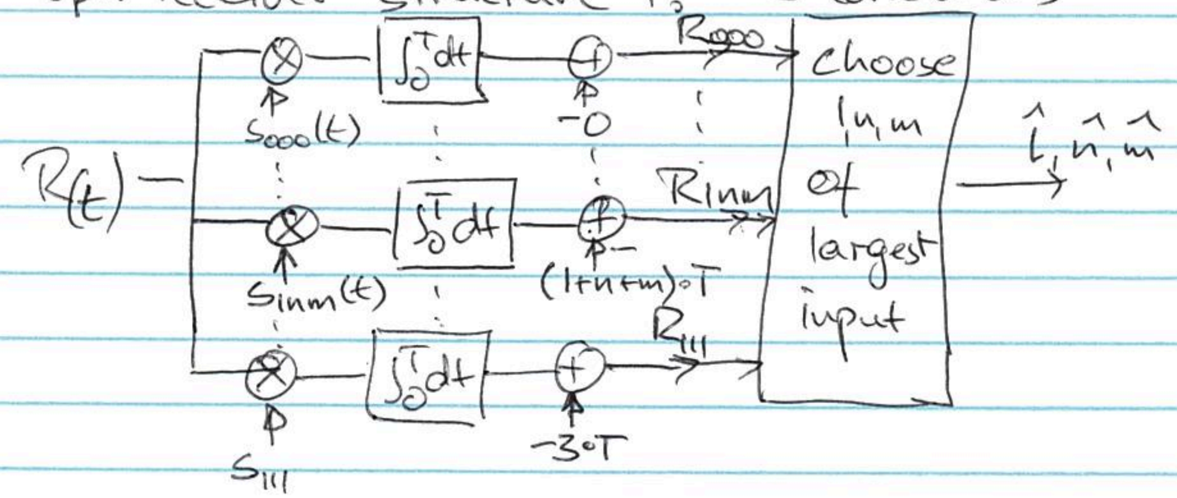
$$P_e = 1 - \left(1 - Q\left(\sqrt{\frac{2E}{N_0}}\right)\right)^N$$

$$= \sum_{n=1}^{N} (-1)^n \binom{N}{n} Q^n\left(\sqrt{\frac{2E}{N_0}}\right)$$

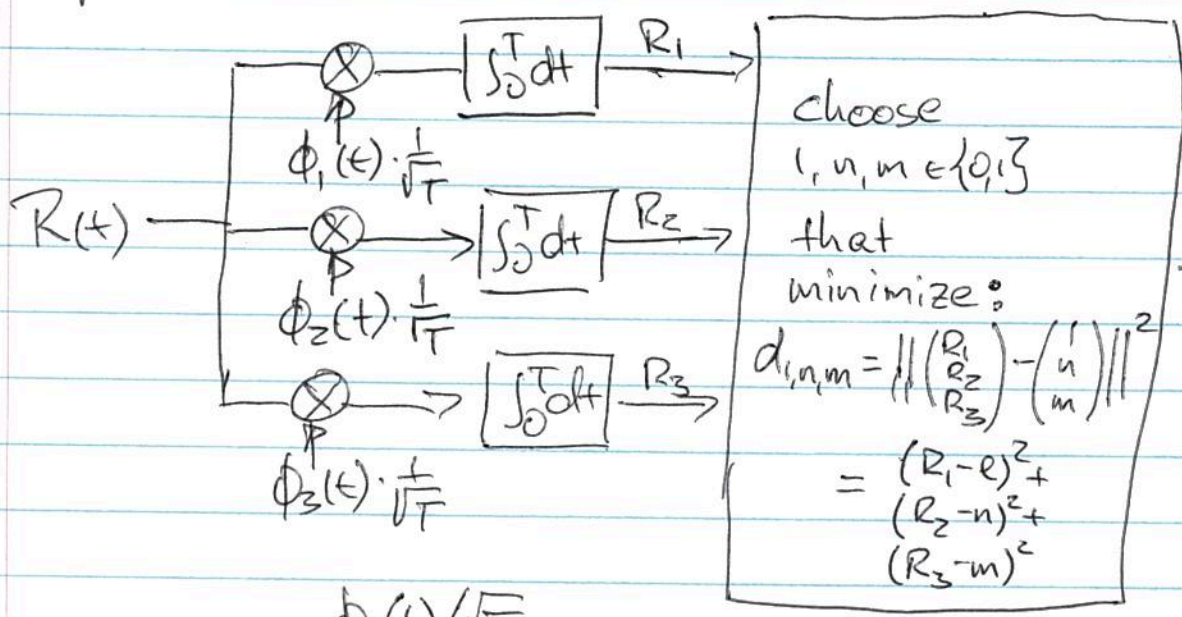
4a)



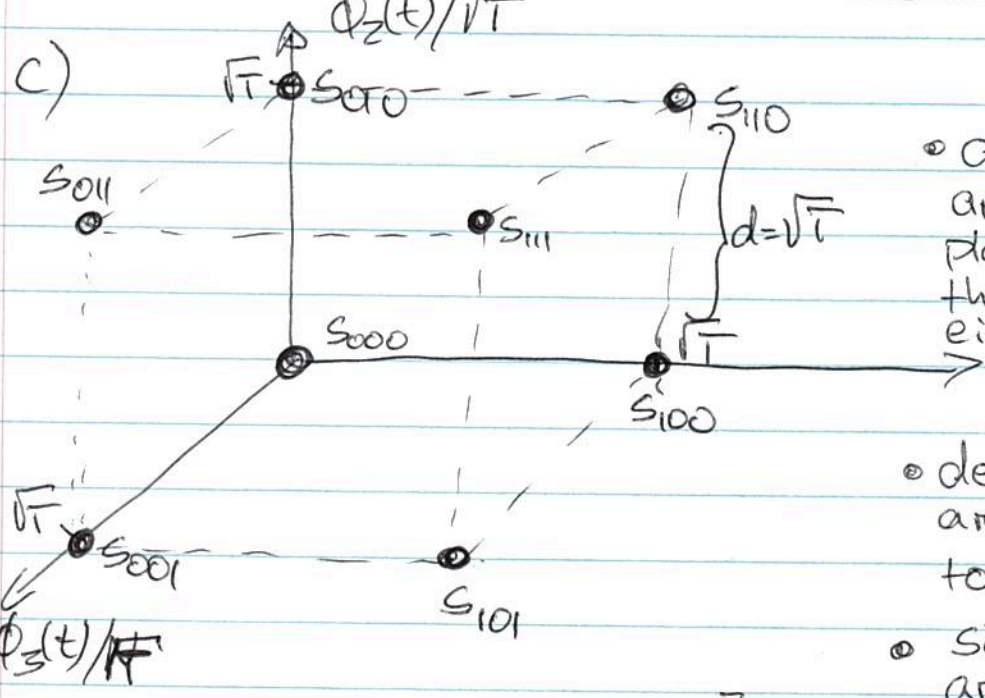
b) Opt. receiver structure 1: 8 correlators



opt. receiver structure 2: 3 correlators



c)



- decision boundaries are the three planes that cut the cube into eight equal pieces
- decision boundaries are perpendiclurar to sides of cube
- sides of cube are length $d = \sqrt{T}$

d) $P_e = 1-\left(1-Q\left(\sqrt{\frac{T}{2N_0}}\right)\right)^3$

$$= 3Q\left(\sqrt{\frac{T}{2N_0}}\right)-3Q^2\left(\sqrt{\frac{T}{2N_0}}\right)+Q^3\left(\sqrt{\frac{T}{2N_0}}\right)$$

e) $s_{l,n,m}(t) = \sqrt{2}\cdot l\cdot\cos(20\pi t/T) + \sqrt{2}\cdot n\cdot\cos(40\pi t/T) + \sqrt{2}\cdot m\cdot\cos(60\pi t/T)$ $\quad l,n,m \in \{0,1\}$

(see P. 360)