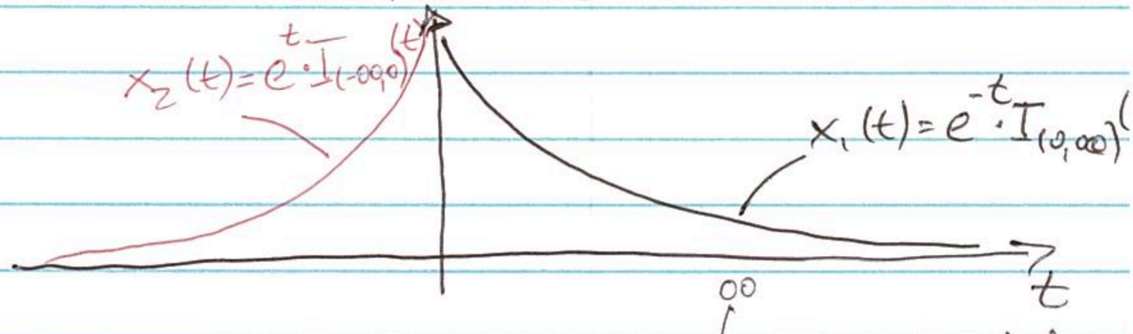
ECE 460
Dr. Paris

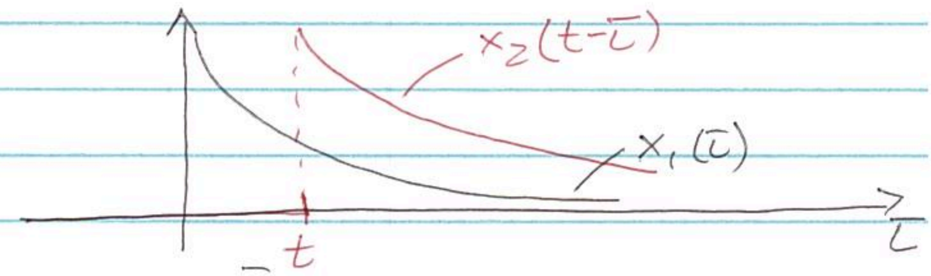
HW#1 - Solution

2.2. a) Since the signals involved are exponential signals, this problem is best solved in the time-domain.



The convolution $x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$ must distinguish two cases:

(1) $t \geq 0$



with $x_1(\tau) = e^{-\tau} \cdot I_{(0,\infty)}(\tau)$

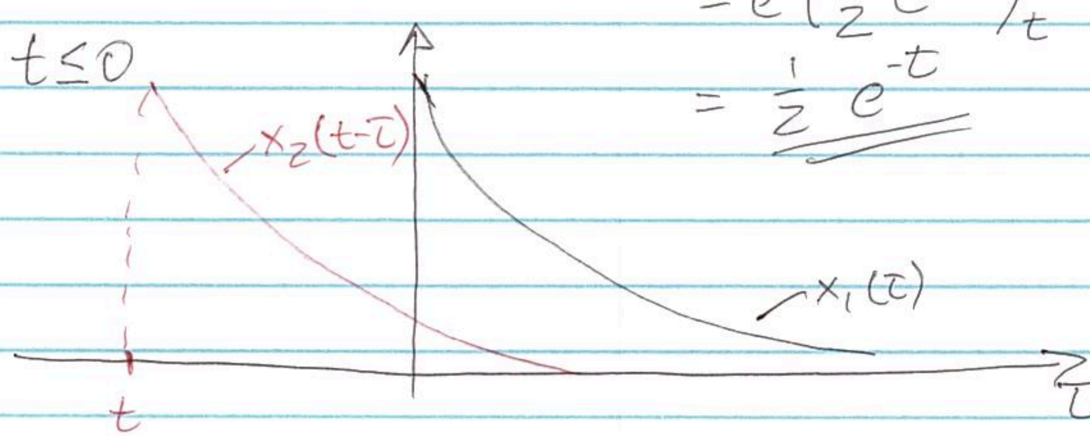
and $x_2(t-\tau) = e^{(t-\tau)} \cdot I_{(t,\infty)}$

$$x_1(t) * x_2(t) = \int_t^{\infty} e^{-\tau} \cdot e^{t-\tau}\, d\tau = e^{t} \cdot \int_t^{\infty} e^{-2\tau}\, d\tau$$

$$= e^{t} \left(\frac{-1}{2} e^{-2\tau} \right)_t^{\infty}$$

$$= \underline{\underline{\frac{1}{2} e^{-t}}}$$

(2) $t \leq 0$



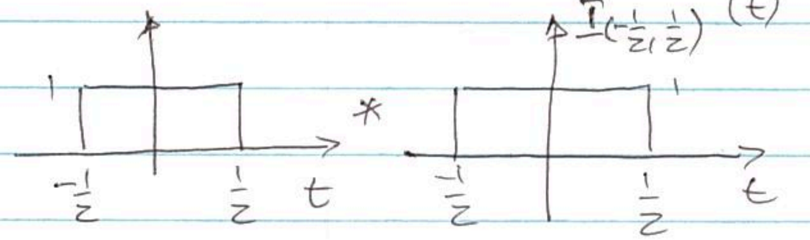
$$x_1(t) * x_2(t) = \int_0^{\infty} e^{-\tau} \cdot e^{t-\tau}\, d\tau = e^{t} \cdot \int_0^{\infty} e^{-2\tau}\, d\tau$$

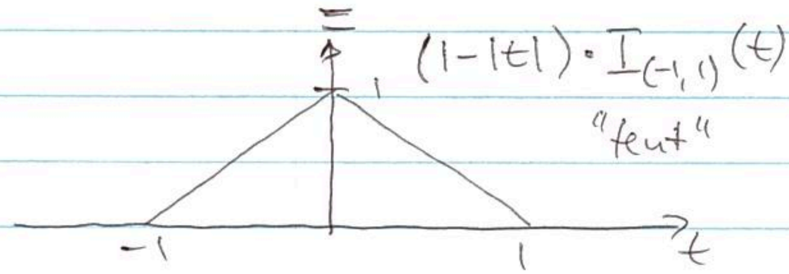
$$= \frac{1}{2} e^{t}$$

In summary

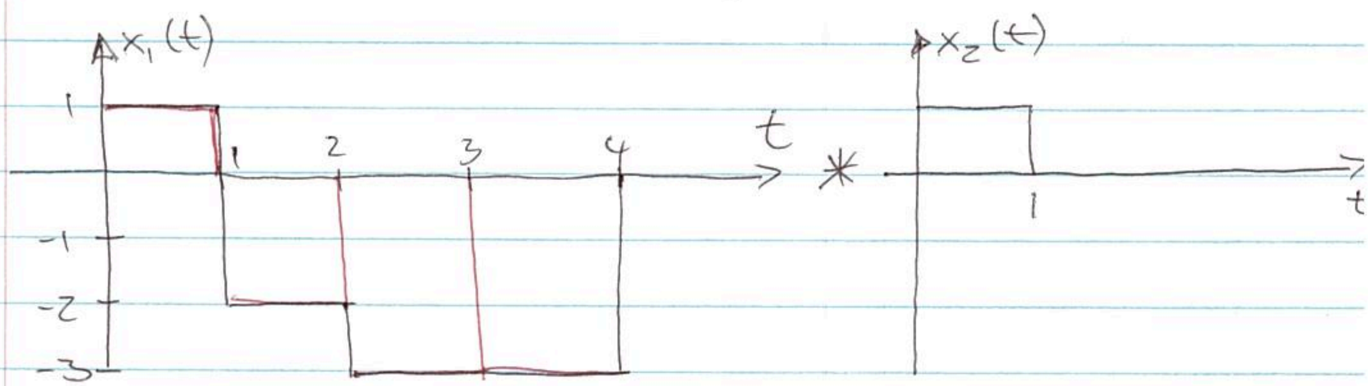
$$x_1(t) * x_2(t) = \begin{cases} \frac{1}{2} e^{t} & \text{if } t \le 0 \\ \frac{1}{2} e^{-t} & \text{if } t \ge 0 \end{cases} = \frac{1}{2} e^{-|t|}$$

b) Recall that:

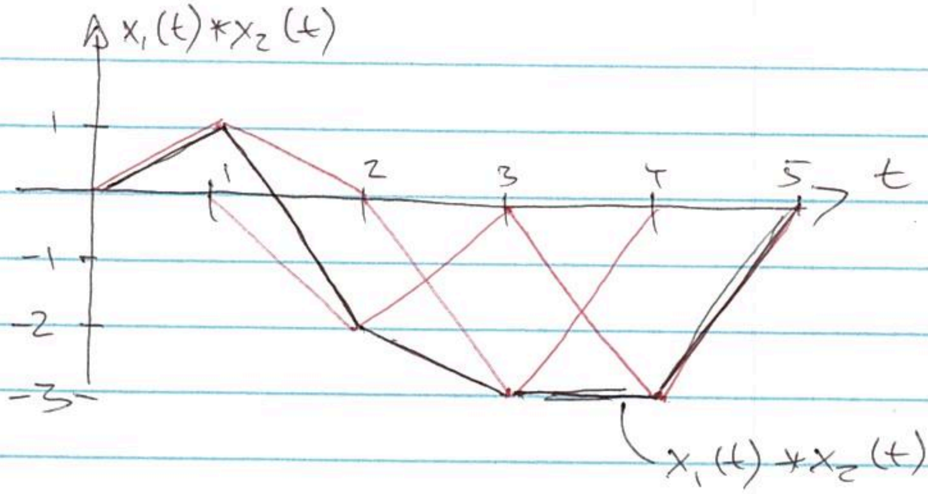




Break $x_1(t)$ into 4 rectangular segments:



Each of the 4 rectangular segments, produce a "tent" signal. The result of the convolution is the sum of the "tents"



2.4. a) "Tent" signal is convolution of rectangles

$$u(t) = (1-|t|)\cdot I_{(-1,1)}(t)$$

$$= I_{(-\frac{1}{2},\frac{1}{2})}(t) \quad * \quad I_{(-\frac{1}{2},\frac{1}{2})}(t)$$

Fourier transform:

$$\updownarrow \qquad\qquad \updownarrow$$

$$\mathrm{sinc}(f) \quad \cdot \quad \mathrm{sinc}(f)$$

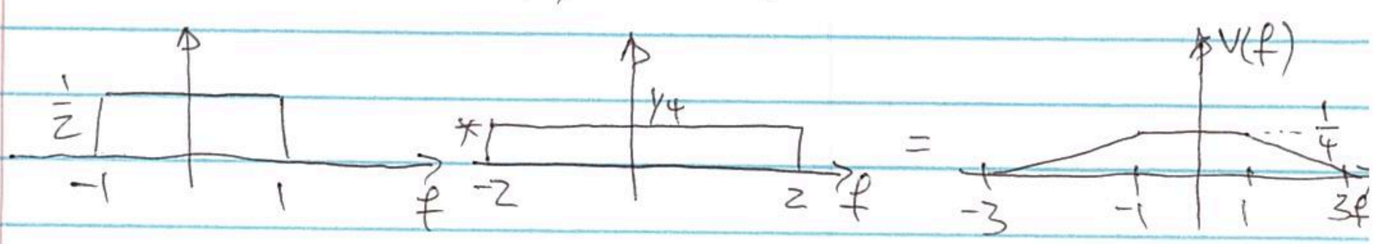
$$= \mathrm{sinc}^2(f) = U(f)$$

b) Fourier transform pair: $\mathrm{sinc}(f_0 t) \longleftrightarrow \frac{1}{f_0}\cdot I_{(-\frac{f_0}{2},\frac{f_0}{2})}(f)$

$$v(t) = \mathrm{sinc}(2t) \cdot \mathrm{sinc}(4t)$$

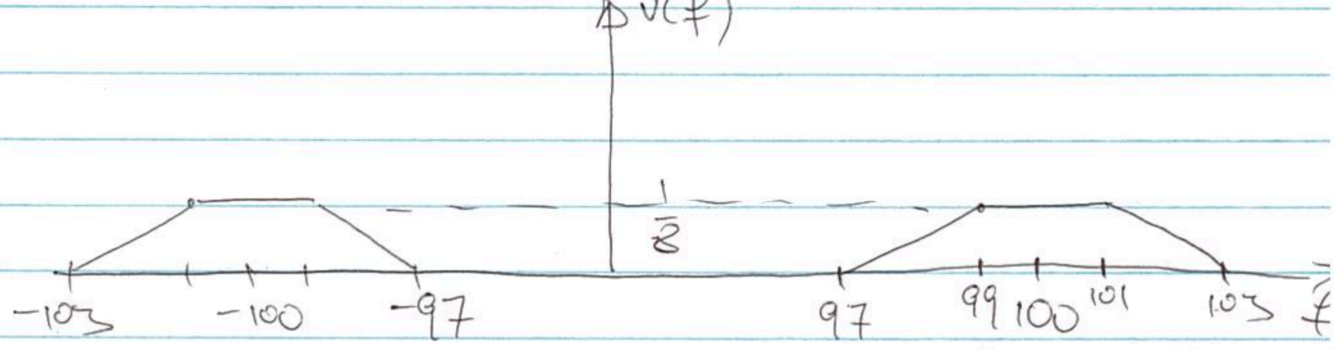
$$\updownarrow \qquad\qquad \updownarrow$$

$$\frac{1}{2}\cdot I_{(-1,1)} \quad * \quad \frac{1}{4} I_{(-2,2)} = V(f)$$



c) By the modulation property:

$$S(f) = \frac{1}{2} V(f-100) + \frac{1}{2} V(f+100)$$



d) • the signals in a) and b) are baseband signals

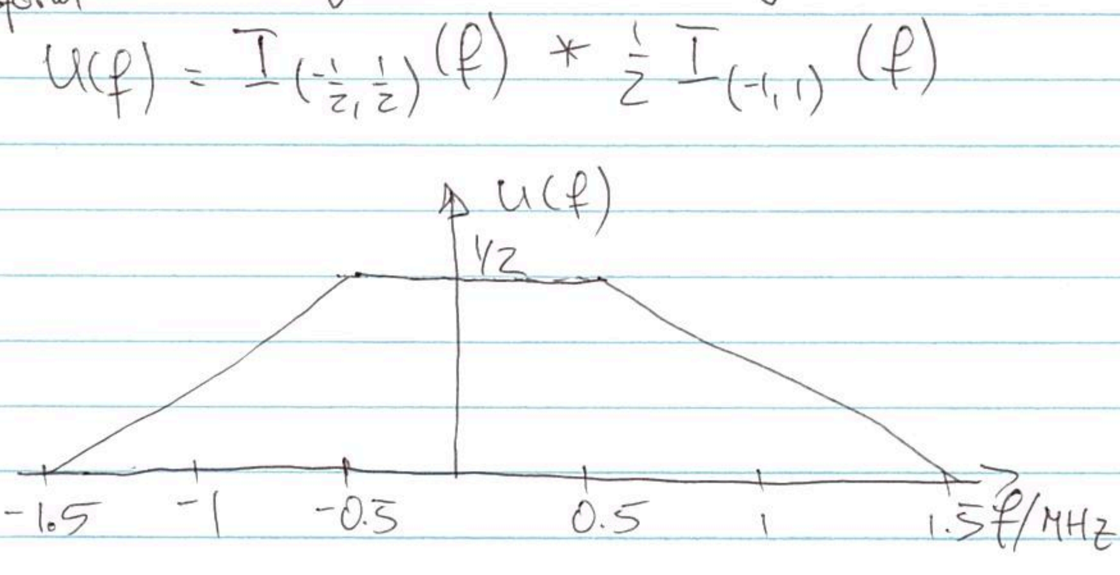
• the signal c) is passband

2.6. a)

$$u(t) = \mathrm{sinc}(t) \cdot \mathrm{sinc}(2t)$$

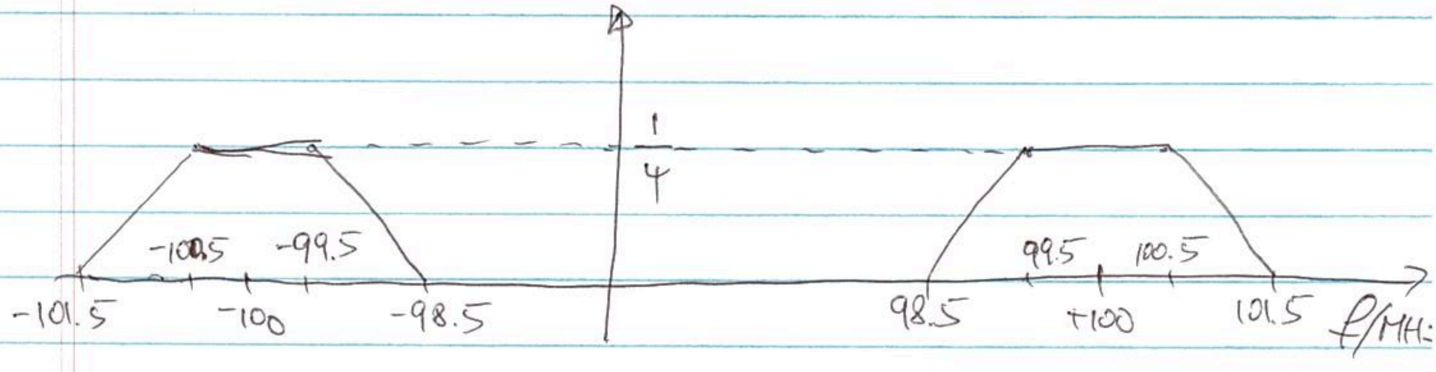
Fourier transform

$$U(f) = I_{(-\frac{1}{2},\frac{1}{2})}(f) * \frac{1}{2} I_{(-1,1)}(f)$$



b) By the modulation property:

$$S(f) = \frac{1}{2}\, U(f-100) + \frac{1}{2}\, U(f+100)$$



2.7. In the frequency domain:

$$Y(f) = S(f) \cdot H(f)$$

$$s(t) = \operatorname{sinc}(4t) \longleftrightarrow S(f) = \frac{1}{4} \cdot I_{(-2,2)}(f)$$

$$h(t) = \operatorname{sinc}^2(t) \cdot \cos(4\pi t) \longleftrightarrow H(f) = \frac{1}{2} \cdot (1 - |t|) \cdot I_{(-1,1)} * \left(\delta(f-2) + \delta(f+2)\right)$$