ECE 732: Mobile Communication Systems
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Solution to Homework 2

1. Problem 1

   (a)

   $$S_{R(dBm)} = \frac{E_s}{N_0}_{(dB)} + kT_{0(dBm/Hz)} + F_{(dB)} + L_{R(dB)} + R_{S(dBHz)} = -100dBm$$

   (b) Maximum allowed path loss

   $$L_{P,max(dB)} = P_{t(dBm)} + G_{T(dB)} + G_{R(dB)} - S_{R(dBm)} = 130dB.$$

   From path loss model and requirement that

   $$L_{P(dB)}(d_{max}) = 130dB$$

   we find $d_{max} = 10^3 m$.

   (c) Additional path loss due to increased distance:

   $$\Delta L_{P(dB)} = 30dB \cdot 10\log_{10}(\frac{2km}{1km}) = 9dB.$$

   This must be compensated by increasing transmit power by 9 dB to 8 W.

   (d) Sensitivity increases by

   $$\Delta S_{R(dB)} = 10\log_{10}(\frac{1MHz}{100KHz}) = 10dB.$$

   Hence, path loss may increase by

   $$\Delta L_{P(dB)} = 30dB \cdot 10\log_{10}(\frac{ad}{d}) = 10dB$$

   which indicates that the range may increase by factor $a = 10^{1/3}$ to 2.15 km.

   (e) With the given values there is no margin. Hence, the probability that the link is not closed is 50%.

(f) The required margin M must satisfy

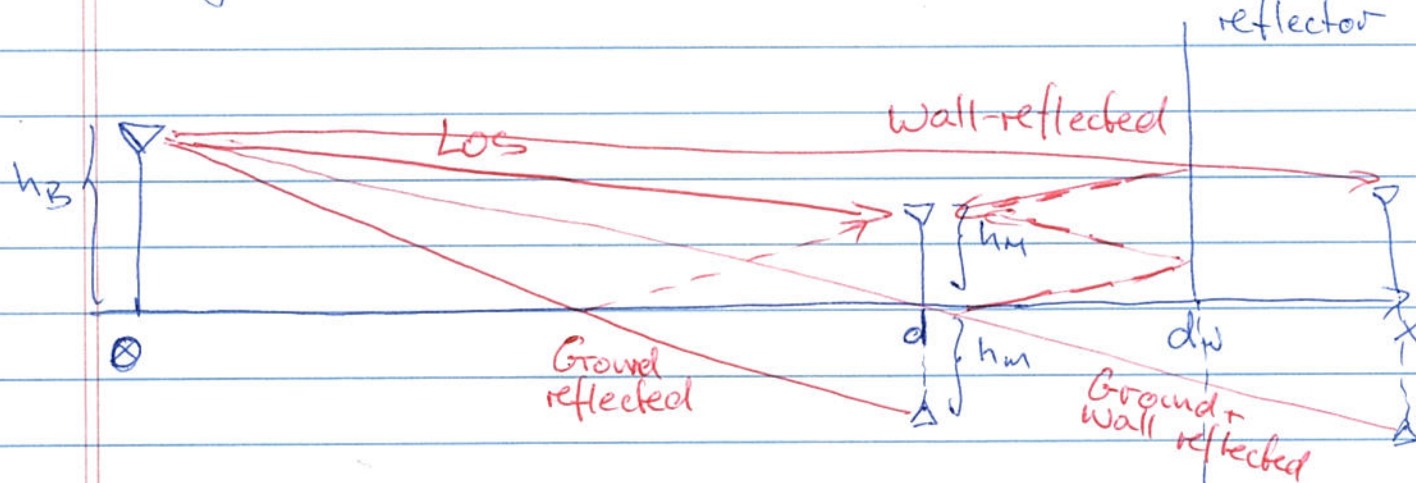
$$\Pr(\text{Fading} > M) = 10^{-3},$$

which implies

$$M = Q^{-1}(10^{-3})\sigma = 12dB.$$

2. Problem 2

You can put the second reflector beyond the receiver:



- There are 4 paths from base to receiver

- MATLAB not provided

3. Problem 2.18

Simplified model: (let $d_0 = 1m$)

$$P_{r,dBm} = P_{t,dBm} + K_{dB} - 10\gamma \log_{10}\left(\frac{d}{d_0}\right)$$

a) <u>Objective:</u> Find model parameters K_{dB} and γ to fit measurements.

<u>Least squares:</u> $\min\limits_{K,\gamma} \sum\limits_{n=1}^{N_M} \left| P_{r,n} - K_{dB} + 10\gamma \log_{10}\left(\frac{d_n}{d_0}\right)\right|^2$

Derivatives w.r.t. K_{dB} and γ:

$$\frac{\partial}{\partial K_{dB}} : -2 \sum_{n=1}^{N_M} \left(P_{r,n} - K_{dB} + 10\gamma \log_{10}\left(\frac{d_n}{d_0}\right)\right) \stackrel{!}{=} 0$$

$$\Rightarrow K_{dB} = \frac{1}{N_M}\sum_{n=1}^{N_M}\left(P_{r,n} + 10\gamma \log_{10}\left(\frac{d_n}{d_0}\right)\right)$$

$$= \frac{1}{N_M}\sum_{n=1}^{N_M} P_{r,n} + \gamma \cdot \frac{1}{N_M}\sum_{n=1}^{N_M} \log_{10}\left(\frac{d_n}{d_0}\right) \quad (*)$$

$$\frac{\partial}{\partial \gamma} : -2 \cdot \sum_{n=1}^{N_M} 10 \log_{10}\left(\frac{d_n}{d_0}\right) \cdot \left(P_{r,n} - K_{dB} + 10\gamma \cdot \log_{10}\left(\frac{d_n}{d_0}\right)\right) \stackrel{!}{=} 0$$

plug in $(*)$: $-2 \sum\limits_{n=1}^{N_M} 10 \log_{10}\left(\frac{d_n}{d_0}\right) \cdot \left(P_{r,n} - \frac{1}{N_M}\sum\limits_{k=1}^{N_M}\left(P_{r,k} + 10\gamma \log_{10}\left(\frac{d_k}{d_0}\right)\right) + \gamma \cdot \log_{10}\left(\frac{d_n}{d_0}\right)\right) \stackrel{!}{=} 0$

$$\Rightarrow \gamma = - \frac{\sum\limits_{n=1}^{N_M} 10 \cdot \log_{10}\left(\frac{d_n}{d_0}\right) \cdot \left(P_{r,n} - \frac{1}{N_M}\sum\limits_{k=1}^{N_M} P_{r,k}\right)}{\sum\limits_{n=1}^{N_M} 10 \cdot \log_{10}\left(\frac{d_n}{d_0}\right) \cdot \left(10 \log_{10}\left(\frac{d_n}{d_0}\right) - \frac{1}{N_M}\sum\limits_{k=1}^{N_M} 10 \log_{10}\left(\frac{d_k}{d_0}\right)\right)} \quad (**)$$

Plugging P_t and the values from Table 2.2 into $(*)$ and $(**)$ yields:

$\gamma = 4.04$, $K_{dB} = -29.7\,dB$

The residuals (errors) of the model equation with these value have a standard deviation of $\underline{\sigma = 3.3\,dB}$

b) Path Loss at $d = 2000$ m:

$$-K_{dB} + 10\gamma \cdot \log_{10}\left(\frac{2000}{1}\right) = 163\,dB$$

c) Problem statement implies that fade margin is 10dB

$$\Rightarrow P_{out} = Q\left(\frac{10dB}{\sigma}\right) = Q(3) \approx 10^{-3}$$

MATLAB for calculations:

```
>> d = [5 25 65 110 400 1000];
Pr = [-60 -80 -105 -115 -135 -150];
mPr = mean(Pr);
>> d_dB = 10*log10(d);
>> md_dB = mean(d_dB);
>> gamma = -sum(d_dB .* (Pr-mPr))/sum(d_dB.*(d_dB-md_dB))
K = mPr + gamma*md_dB

gamma =

    4.0394


K =

  -29.7200

>> sigma = std(Pr - K + gamma*d_dB)

sigma =

    3.3310

>> PL = -K + gamma*10*log10(2000)

PL =

  163.0609

>> Q(3)

ans =

    0.0013
```

4. Problem 2.23

<u>Given:</u> $P_L = -K + 10\gamma \cdot \log_{10}\left(\frac{d}{d_0}\right)$, with $K = 0\,dB$, $\gamma = 3$, $d_0 = 1m$

$P_t = 80mW = 19dBm$

$\Rightarrow \overline{P_r}(R) = 19dBm + 0dB - 10 \cdot 3 \cdot \log_{10}\left(\frac{100m}{1m}\right) = -41dBm$

this is much greater than $P_{min} = -100dBm$

Equation (2.59):

$$a = \frac{P_{min} - \overline{P_r}(R)}{\sigma} = \frac{-100-(-41)}{4} \approx -15$$

$$b = \frac{10 \cdot \gamma \log_{10}(e)}{\sigma} = \frac{30 \cdot \log_{10}(e)}{4} \approx 3.25$$

Cell coverage area: (2.60)

$$C = Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) \cdot Q\left(\frac{2-ab}{b}\right) = 1$$

(this is pretty obvious with nearly 60dB margin at the cell edge)

5. Problem 2.25

- Problem statement implies that $\overline{P_r}(R) = P_{min}$ ($\Rightarrow a = 0$)
- $b = \frac{10 \cdot \gamma \cdot \log_{10}(e)}{\sigma} = \frac{\gamma}{\sigma} \cdot 10 \cdot \log_{10}(e)$ $\quad C = \frac{1}{2} + e^{2/b^2} \cdot Q\left(\frac{2}{b}\right)$

Note that C is monotonically increasing in b. (This may not be obvious; plot it or use $Q(x) \approx \frac{1}{\sqrt{2\pi}x} e^{-x^2/2}$)

$\Rightarrow$ best coverage when b is large ($\gamma = 6, \sigma = 4$) $\Rightarrow C = 0.72$
worst coverage when b is small ($\gamma = 2, \sigma = 12$) $\Rightarrow C = 0.533$