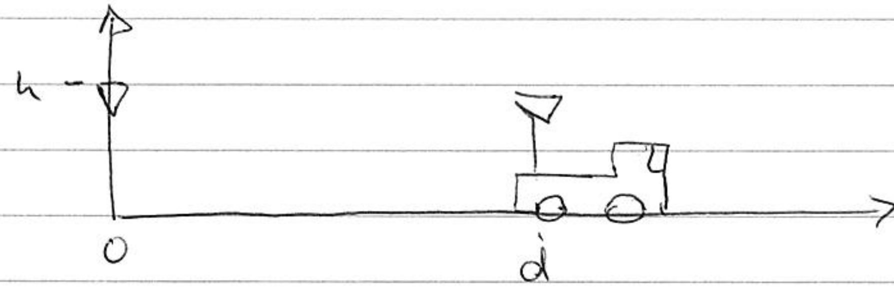
P 3.1



<u>direct path</u>: distance : $d = v \cdot t$

delay : $\tau_1 = \frac{d}{c} = \frac{v \cdot t}{c}$

$\Rightarrow$ phase: $\phi_1 = -2\pi f_c \cdot \tau_1 = -2\pi f_c \cdot \frac{vt}{c}$

$= -2\pi f_d \cdot t$

$f_d = \frac{f_c \cdot v}{c}$ (Doppler shift)

path loss : $L_1 = \left(\frac{\lambda}{4\pi d}\right)^2 = \left(\frac{1}{4\pi f_d \cdot t}\right)^2$

<u>reflected path</u>: distance : $d_R = \sqrt{d^2 + (2h)^2} \approx d + \frac{2h^2}{d} \approx vt + \frac{2h^2}{vt}$

delay : $\tau_R = \frac{d_R}{c} \approx \frac{vt}{c} + \frac{2h^2}{v \cdot t \cdot c}$

$\Rightarrow$ phase:

$\phi_R = -2\pi f_c \cdot \tau_R \approx -2\pi f_d \cdot \tau - 4\pi f_c \frac{h^2}{c v t}$

$= -2\pi f_d \cdot \tau - 4\pi \frac{h^2}{\lambda \cdot vt}$

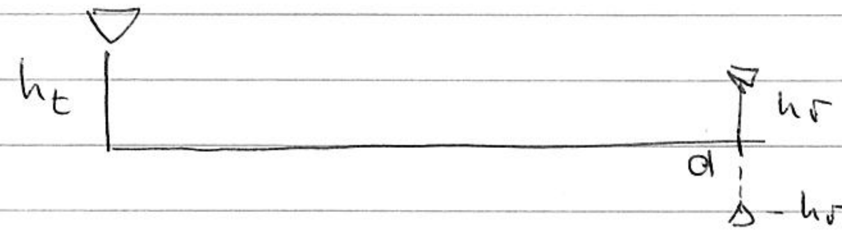
path loss: $L_R = \left(\frac{\lambda}{4\pi d_R}\right)^2 \approx \frac{\lambda}{4\pi} \cdot \left(\frac{1}{d^2 + (2h)^2}\right)$

$$\Rightarrow h(t, \tau) \approx \frac{\lambda}{4\pi f_d \cdot t} \cdot e^{-j2\pi f_d \cdot t} \cdot \delta\left(\tau - \frac{vt}{c}\right) +$$

$$e^{j\pi} \cdot \frac{\lambda}{4\pi \cdot \left(d + \frac{2h^2}{d}\right)} \cdot e^{-j2\pi\left(f_d \cdot t + \frac{2h^2}{\lambda \cdot vt}\right)} \cdot \delta\left(\tau - vt - \frac{2h^2}{vt}\right)$$

reflection coefficient ≈ -1 $\rightarrow e^{j\pi}$

P3.2



direct path delay: $\frac{1}{c}\sqrt{d^2+(h_t-h_r)^2}$

reflected path delay: $\frac{1}{c}\sqrt{d^2+(h_t+h_r)^2}$

$\Rightarrow$ delay spread: $T_m = \frac{1}{c}\left(\sqrt{d^2+(h_t+h_r)^2} - \sqrt{d^2+(h_t-h_r)^2}\right)$

$$= \frac{d}{c}\cdot\left(\sqrt{1+\frac{(h_t+h_r)^2}{d^2}} - \sqrt{1+\frac{(h_t-h_r)^2}{d^2}}\right)$$

with:
$\sqrt{1+x^2} \approx 1+\frac{1}{2}x^2$ for small x : $T_m \approx \frac{d}{c}\cdot\left(1+\frac{1}{2}\frac{(h_t+h_r)^2}{d^2} - \left(1+\frac{1}{2}\frac{(h_t-h_r)^2}{d^2}\right)\right)$

$$= \frac{d}{c}\cdot\left(1+\frac{1}{2}\frac{h_t^2+2h_th_r+h_r^2}{d^2} - \left(1-\frac{1}{2}\frac{h_t^2-2h_th_r+}{d^2}\right)\right)$$

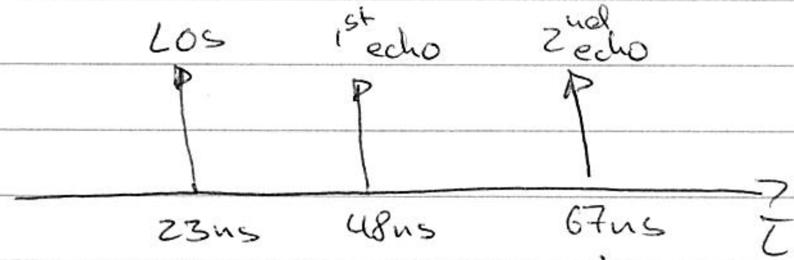
$$= \frac{d}{c}\cdot\left(\cancel{1+}\ \frac{2\cdot h_t\cdot h_r}{d^2}\right)$$

$$= \frac{2\cdot h_t\cdot h_r}{c\cdot d}$$

For $d = 100\,m$, $h_t = 10\,m$, $h_r = 4\,m$

$$T_m \approx \frac{2\cdot 10\cdot 4}{3\cdot 10^8\cdot 100}\,s = \frac{8}{3}\cdot 10^{-9}\,s \approx 3\,ns$$

P 3.3



delay spread: (67-23) ns = 44 ns

regardless of which arrival we sync to.