ECE 630: Statistical Communication Theory
Prof. B.-P. Paris
Homework 3
Due: February 19, 2019

Reading Madhow: Section 3.3.

Problems

1. Let $X_t(\omega)$ be a random process defined on $\Omega = \{\omega_1, \ldots, \omega_4\}$ having probability assignments $\Pr\{\omega_i\} = \frac{1}{4}$ for $i = 1, 2, 3, 4$. The sample functions are

$$\begin{array}{ll} X_t(\omega_1) = t & X_t(\omega_2) = -t \\ X_t(\omega_3) = \cos 2\pi t & X_t(\omega_4) = -\cos 2\pi t \end{array}$$

   (a) Compute the joint probability $\Pr\{X_0(\omega) = 1, X_1(\omega) = 1\}$.
   (b) Compute the conditional probability $\Pr\{X_1(\omega) = 1 | X_0(\omega) = 0\}$.
   (c) Compute the mean and correlation function of $X_t(\omega)$.

2. Prove the following properties of a random process:
   (a) $R_X(t, t) \geq 0$
   (b) $R_X(t, u) = R_X(u, t)$ (symmetry)
   (c) $|R_X(t, u)| \leq \frac{1}{2}(R_X(t, t) + R_X(u, u))$
   (d) $|R_X(t, u)|^2 \leq R_X(t, t) \cdot R_X(u, u)$

3. A random process is defined by

$$X_t = \cos 2\pi F t$$

   where the frequency F is uniformly distributed over the interval $[0, f_0]$.

   (a) Find the mean and correlation function of X_t.
   (b) Show that this process is non-stationary.

   Now suppose we redefine the process X_t to be

$$X_t = \cos(2\pi F t + \Theta)$$

   where F and Θ are statistically independent random variables. Θ is uniformly disributed over $[-\pi, \pi)$ and F is distributed as before.

   (c) Compute the mean and correlation function of X_t.
   (d) Is X_t wide-sense stationary? Show your reasoning.
   (e) Find the first order density $p_{X_t}(x)$.