Estimating Parameter of a Sinusoid from Samples

AN EXAMPLE SOLUTION

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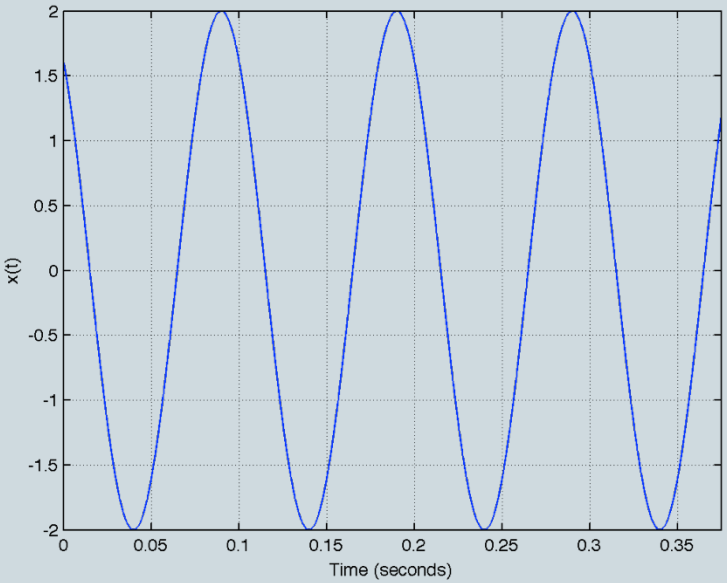
Problem Statement

- Given:
  - A set of samples from a sinusoidal signal, e.g.,
    ```
    sig = [1.6180,
    1.6164, 1.6147, …];
    ```
  - A sampling frequency f_s, e.g.,
    ```
    fs = 44100;
    ```
- Objective:
  - Determine signal's amplitude, frequency, and phase.

Plot of a sinusoidal signal

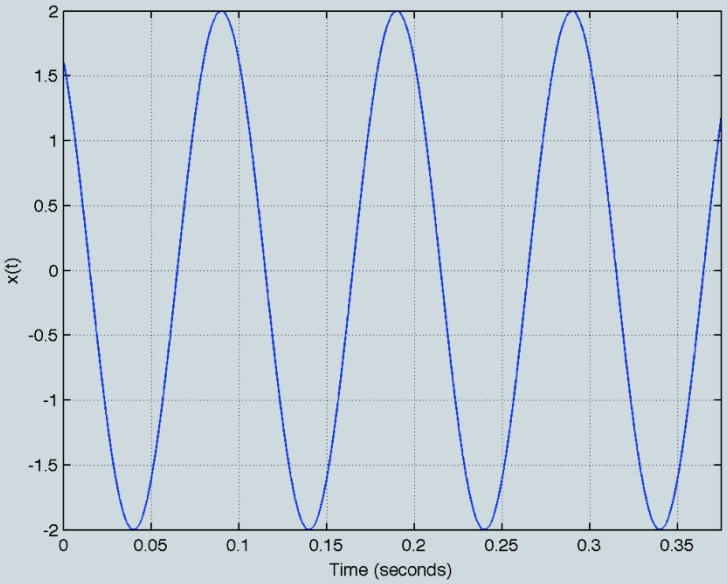


$$x(t) = A\cos(2\pi ft + \phi)$$

Amplitude Estimation

- For amplitude estimation, the overall maximum value of a samples is used.
- In MATLAB, this is accomplished via the `max` function:

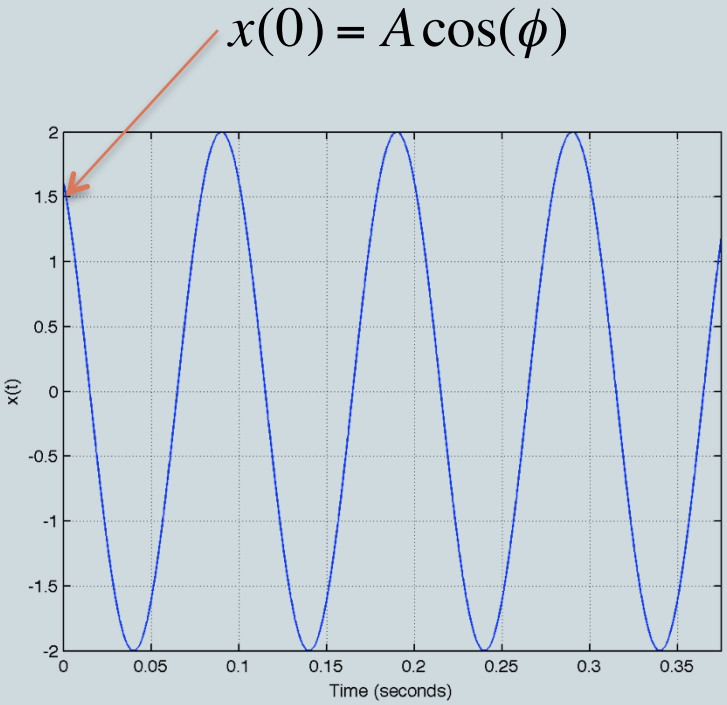
```
amp = max(sig);
```



The maxima of a sinusoid equal the amplitude of a sinusoid.

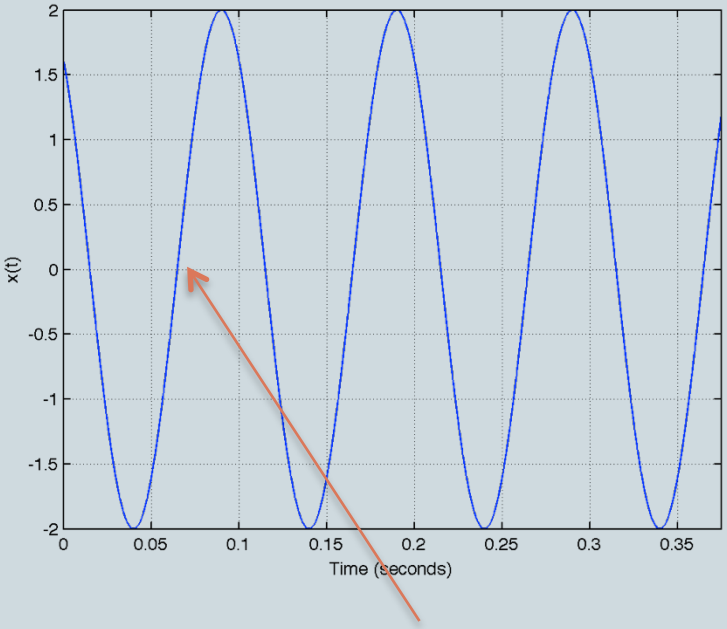
Phase Estimation

- The phase can be determined from the first sample.
  - First sample, `sig(1)`, corresponds to *x(0)*.
  - We have $x(0)=A\cos(\phi)$.
  - MATLAB:
    ```
    phi = acos(sig(1)/amp);
    ```
    - This is always positive!
- Phase `phi` may be negative.
  - Occurs when signal is initially increasing.
  - MATLAB:
    ```
    if sig(2) > sig(1)
       phi = -phi;
    end
    ```



Frequency Estimation

- Frequency estimation is based on the slope of a sinusoid at a zero crossing.
- First step: find first zero crossing.
  - Using MATLAB's `find` function.
- Second step: determine the slope near the zero-crossing.
  - Compute "rise-over-run"
- Solve for frequency.



The slope at a zero-crossing equals: $2A\pi f$

Summary and Conclusions

- Presented methods for estimating amplitude, frequency, and phase from samples of a signal.
- Estimates are reasonable as they are derived directly from mathematical properties of sinusoidal signals.
- Room for Improvement:
  - Estimates should use all samples – phase and frequency estimates are based on very, few samples.
    - This will likely lead to poor results if sinusoids are not perfect (e.g., corrupted by noise).
  - Phase estimate relies on amplitude estimate – if amplitude is wrong, then phase will be wrong.