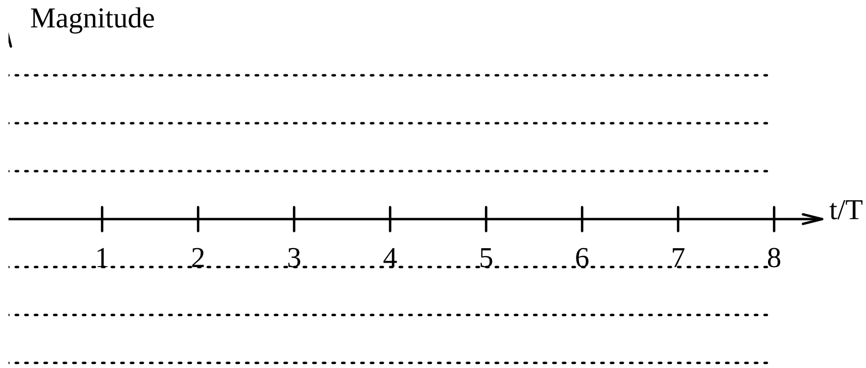
1. **Modulation** The symbol sequence $b = \{-1, 1, 1, -1, 1, 1, -1, -1\}$ is to be transmitted at a rate of one symbol per T seconds. For each of the following modulation formats sketch in the space provided below

   - the magnitude,
   - the phase, and
   - the modulated signal assuming a carrier frequency of $1/T$.

   (a) Binary Phase Shift Keying (BPSK) with rectangular pulses.


Magnitude
t/T
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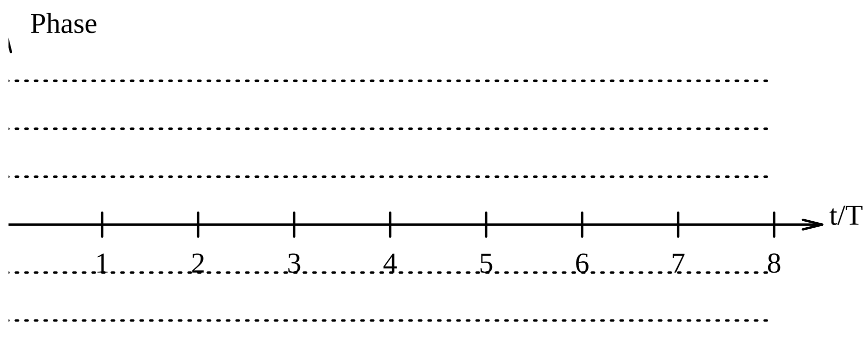
Phase
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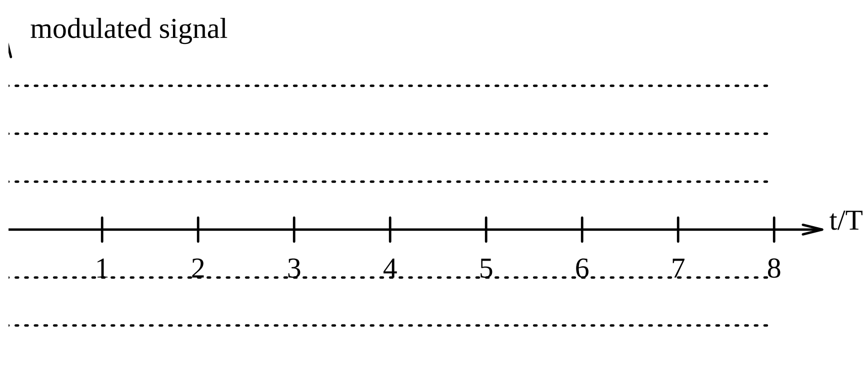
modulated signal
t/T
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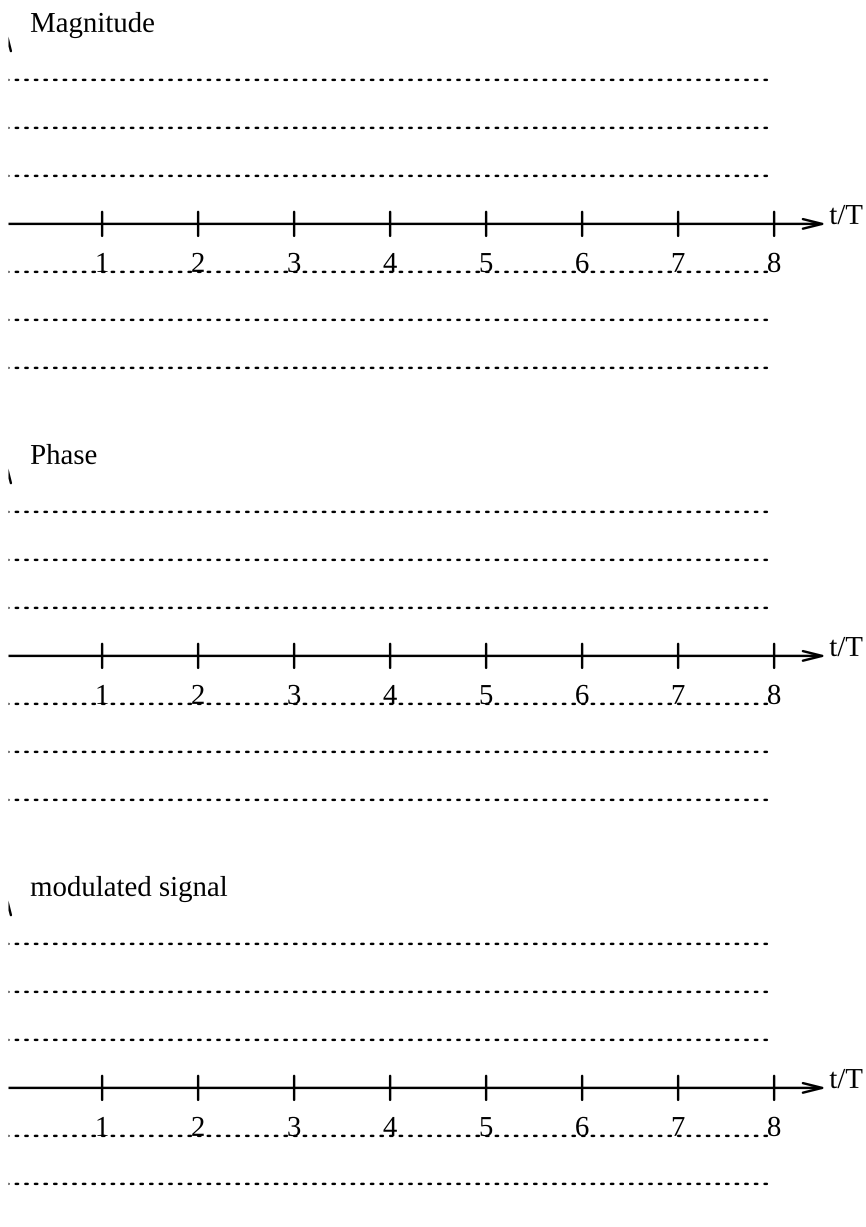
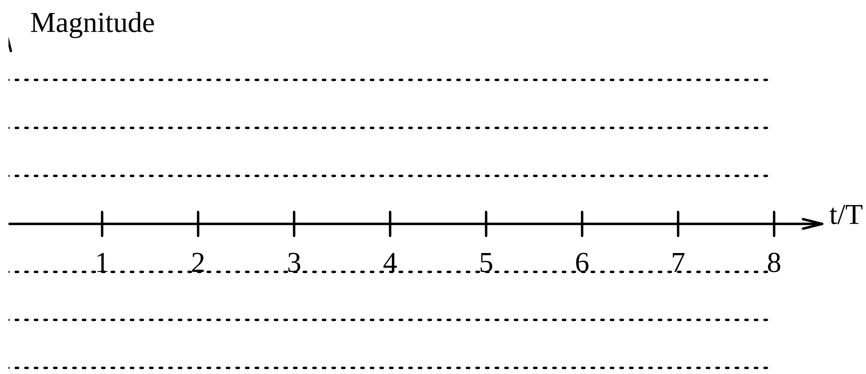
(b) Quaternary Phase Shift Keying (QPSK) with triangular pulses.


Magnitude
t/T
1 2 3 4 5 6 7 8
Phase
t/T
1 2 3 4 5 6 7 8
modulated signal
t/T
1 2 3 4 5 6 7 8

(c) Full-response Continuous Phase Modulation (CPM) with modulation index $h = 1/2$ and rectangular pulses.


Magnitude
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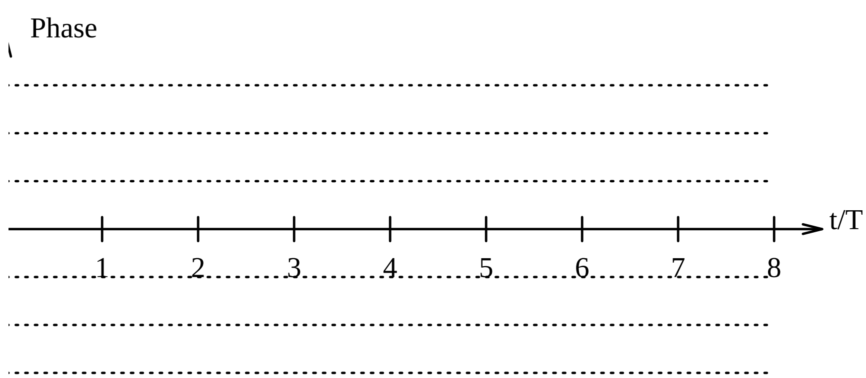
Phase
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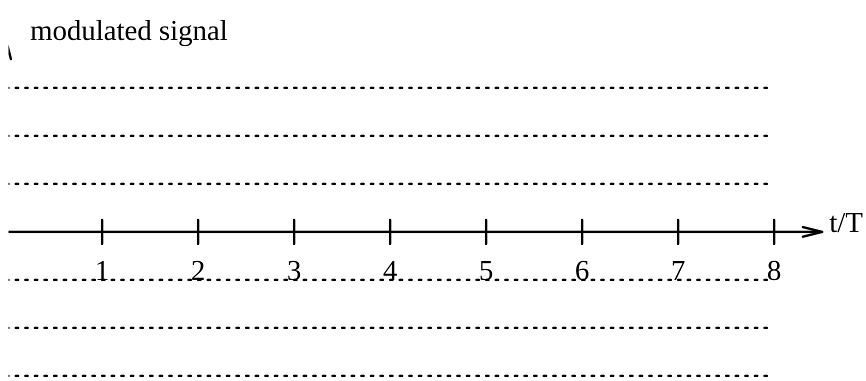
modulated signal
t/T
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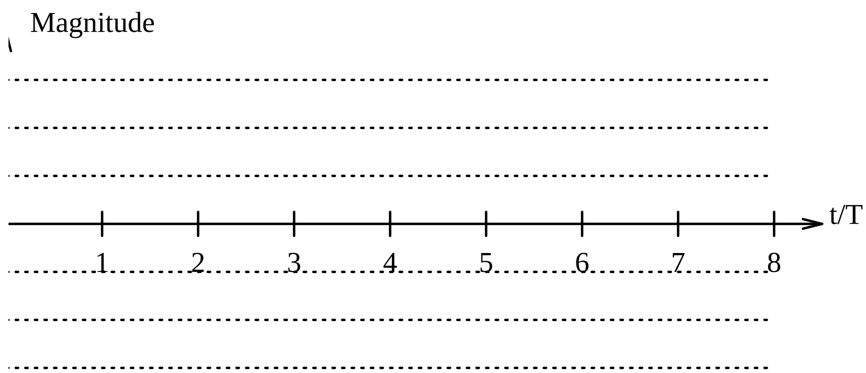
(d) Partial-response ($L = 2$) Continuous Phase Modulation (CPM) with modulation index $h = 1/2$ and rectangular pulses.


Magnitude
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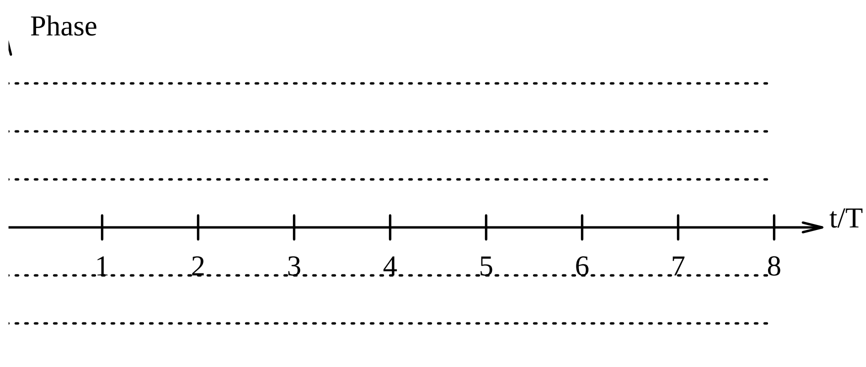
Phase
t/T
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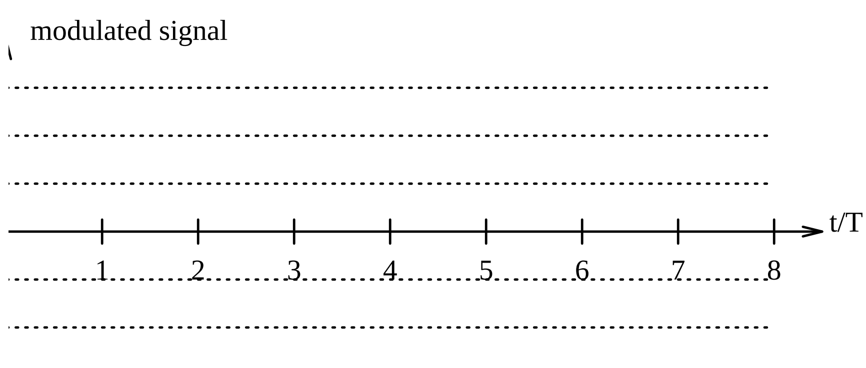
modulated signal
t/T
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(e) Comment on the bandwidth requirements for each of these modulation formats. Which modulation format has the narrowest (widest) main-lobe? Which modulation format provides the fastest (slowest) side-lobe decay?

2. **Multi-Path Channel and Tapped Delay-Line Model**

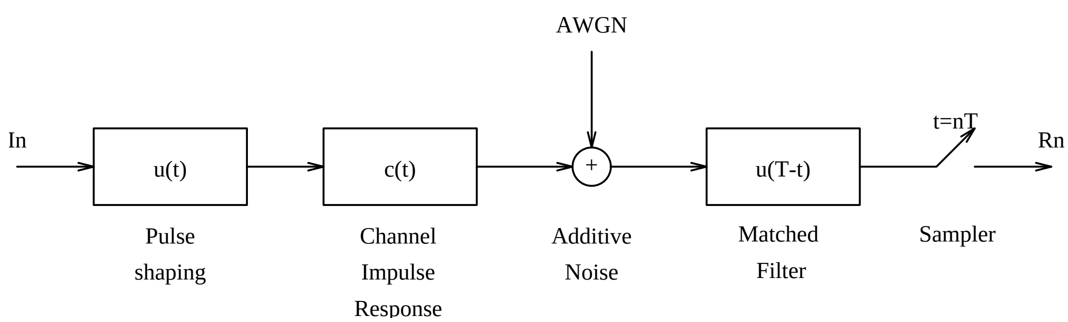
A mobile communication system employs binary phase shift keying with rectangular pulses

$$u(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq T \\ 0 & \text{else} \end{cases}$$

to transmit information at a rate of one bit per T seconds over a channel with (time-invariant) baseband-equivalent impulse response

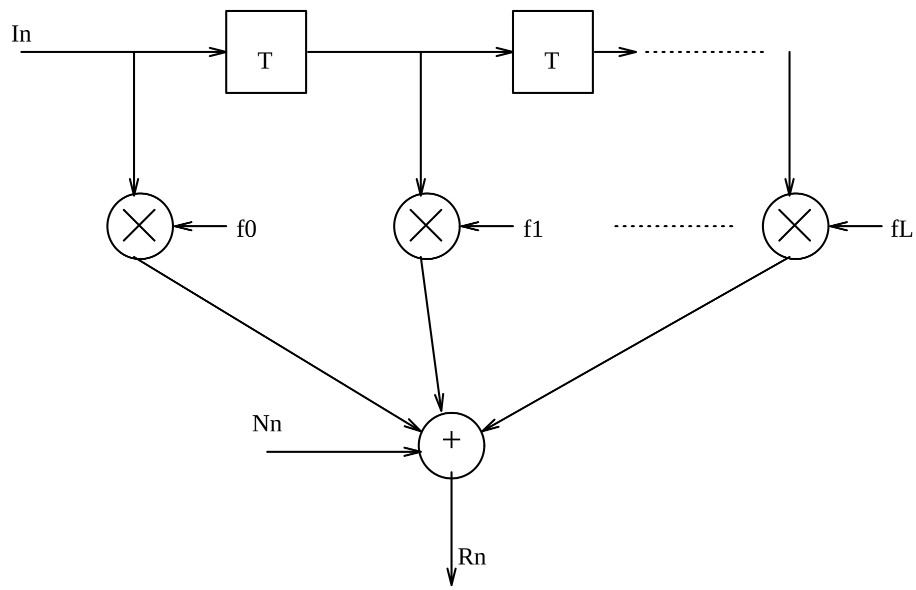
$$c(t) = \begin{cases} A & \text{for } 0 \leq t \leq 2T \\ 0 & \text{else.} \end{cases}$$

Furthermore, the channel adds white Gaussian noise N_t of spectral height $\frac{N_0}{2}$. The receiver is designed under the assumption that $c(t) = \delta(t)$ and hence is simply a filter matched to the transmitted pulses $u(t)$. In summary, the communication system can be described by the following block diagram.



The objective of this problem is to determine the parameters of the equivalent tapped delay-line model between the input sequence I_n and the output sequence R_n. A generic block diagram for a tapped delay-line model is given below.

(a) Compute the convolution of the pulse shaping filter $u(t)$ with the channel impulse response $c(t)$.



(b) How many memory elements L are there in the tapped delay-line?

(c) What are values for the coefficients in the tapped delay-line model?

(d) Find the distribution (type, mean, variance, etc.) of the additive noise N_n samples in the tapped delay line model.

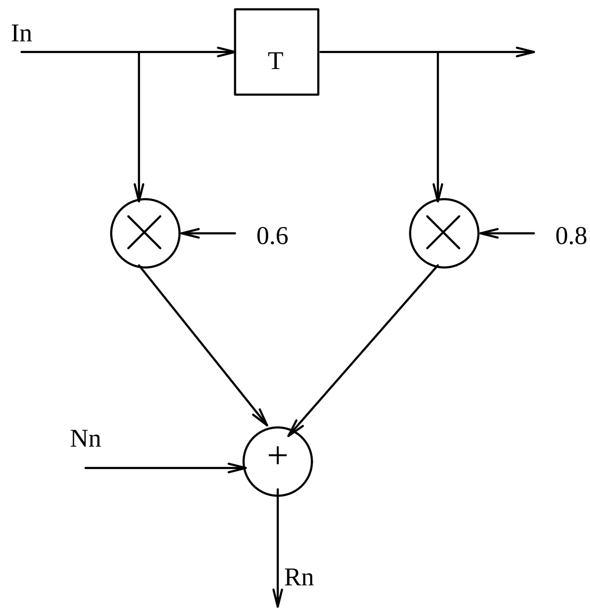
3. **Maximum Likelihood Sequence Estimation** A binary sequence of five symbols $I_n, n = 1, \ldots, 5$ (elements are drawn from$I_n \in \{-1, 1\}$) is transmitted over a channel which is characterized by tapped delay-line with coefficients $\underline{f} = \{\frac{3}{5}, \frac{4}{5}\}$. The observation is further corrupted by additive white Gaussian noise. The following sequence R_n is observed at the output of the tapped delay line

$$R_n = \{1, \frac{3}{2}, \frac{1}{4}, -\frac{5}{4}, -\frac{1}{2}, \frac{3}{2}\}$$

(a) Given the observed sequence R_n, determine the most likely input sequence I_n. Clearly, show how you arrived at your solution.

(b) Draw and clearly label a trellis diagram and indicate the path through the trellis which corresponds to the most likely sequence.

(c) For the remainder of the problem the coefficients $\underline{f}$ of the tapped delay-line are unknown. However it is known that the first five input symbols



are

$$I_n = \{1, -1, 1, -1, -1\}.$$

In total, the input sequence is eight symbols long and the observed output sequence is

$$R_n = \{1, 0.1, 0.3, 0, -1.4, 0, 1.8, -0.4, -0.4\}$$

Find the best estimate for the channel coefficients based on the knowledge of the first five symbols in the sequence.

(d) Use your estimate for the channel coefficients to determine the most likely sequence of the remaining three symbols.

4. **Continuous Phase Modulation**
Equally likely random symbols $I_n \in \{-1, 1\}$ are to be transmitted at a rate of one symbol per T seconds over an additive white Gaussian noise channel. The symbols are modulated using partial response ($L = 2$) continuous phase modulation with modulation index $h = \frac{1}{2}$ and rectangular pulses.

(a) Sketch the phase of the transmitted signal resulting from the sequence

$$\underline{I} = \{1, -1, 1, 1 - 1, -1, -1, 1\}.$$

(b) Show that for an arbitrary input sequence $\underline{I}$ the phase can be expressed as

$$\phi(t, \underline{I}) = \theta_n + 2\pi h \sum_{k=0}^{L-1} I_{n-k} q(t-(n-k)T) \quad \text{for } nT \leq t < (n+1)T.$$

Provide an explicit expression for θ_n.

(c) Which values can θ_n take on?

(d) Phase transitions between times nT and $(n+1)T$ depend only on θ_n, I_n, and I_{n-1}. Hence, the evolution of the phase $\phi(t, \underline{I})$ can be described as a path through a suitably chosen trellis. How would you define the states of the trellis?

(e) Draw a trellis diagram showing all possible state transitions between times nT and $(n+1)T$.

(f) To employ the Viterbi algorithm for demodulating CPM signals the metrics (distances) associated with the state transitions are required. Define these distances precisely and explain how they can be obtained.

5. **Time-Varying Channels**
Assume the time-varying impulse response of a channel is given by

$$h(t, \tau) = \delta(t) + \alpha(\tau) \cdot \delta(t - T_c).$$

More precisely, $h(t, \tau)$ is the response of the channel to an impulse applied at time τ. The time-varying amplitude of the delayed component is given by

$$\alpha(\tau) = A \cdot \cos(2\pi f_d \tau).$$

(a) Find the response of the channel if an impulse is applied at time

i. $\tau = 0$

ii. $\tau = \frac{1}{4f_d}$

iii. $\tau = \frac{1}{2f_d}$

(b) Find the response of the channel to a constant signal, $s(t) = 1$ for all t. I.e., compute

$$s(t) * h(t, \tau) = \int s(\tau) h(t - \tau, \tau) d\tau.$$

(c) Sketch the resulting signal.

(d) A packet of N bits is to be transmitted over the channel using BPSK modulation. The bit period is T_b seconds. For each of the following cases explain *qualitatively* the influence of the channel on the transmitted signal:

i. $T_b \gg T_c$ and $NT_b \ll \frac{1}{f_d}$

ii. $T_b \gg T_c$ and $NT_b \approx \frac{1}{f_d}$

iii. $T_b \approx T_c$ and $NT_b \ll \frac{1}{f_d}$

(e) For each of the three cases, explain which provisions must be made in the receiver to ensure reliable communication.

6. **Control Channels in Mobile Communication Systems**
To ensure proper functioning of mobile communication systems the following transactions must be supported by the employed protocol:

- (mobile originating/terminating) Call establishment and release
- Handover
- Periodic reporting of signal quality and strength measurements
- Location updating

(a) Explain under what conditions each of these transactions has to be performed. Estimate how many control messages are necessary in both the up-link and down-link direction to complete each transaction. Justify your answers.

(b) For the remainder of the problem, the following assumptions regarding the traffic to be supported by the mobile network are made:

- There is an equal number of users from two classes: the first moves at a speed of 50 km/h and the second is moving at a speed of 3 km/h.
- Each cell has 10 traffic channels which are in continuous use[1].
- The average call duration is 2 minutes.
- A location area comprises 10 cells.
- Measurements are reported every 10 seconds for every active call.
- The cell radius is 1 km for all cells.

[1] This is a pessimistic assumption but it avoids the use of queuing theoretic methods in this problem.

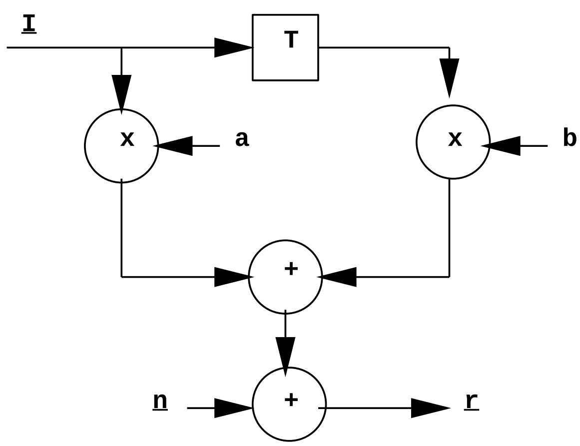
Estimate the total number of control messages which have to be exchanged in support of the transactions above in one hour in a location area (10 cells).

(c) Assuming each message contains 100 bits, how much channel capacity is required to carry the control traffic for the location area.

(d) How do your answers change if the cell radius were reduced to 100 m? Would a measurement reporting interval of 10 seconds still be sufficient? Explain.

7. **Maximum Likelihood Sequence Estimation** A binary sequence of five symbols $\underline{I}$ (elements are drawn from$I_n \in \{-1, 1\}$) is transmitted over a channel which is characterized by tapped delay-line with coefficients $a = 5$ and $b = -4$. The observation is further corrupted by additive white Gaussian noise. The following sequence $\underline{r}$ is observed at the output of the tapped delay line

$$\underline{r} = \{7, -1, -4, -5, 7, -2\}$$



(a) Given the observed sequence $\underline{r}$, determine the most likely input sequence $\underline{I}$. Clearly, show how you arrived at your solution.

(b) Draw and clearly label a trellis diagram and indicate the path through the trellis which corresponds to the most likely sequence.

(c) What is the Euclidean distance associated with the two sequences $\underline{I}_1 = \{1, 1, -1, -1, 1\}$ and $\underline{I}_2 = \{1, 1, 1, -1, 1\}$, respectively? Explain.

(d) For the remainder of the problem the coefficients of the tapped delay-line are time-varying. In the first, third, and fifth symbol period the channel coefficients are $a = 5$ and $b = -4$, in the second, fourth, and sixth symbol period the channel coefficients are $a = 4$ and $b = -5$.

Explain how the Viterbi Algorithm must be modified to account for the time-varying coefficients.

(e) The observed sequence is $\underline{r} = \{6, -8, 8, 0, -8, 4\}$. Find the most likely input sequence $\underline{I}$.

8. **Minimum-Shift Keying (MSK)**
As discussed in class, MSK is full response, continuous phase modulation with modulation index $h = \frac{1}{2}$, using rectangular pulse-shaping.

(a) Draw and accurately label a block diagram for a MSK transmitter which indicates clearly how the transmitted signal is generated from the sequence of input bits.

(b) Sketch the baseband equivalent (magnitude and phase) MSK signal for the input sequence $\{1, -1, 1, 1, -1, -1\}$.

(c) Draw a block diagram of the receiver which minimizes the probability of error if a coherent phase reference is available. Compute the probability of error achievable with this receiver.

(d) Is it possible to use a non-coherent receiver with MSK, i.e., a receiver which does not require knowledge of the phase of the received signal? If your answer is no, explain why not. If your answer is yes, draw a block diagram of the non-coherent detector.

(e) Gaussian Minimum Shift Keying (GMSK) employs a Gaussian pulse for filtering the input sequence. Draw a block diagram of the transmitter for GMSK which shows clearly where the Gaussian filter is located.

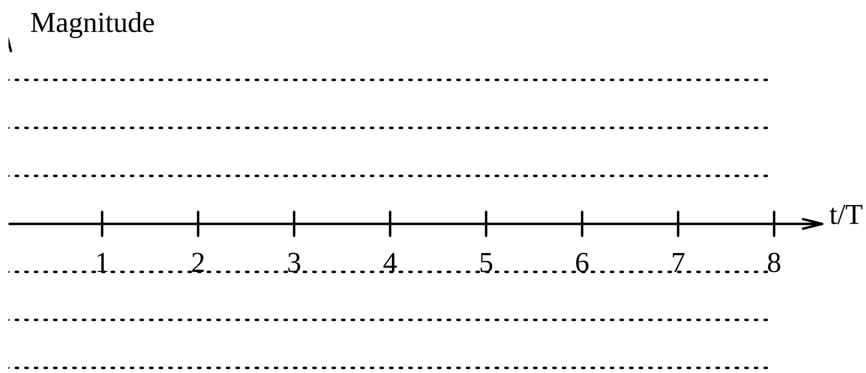
(f) Explain the influence of this filter on system parameters such as bandwidth and error rate and discuss how the receiver must be modified to reflect the influence of the Gaussian filter.

9. **Doppler Shift**
What is the maximum Doppler shift for the GSM mobile cellular system on the “down-link” from the base station to the mobile (935 to 960 MHz)? What is it on the “up-link,” or mobile to base direction (890 to 915 MHz)? Assume that the mobile is driven at a speed of 180km/h on the *autobahn*. Comment on the validity of the assumption that the channel is constant during the transmission of a burst (approximate duration 0.5 ms).

10. **Modulation**
The symbol sequence $b = \{-1, 1, 1, -1, 1, 1, -1, -1\}$ is to be transmitted at a rate of one symbol per T seconds. For each of the following modulation formats sketch in the space provided below

- the magnitude,
- the phase, and
- the modulated signal assuming a carrier frequency of $1/T$.

(a) Binary Phase Shift Keying (BPSK) with raised cosine pulses.


Magnitude
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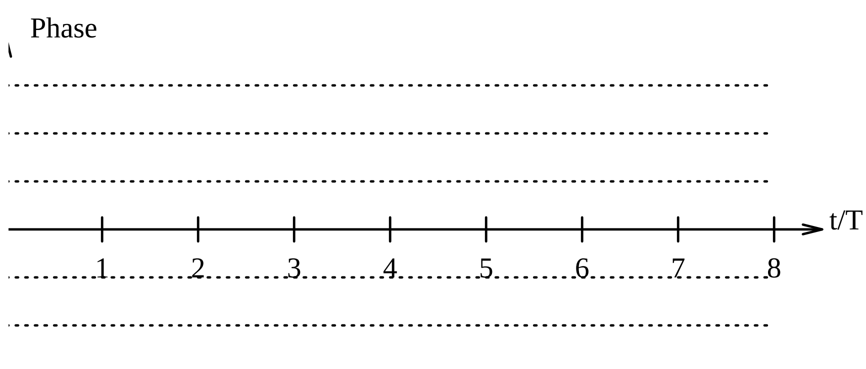
Phase
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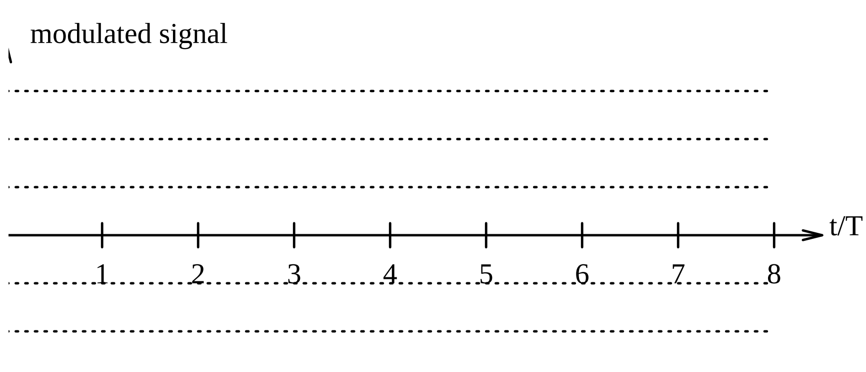
modulated signal
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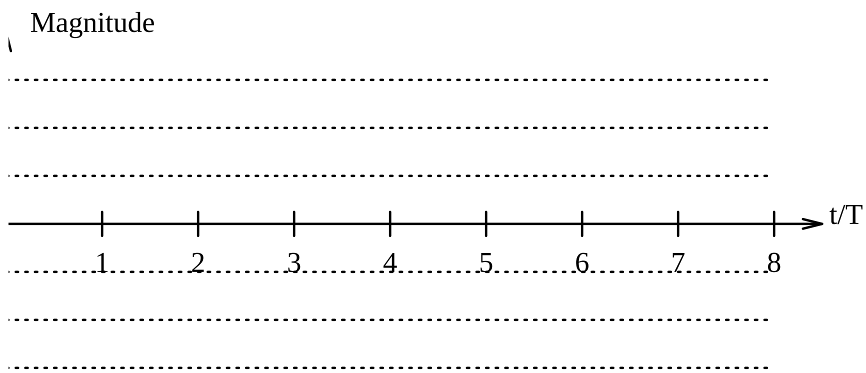
(b) Full-response Continuous Phase Modulation (CPM) with modulation index $h = 1$ and rectangular pulses.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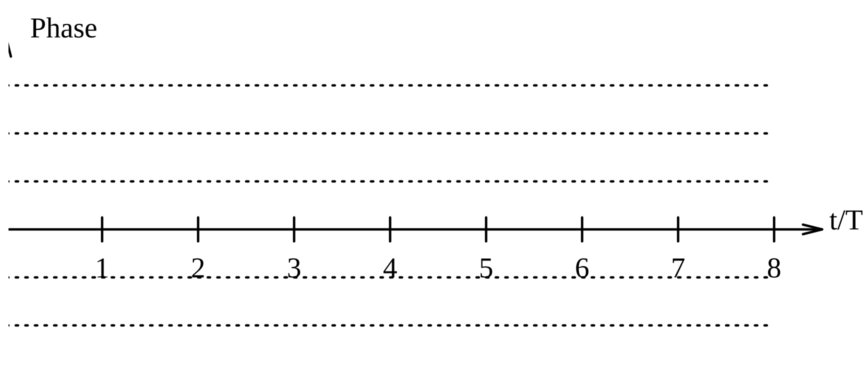
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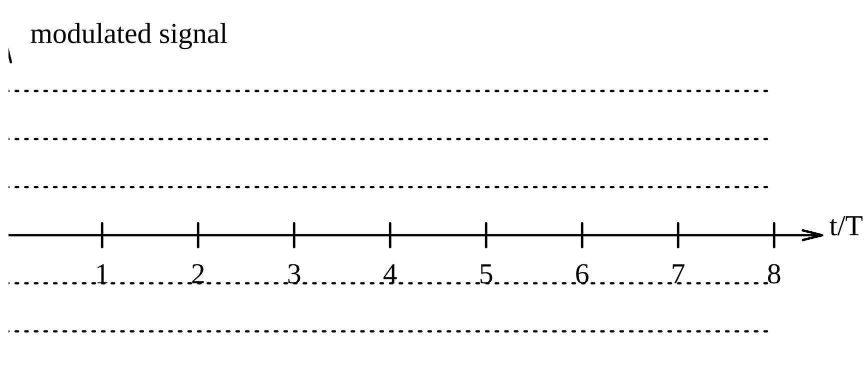
modulated signal
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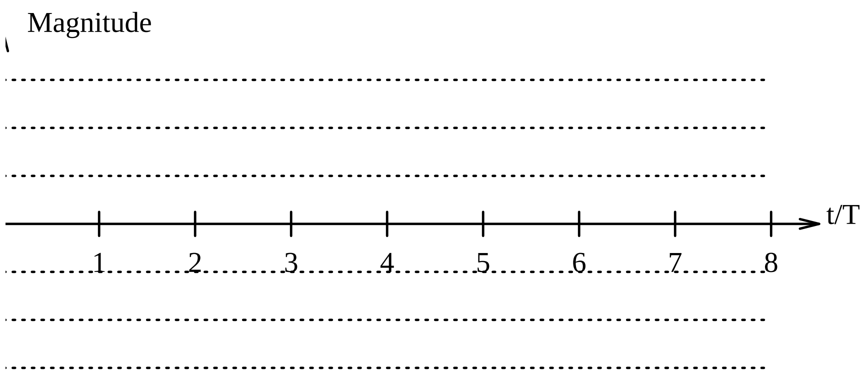
(c) Partial-response ($L = 2$) Continuous Phase Modulation (CPM) with modulation index $h = 1$ and rectangular pulses.


Magnitude
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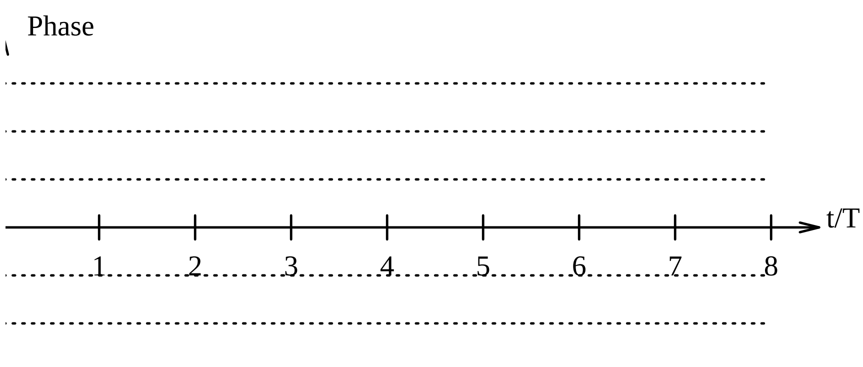
Phase
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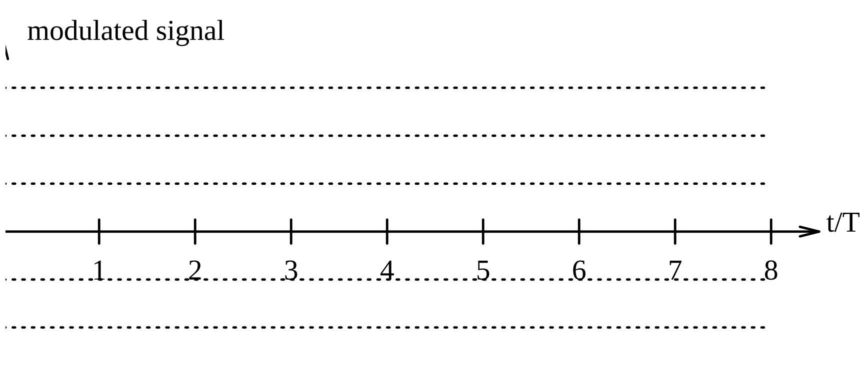
modulated signal
t/T
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(d) Comment on the bandwidth requirements for each of these modulation formats. Which modulation format has the narrowest (widest) main-lobe? Which modulation format provides the fastest (slowest) side-lobe decay?

11. **Maximum-Likelihood Sequence Estimation for Convolutional Codes**

Consider a rate $\frac{1}{2}$ convolutional coder with constraint length $K = 3$ and generator polynomials in octal representation $g_1 = 5$ and $g_2 = 7$.

(a) Draw a block diagram of this encoder and label it accurately.

(b) Determine the output from the encoder if the input sequence is $I = \{1, 0, 0, 1, 1\}$. Assume that the register is initially loaded with 0-bits and that 0-bits are transmitted after the sequence as stop bits .

(c) Assume now that the sequence $V = \{1, 1, 0, 1, 1, 0, 0, 1, 1, 1\}$ is received. Again 0-start and stop bits have been added to the sequence to force the encoder into the all 0-state at the beginning and end of the transmission. Thus, there are three information bits between the start and stop bits.

Draw a trellis diagram, showing clearly the possible states of the encoder, the possible transitions and the encoder outputs associated with the transitions.

(d) Determine the most likely input sequence.

12. **Coding and Interleaving**

Consider a TDMA system that transmits information in frames of 8 bits. The probability that a frame is subject to fading is $P_f = 0.1$. If a frame is subject to fading the probability that a bit is in error is $P_{e|f} = 0.5$. Otherwise if there is no fading the bit error probability is $P_{e|n} = 0$.

(a) What is the probability that a frame is received that does not contain any errors?

(b) Assume now that the information has been coded such that one bit error per frame can be corrected. No interleaving is employed. Compute the probability that a decoded frame does not contain any bit errors.

(c) Assume now that bits are interleaved over 8 frames such that each deinterleaved frame contains exactly one bit from each of the transmitted, interleaved frames. What is the probability of a bit error if no coding is used?

(d) Compute the probability that a frame does not contain any errors if both coding and interleaving are employed.

13. **Power Control in CDMA Systems**

    As discussed in class, power control is used in CDMA systems to limit the detrimental effects of multiple-access interference. It aims to ensure that signals transmitted by the mobiles arrive at the base station at the same power level. This problem addresses some of the network management aspects associated with power control.

    (a) Approximately how long does the received power level at the base station in a cellular communication network remain constant? On which parameters does your answer depend? What is a typical value for the time the received power remains constant?

    (b) What does your answer imply regarding the frequency with which power control commands have to be issued from the base station to a mobile?

    (c) Assume that the received power level can vary as much as 80 dB and that the received power level has to be controlled within 1 dB. How many bits are required at the minimum in a power control message?

    (d) Assume that power control commands are issued every 3 ms and that a power control message contains 10 bits. What is the required bit rate for power control message between the base station and a mobile? What is the total bit rate for power control messages if a base stations communicates with 100 mobile stations?

    (e) Explain why power control in the reverse directions, from the base station to the mobile, is much less of a concern.

14. **Mobile Communication Channels**
    Assume the impulse response of a channel is given by

$$h(t, \tau) = \delta(t) + \alpha(\tau) \cdot \delta(t - T_c).$$

I.e., the response of the channel to a signal $s(t)$ is given by

$$s(t) * h(t, \tau) = \int s(\tau) h(t - \tau, \tau) d\tau.$$

The amplitude of the delayed component is given by

$$\alpha(\tau) = A \cdot \cos(2\pi f_d \tau).$$

(a) Find the response of the channel if the following impulses are applied as the input to the channel

i. $\delta(t)$

ii. $\delta(t - \frac{1}{4f_d})$

iii. $\delta(t - \frac{1}{2f_d})$

Is the channel time-varying or time-invariant.

(b) Find the response $r(t)$ of the channel to a constant signal, $s(t) = 1$ for all t. Then, sketch the resulting signal $r(t)$.

(c) Compute the Fourier transform $R(f)$ of the signal $r(t)$ and explain what conclusions you can draw from $R(f)$.

(d) Estimate the coherence-time and the coherence-bandwidth of the channel in terms of the parameters T_c and f_d.

(e) The above channel is used to transmit digitally modulated data at a rate $\frac{1}{T_b}$. Data are transmitted in packets of N symbols. For each of the following cases indicate if the channel is frequency selective or non-selective and fast or slow fading.

i. $T_b \gg T_c$ and $NT_b \ll \frac{1}{f_d}$

ii. $T_b \gg T_c$ and $NT_b \approx \frac{1}{f_d}$

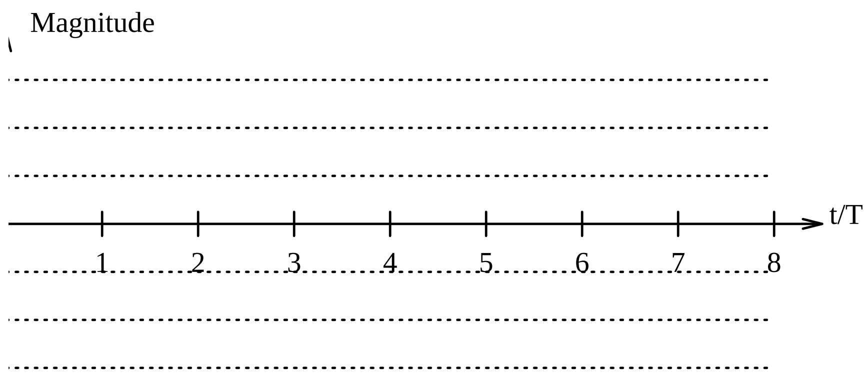
iii. $T_b \approx T_c$ and $NT_b \ll \frac{1}{f_d}$

(f) For each of the three cases, explain which provisions must be made to ensure reliable communication.

15. **Modulation** The symbol sequence $b = \{1, 1, 1, -1, -1, 1, -1, -1\}$ is to be transmitted at a rate of one bit per T seconds. For each of the following modulation formats sketch in the space provided below

- the magnitude,
- the phase, and
- the modulated signal assuming a carrier frequency of $1/T$.

(a) Binary Phase Shift Keying (BPSK) with triangular pulses.


Magnitude
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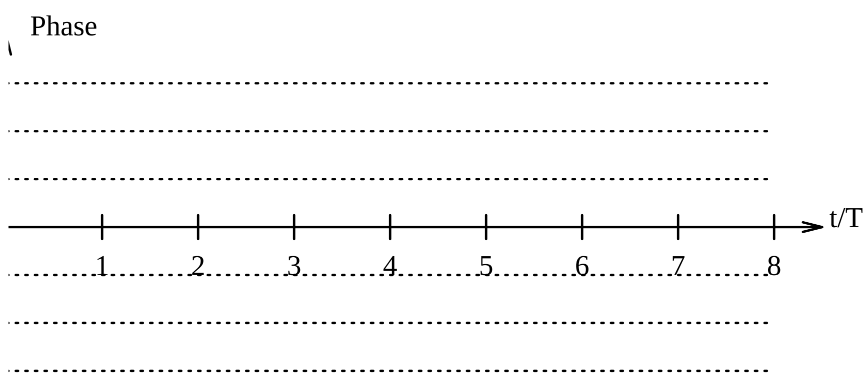
Phase
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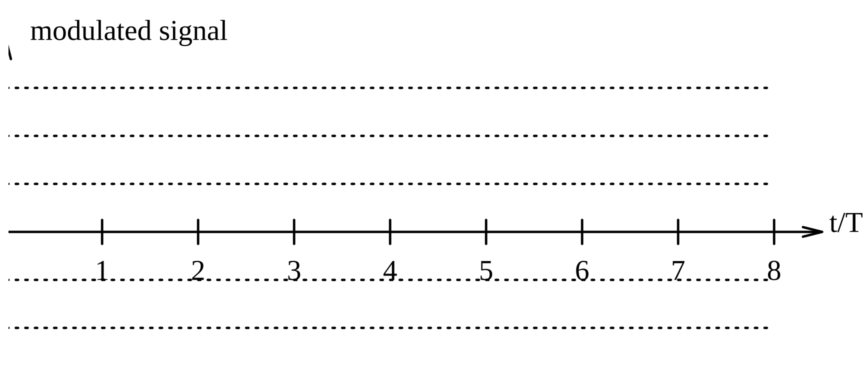
modulated signal
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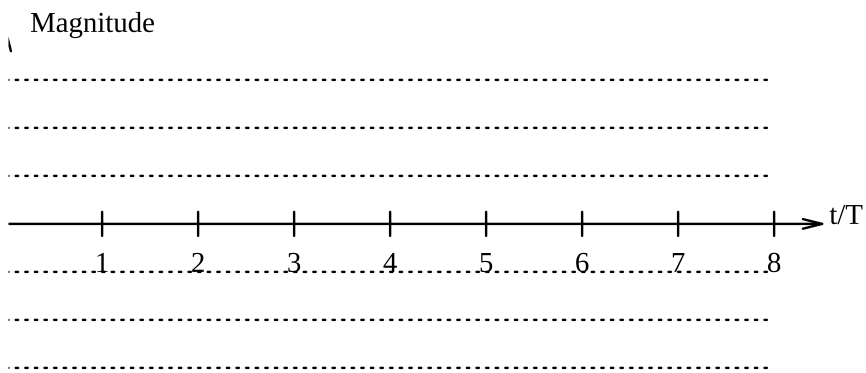
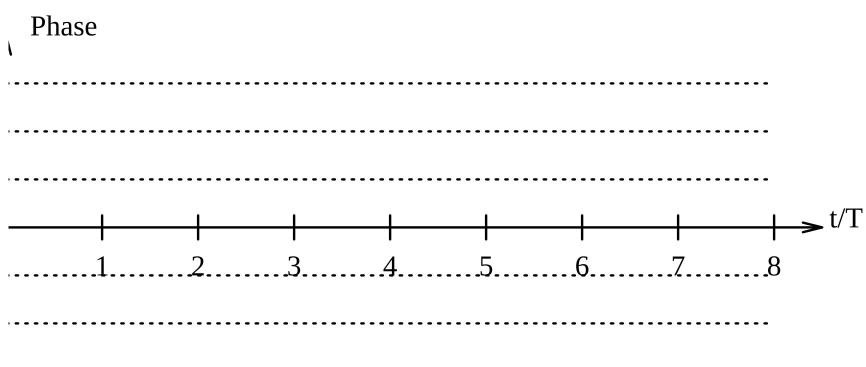
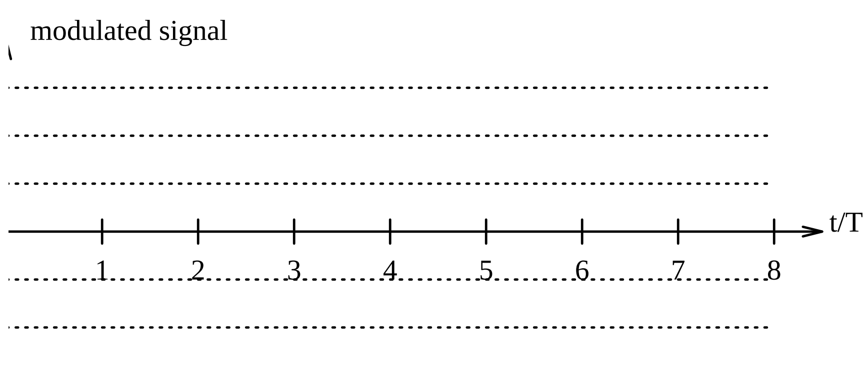
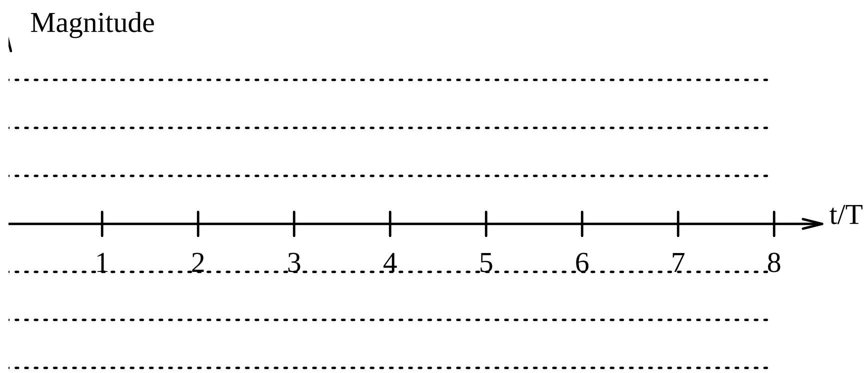
(b) Offset-Quaternary Phase Shift Keying (OQPSK) with rectangular pulses.

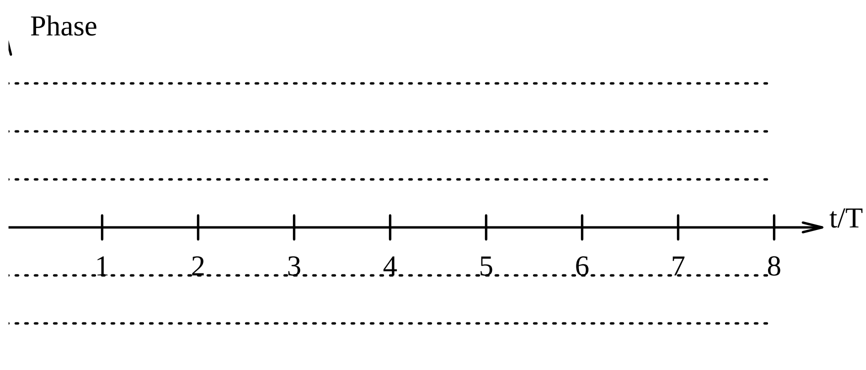

Magnitude
t/T
1
2
3
4
5
6
7
8

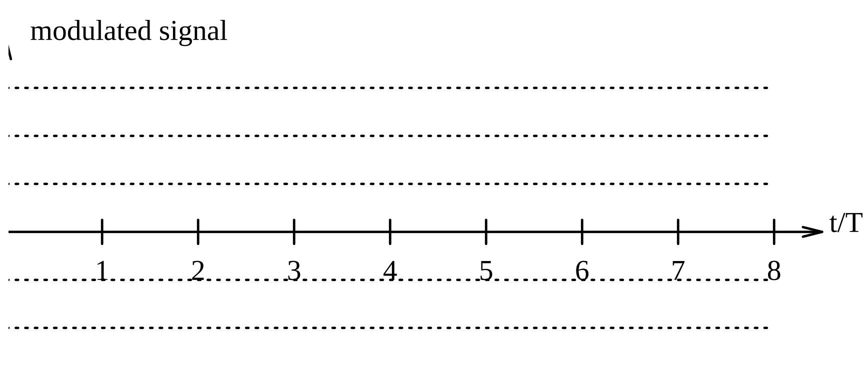

Phase
t/T
1
2
3
4
5
6
7
8


modulated signal
t/T
1
2
3
4
5
6
7
8

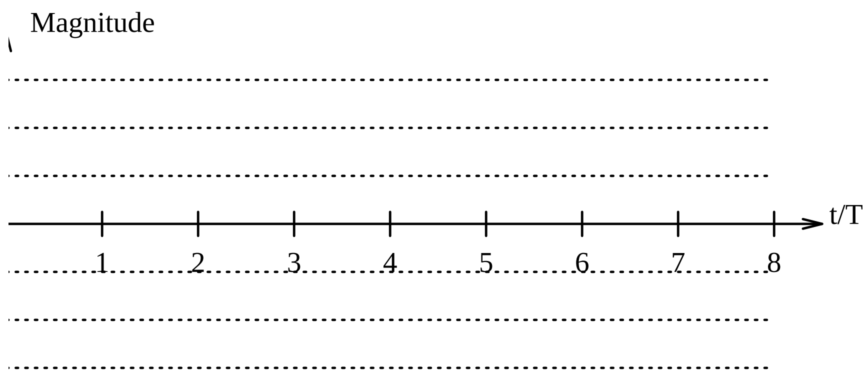
(c) Full-response Continuous Phase Modulation (CPM) with modulation index $h = 1/4$ and rectangular pulses.

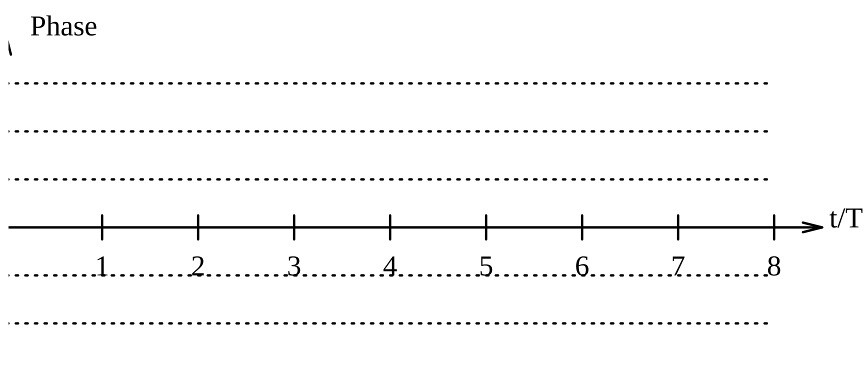

Magnitude
t/T
1 2 3 4 5 6 7 8

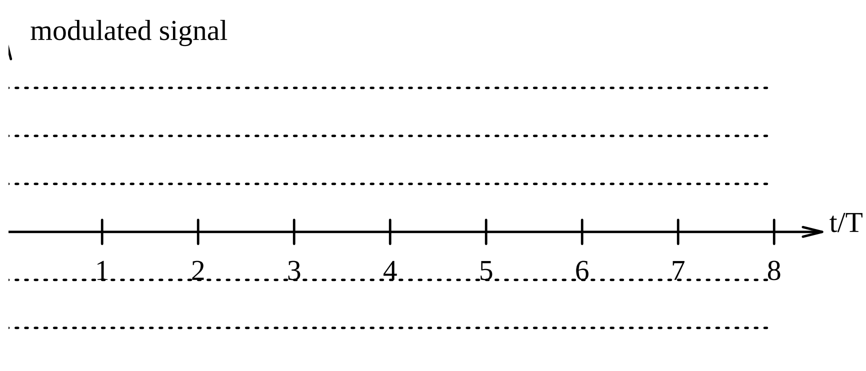

Phase
t/T
1 2 3 4 5 6 7 8


modulated signal
t/T
1 2 3 4 5 6 7 8

(d) Partial-response ($L = 2$) Continuous Phase Modulation (CPM) with modulation index $h = 1/2$ and rectangular pulses.


Magnitude
t/T
1
2
3
4
5
6
7
8


Phase
t/T
1
2
3
4
5
6
7
8

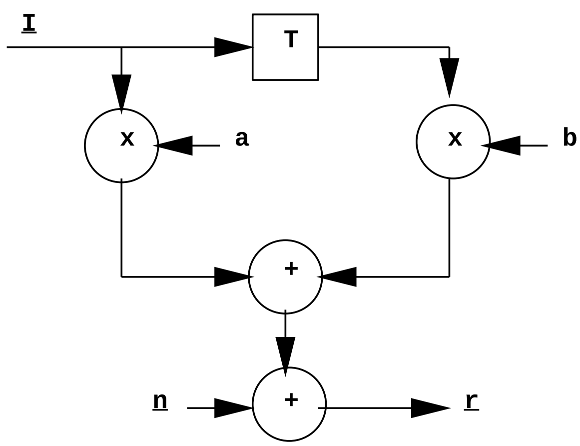

modulated signal
t/T
1
2
3
4
5
6
7
8

(e) Comment on the bandwidth requirements for each of these modulation formats. Which modulation format has the narrowest (widest) main-lobe? Which modulation format provides the fastest (slowest) side-lobe decay?

16. **Maximum Likelihood Sequence Estimation**

A binary sequence of five symbols $\underline{I}$ (elements are drawn from$I_n \in \{-1, 1\}$) is transmitted over a channel which is characterized by tapped delay-line with coefficients $a = 5$ and $b = -4$. The observation is further corrupted by additive white Gaussian noise. The following sequence $\underline{r}$ is observed at the output of the tapped delay line

$$\underline{r} = \{7, -7, 4, 5, -8, 5\}$$



(a) Given the observed sequence $\underline{r}$, determine the most likely input sequence $\underline{I}$. Clearly, show how you arrived at your solution.

(b) Draw and clearly label a trellis diagram and indicate the path through the trellis which corresponds to the most likely sequence.

(c) What is the Euclidean distance associated with the two sequences $\underline{I}_1 = \{1, -1, 1, 1, -1\}$ and $\underline{I}_2 = \{1, -1, -1, 1, -1\}$, respectively? Explain.

(d) For the remainder of the problem the coefficients of the tapped delay-line are time-varying. In the first, third, and fifth symbol period the channel coefficients are $a = 5$ and $b = -4$, in the second, fourth, and sixth symbol period the channel coefficients are $a = 4$ and $b = -5$.

Explain how the Viterbi Algorithm must be modified to account for the time-varying coefficients.

(e) The observed sequence is $\underline{r} = \{6, -8, 8, 0, -8, 4\}$. Find the most likely input sequence $\underline{I}$.

17. **Coding and Interleaving**

Consider a TDMA system that transmits information in frames of 8 bits. The probability that a frame is subject to fading is $P_f = 0.1$. If a frame is subject to fading the probability that a bit is in error is $P_{e|f} = 0.1$. Otherwise if there is no fading the bit error probability is $P_{e|n} = 0.01$. Assume that frames fade independently.

(a) What is the probability that a frame is received that does not contain any errors?

(b) Assume now that the information has been coded such that one bit error per frame can be corrected. No interleaving is employed. Compute the probability that a decoded frame does not contain any bit errors.

(c) Assume now that bits are interleaved over 8 frames such that each deinterleaved frame contains exactly one bit from each of the transmitted, interleaved frames. What is the probability of a bit error if no coding is used?

(d) Compute the probability that a frame does not contain any errors if both coding and interleaving are employed.

18. **Random Access Channel**

Three users are competing for access to the random access channel (RACH) of a cellular communication system. A simple ALOHA protocol is used to control access to this channel, i.e., in a given time slot a user attempts to transmit with some probability p and the base station informs mobiles upon successful reception via a message on the access grant channel (AGCH).

(a) Assume first that a successful random access can occur if and only if exactly one transmission attempt is made. If three users are executing the above protocol to access the RACH, what is the probability of a success?

(b) Assuming no additional users attempt to access the same RACH, what is the expected number of time slots until all three users have successfully transmitted their random access message?

(c) How would you select the transmit probability p in light of your results?

(d) A more realistic model for the RACH is as follows. A single transmission is successful with probability 0.9 (transmission errors), while one message is successfully received when two messages are transmitted with probability 0.5. When all three users transmit in the same time slot, one of the three messages will be received correctly by the base station with probability 0.3. The non-zero probabilities in the latter two cases are due to a phenomenon called capture.

If $p = 1$, what is the expected number of time slots until all users have successfully accessed the RACH?

(e) For a general p, what is the expected number of time slots until all three users have successfully accessed the RACH?

(f) How would you select p for this channel?

19. **Convolutional Coder**
The following rate $\frac{1}{2}$ convolutional coder is given.

conv_coder.eps

Assume throughout that all registers of the encoder are initially set to zero. Further, zeros will be transmitted after the information sequence to ensure that all registers will contain zeros at the end of the encoding.

(a) How would this encoder be tabulated in literature on convolutional coding?

(b) Assume the following information sequence, including trailing zeros for resetting the encoder, is to be transmitted

$$\underline{i} = \{101100\}.$$

Find the encoded bits $\underline{v}$.
Hint: There should be 12 coded bits.

(c) Now, assume the following sequence of bits is received

$$\underline{v} = \{111010100111\}.$$

Find the most likely information sequence for this observation.

(d) How many (uncoded) bit errors have occurred during transmission?

(e) Illustrate your result in a suitably chosen trellis diagram.

20. **Coding and Interleaving**

Consider a TDMA system that transmits information in frames of 16 bits. The probability that a frame is subject to fading is $P_f = 0.1$. If a frame is subject to fading the probability that a bit is in error is $P_{e|f} = 0.5$. Otherwise if there is no fading the bit error probability is $P_{e|n} = 0$.

(a) What is the probability that a frame is received that does not contain any errors?

(b) Assume now that the information has been coded such that two bit errors per frame can be corrected. No interleaving is employed. Compute the probability that a decoded frame does not contain any bit errors.

(c) Assume now that bits are interleaved over 8 frames such that each deinterleaved frame contains exactly two bits from each of the transmitted, interleaved frames. What is the probability of a bit error if no coding is used?

(d) Compute the probability that a frame does not contain any errors if both coding and interleaving are employed.

21. **Mobile Communication Channels**
Assume the impulse response of a channel is given by

$$h(t) = \delta(t) + \alpha \cdot \delta(t - T_c).$$

The amplitude of the delayed component is constant.

(a) Is the channel time-varying or time-invariant. Explain.

(b) Find the response of the channel if the following signals are applied as the input to the channel

i. $s(t) = 1$ for all t,

ii. $s(t) = \sin(\frac{\pi t}{T_c})$ for all t,

iii. $s(t) = \sin(\frac{\pi t}{2T_c})$ for all t.

(c) Find a general expression for the response of the channel, when the input is $s(t) = \sin(2\pi f t)$ for all t.

(d) Plot the magnitude of the response as a function of the frequency f.

(e) What conclusions you can draw from that plot?

(f) Express the coherence-bandwidth of the channel in terms of the parameters.

(g) The above channel is used to transmit digitally modulated data at a rate $\frac{1}{T_b}$. Data are transmitted in packets of N symbols. For each of the following cases indicate if the channel is frequency selective or non-selective.

i. $T_b \gg T_c$,

ii. $T_b \approx T_c$.

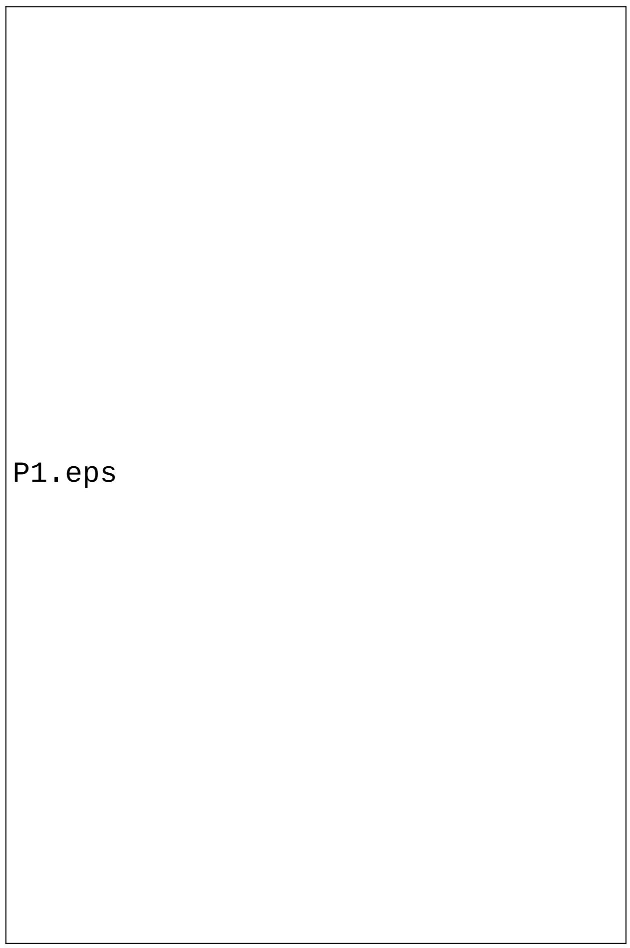
(h) For each of the two cases, explain which provisions must be made to ensure reliable communication.

22. **Modulation** The symbol sequence $b = \{1, 1, 1, -1, -1, 1, -1, -1\}$ is to be transmitted at a rate of one bit per T seconds. For each of the following modulation formats sketch in the space provided below

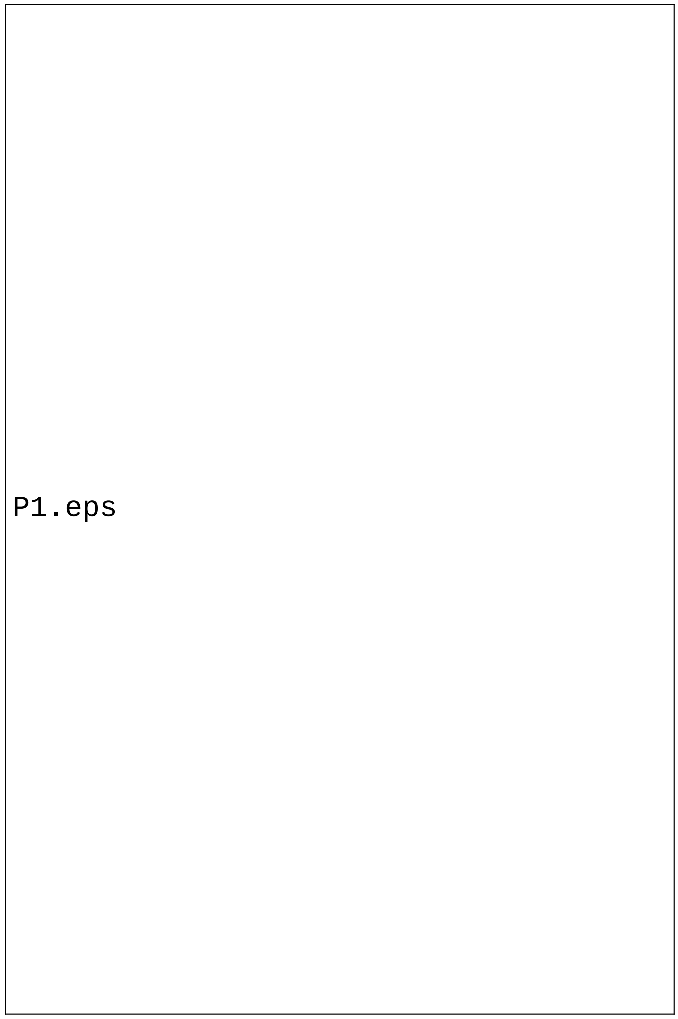
- the magnitude,
- the phase, and
- the modulated signal assuming a carrier frequency of $1/T$.

State any assumptions that you make.

(a) Binary Phase Shift Keying (BPSK) with triangular pulses.


P1.eps

(b) Offset-Quaternary Phase Shift Keying (OQPSK) with rectangular pulses.



(c) Full-response Continuous Phase Modulation (CPM) with modulation index $h = 1/4$ and rectangular pulses.

P1.eps

(d) Partial-response ($L = 2$) Continuous Phase Modulation (CPM) with modulation index $h = 1/2$ and rectangular pulses.

P1.eps

(e) Comment on the bandwidth requirements for each of these modulation formats. Which modulation format has the narrowest (widest) main-lobe? Which modulation format provides the fastest (slowest) side-lobe decay?

(f) Which signals are constant envelopes?

23. **Time-Varying Channels** A wireless channel is specified by the time-varying channel impulse response

$$g(t,\tau) = 1 \cdot \cos(2\pi f_d t), \text{ for } 0 \leq \tau \leq T.$$

(a) Determine the time-varying channel transfer function $T(f,t)$.

(b) Compute the output of the channel when the input $x(t)$ is an impulse at time t_0, i.e., compute the response to the signal

$$x(t) = \delta(t - t_0).$$

(c) What is the output when the input is a constant signal

$$x(t) = 1, \text{ for all } t?$$

(d) What is the output when the input is

$$x(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq T \\ 0 & \text{else?} \end{cases}$$

24. **Channel Correlation Functions** Consider a WSSUS channel with scattering function

$$\Psi_S(\tau,\nu) = \Psi_1(\tau) \cdot \Psi_2(\nu),$$

where

$$\Psi_1(\tau) = \begin{cases} 1 & \text{for } 0 \leq \tau \leq T_m \\ 0 & \text{else} \end{cases}$$

and

$$\Psi_2(\nu) = \begin{cases} \frac{1}{f_m}(1 - (\frac{\nu}{f_m})^2) & \text{for } 0 \leq |\nu| \leq f_m \\ 0 & \text{else.} \end{cases}$$

(a) Find the Power Delay Profile $\Psi_g(\tau)$.

(b) Determine the Doppler PSD $\Phi_H(\nu)$.

(c) Compute the mean delay τ_m and the delay spread σ_τ.

(d) Compute the Doppler spread σ_ν.

(e) What are the coherence time and coherence bandwidth?