ECE 460: Communication & Information Theory
Midterm Exam Solution – Problem 2
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April 1, 2002

AM Modulation

Throughout this problem assume that the message signal $m(t)$ is given as $m(t) = \sin(2\pi f_m t)$. The carrier frequency f_c is much higher than f_m.

(a) Assume $m(t)$ is modulated using conventional DSB AM. Sketch the resulting time domain signal.

$$x(t) = \left[A + m(t)\right].\cos(2\pi f_c t)$$

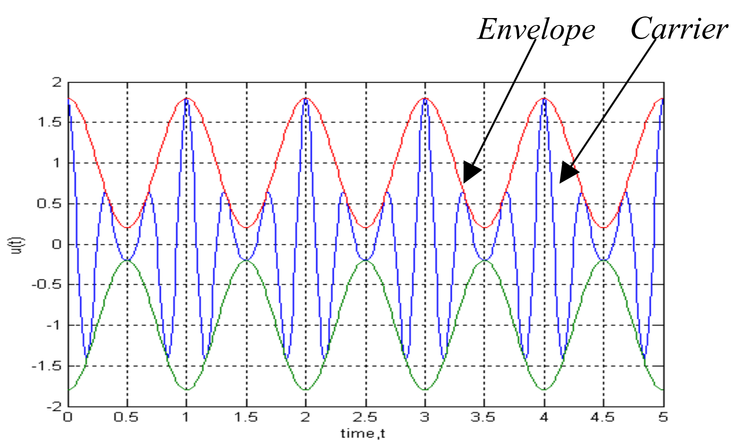

Figure 1

$$x(t) = \left[A + \cos(2\pi f_m t)\right].\cos(2\pi f_c t)$$

Then we simplify it by using the identity $\cos(x)\cos(y) = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$

$$x(t) = A.\cos(2\pi f_c t) + \cos(2\pi f_m t).\cos(2\pi f_c t)$$
$$x(t) = A.\cos(2\pi f_c t) + \frac{1}{2}\left[\cos(2\pi(f_c - f_m)t) + \cos(2\pi(f_c + f_m)t)\right]$$

(b) Compute and sketch the Fourier transform of the modulated signal

Taking the Fourier transform of $x(t)$ results in the following equation:

$$X(f)=\frac{A}{2}[\delta(f-f_c)+\delta(f+f_c)]+\frac{1}{4}[\delta(f-(f_c-f_m))+\delta(f+(f_c-f_m))]$$
$$+\frac{1}{4}[\delta(f-(f_c+f_m))+\delta(f+(f_c+f_m))]$$

Below is the Fourier transform plot of the modulated signal.

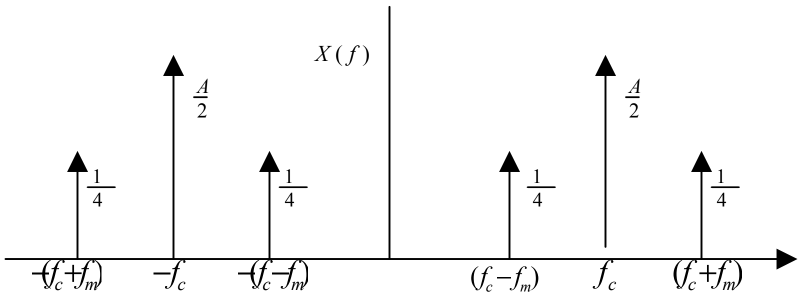

Figure 2

(c) Repeat parts (a) and (b) if DSB-SC AM is used.
Taking the same steps as for part (a) we determine the modulated signal $x(t)$ using a DSB-SC AM.

$$x(t)=m(t).\cos(2\pi f_c t)=\cos(2\pi f_m t).\cos(2\pi f_c t)$$

$$x(t)=\frac{1}{2}[\cos(2\pi(f_c-f_m)t)+\cos(2\pi(f_c+f_m)t)]$$

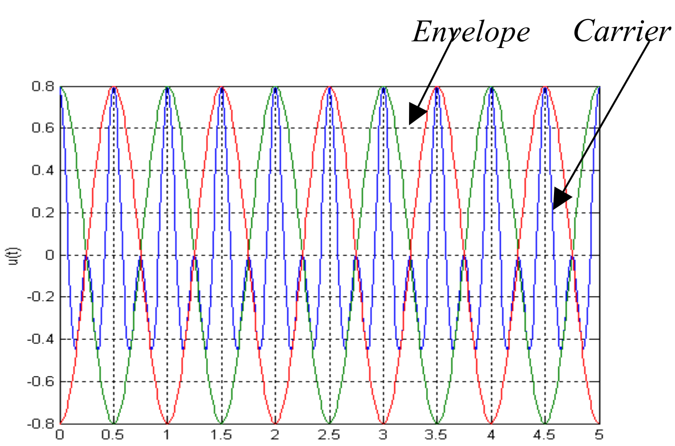

Figure 3

Then we find the Fourier transform of the modulated signal $x(t)$ as shown below

$$X(f)=\frac{1}{4}\left[\delta(f-(f_c-f_m))+\delta(f+(f_c-f_m))\right]$$
$$+\frac{1}{4}\left[\delta(f-(f_c+f_m))+\delta(f+(f_c+f_m))\right]$$

The Fourier transform plot of the DSB-SC AM signal is shown in figure 4 below:

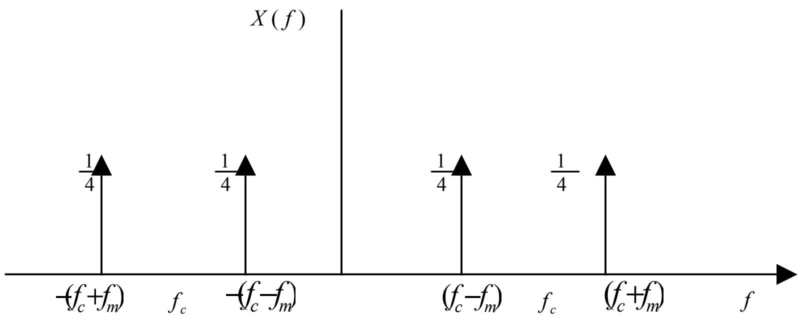

Figure 4

(d) For the remainder of the problem, assume that $m(t)$ is the input to the system in Figure 1. The cutoff frequency of the lowpass filters equals f_m.
Compute $x_1(t)$ and as well as their Fourier transform $X_1(f)$ and $X_2(f)$.

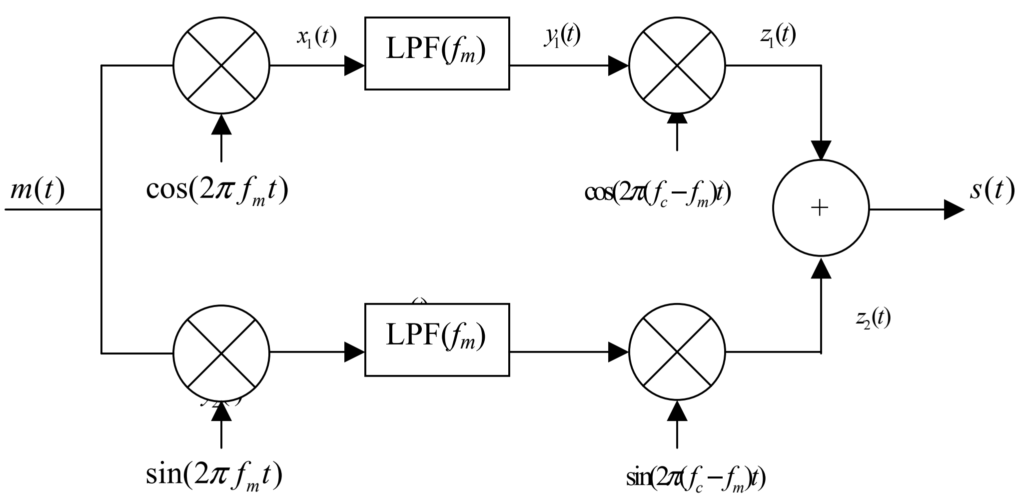

Figure 5: AM Modulation

First, we simplify $x_1(t)$ by using the identity $\cos(x)\cos(y)=\frac{1}{2}[\cos(x-y)+\cos(x+y)]$, and take its Fourier transform as follows.

$$x_1(t) = m(t).\cos(2\pi f_m t) = \cos(2\pi f_m t)\cdot\cos(2\pi f_m t)$$
$$x_1(t) = \cos^2(2\pi f_m t) = \frac{1}{2}(1+\cos(4\pi f_m t)$$
$$X(f) = \frac{1}{4}[\delta(f-2f_m)+\delta(f+2f_m))]$$

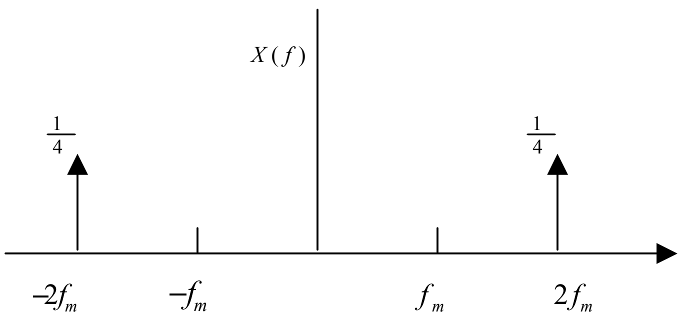

Figure 6

(e) Compute the outputs of the lowpass filters, $y_1(t)$ and $y_2(t)$, as well as their Fourier transforms $Y_1(f)$ and $Y_2(f)$.

When the signals $x_1(t)$ and $x_2(t)$ are passed through the lowpass filters, the filter cuts off all-higher frequencies signals and the following signals result:

$$Y_1(f) = \frac{1}{2}\delta(f)$$
$$y_1(t) = \frac{1}{2}$$

$$Y_2(f) = 0$$

$$y_2(t) = 0$$

(f) Compute the output signal $s(t)$ and its Fourier transform $S(f)$.

First we determine from the block diagram that $s_2(t) = z_1(t) - z_2(t)$, which results as follows.

$$z_1(t) = y_1(t) \cdot \cos(2\pi(f_c - f_m)t) = \frac{1}{2} \cdot \cos(2\pi(f_c - f_m)t)$$

$$z_2(t) = y_2(t) \cdot \sin(2\pi(f_c - f_m)t) = 0$$

$$s(t) = \frac{1}{2} \cdot \cos(2\pi(f_c - f_m)t)$$

Ultimately we determine we the Fourier transform of $s(t)$

$$S(f) = \frac{1}{4}\left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))\right]$$

(g) How would you describe what this system does ?

This system is a LSB-SSB AM Modulator .